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Impact of Membership Function Design on Grammatical Acceptability in Fuzzy Grammar Models

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ABSTRACT

Grammatical acceptability, the extent to which a sentence conforms to the structural and usage norms of a language, has long been recognized as a gradient phenomenon rather than a binary distinction. Linguistic research on gradient grammaticality has examined how factors such as syntactic configuration, word order, and clause integration influence native speaker judgments. This study adopts a fuzzy grammar framework to model such gradience and investigates how the design of membership functions influences the evaluation of sentence acceptability. A curated dataset of five English sentences was selected to represent a range of linguistic structures, including canonical declaratives, syntactic inversion, passive voice, and clausal subordination. For each sentence, rule-based violation scores were assigned for three linguistic dimensions: subject–verb agreement, phrase structure and word order, and clause integration and cohesion. Four types of membership functions, Linear, Sigmoid, Gaussian, and Trapezoidal, were applied to transform these scores into fuzzy membership degrees, which were then aggregated into overall acceptability judgments. Results reveal that while the relative ranking of sentences by acceptability remains stable across functions, the absolute scores vary substantially, with Gaussian producing the most conservative evaluations and Trapezoidal yielding plateau effects. These differences have

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direct implications for how fuzzy models capture subtle linguistic variation and for the interpretability of computational tools used in grammaticality assessment. The findings highlight the necessity of treating membership function selection as a theoretically motivated decision in fuzzy linguistic modeling, thereby contributing to more transparent and linguistically grounded applications in both theoretical and applied language studies.

Keywords: Fuzzy Grammar; Fuzzy Logic; Grammatical Acceptability; Interpretability; Linguistic Modeling; Membership Function Design; Natural Language Processing; Rule-Based Evaluation

1. Introduction

Fuzzy logic has long been recognized as a powerful framework for modeling vagueness and uncertainty inherent in natural language processing (NLP) tasks^[1]. Unlike classical logic, which deals with binary true/false evaluations, fuzzy logic enables the representation of partial truth values, allowing for more flexible and human-like reasoning^[2]. One domain where this advantage becomes particularly apparent is grammaticality judgment, where acceptability is not always binary, but rather exists on a spectrum of well-formedness^[3].

Grammatical acceptability—the extent to which a sentence conforms to the rules and norms of a language—is a complex construct that often defies rigid classification. Sentences can be fully acceptable, marginal, or outright unacceptable depending on factors such as syntactic complexity, semantic coherence, contextual appropriateness, and native speaker intuition^[4]. To model this gradience, fuzzy grammar models have emerged as an alternative to classical rule-based or probabilistic grammars, enabling researchers to assign degrees of membership to linguistic units and constraints^[5].

At the heart of fuzzy grammar models lies the membership function, which defines how linguistic elements map to degrees of rule satisfaction^[6]. The shape of this function—whether linear, sigmoid, Gaussian, or trapezoidal—determines the sensitivity and interpretability of the resulting grammaticality score^[7]. Despite its foundational role, the choice of membership function is often treated as a neutral or secondary modeling decision, with many studies defaulting to linear forms without justification^[8].

This oversight is significant because different membership functions embody distinct assumptions about linguistic gradience. A linear function assumes a uniform rate of transition from low to high membership, while a sigmoid function

implies a threshold-based transition with soft boundaries^[9]. A Gaussian function emphasizes centrality and penalizes deviations symmetrically, and a trapezoidal function introduces flat regions to model categorical plateaus^[10,11]. These characteristics can have meaningful effects on the final grammaticality judgments produced by a model.

In previous work, fuzzy grammars have been used to model syntactic categories^[12], semantic compatibility^[13], grammatical violations^[14], and even pragmatics^[15]. However, few studies systematically compare the impact of membership function design on the outputs of these models. This raises a crucial methodological question: Is the final fuzzy grammar output robust to the choice of membership function, or does the function act as a silent biasing factor?

To address this gap, the present study investigates the impact of membership function shape on fuzzy grammatical acceptability scores. Using a fixed dataset of natural language sentences and a consistent set of syntactic and semantic rules, we apply four widely-used types of membership functions—linear, sigmoid, Gaussian, and trapezoidal—and compare the resulting outputs. We hypothesize that certain functions will yield higher sensitivity to borderline cases, while others will produce more stable or conservative evaluations.

The contributions of this paper are threefold. First, we provide an empirical evaluation of how different membership functions affect fuzzy grammatical acceptability modeling. Second, we highlight the methodological importance of choosing an appropriate function based on linguistic context and model goals. Third, we offer practical recommendations for researchers designing fuzzy grammar systems, encouraging a more transparent and justified selection of membership functions.

Overall, this work challenges the assumption of function neutrality in fuzzy linguistic modeling. By demonstrating that the design of the membership function is not a trivial

modeling choice, we aim to promote a more reflective and rigorous use of fuzzy logic in computational linguistics. In doing so, we contribute to ongoing discussions about interpretability, reproducibility, and the epistemological foundations of fuzzy grammar models^[16].

This paper is structured as follows. Section 2 reviews relevant literature on fuzzy sets, grammatical acceptability, and the role of membership functions in NLP. Section 3 presents the methodology, including the dataset, rule sets, and implementation of the four membership functions. Section 4 reports the results of our experiments and provides a comparative analysis of the outputs. Section 5 discusses the implications of our findings for future fuzzy grammar research, and Section 6 concludes with key takeaways and suggestions for further work.

2. Related Work

The integration of fuzzy logic into natural language processing (NLP) and linguistic modeling has been a prominent research trajectory since the late 20th century^[17]. Fuzzy set theory, introduced by Zadeh in 1965, provides the mathematical basis for reasoning under uncertainty—a key aspect of linguistic interpretation where grammaticality and acceptability are not binary but often gradual and context-sensitive^[18].

A number of studies have employed fuzzy logic to model various linguistic phenomena, including phonological rules^[19], syntactic parsing^[20], semantic similarity^[21], and pragmatic interpretation^[22]. In particular, fuzzy grammar models have emerged as a flexible framework for capturing the graded nature of grammatical acceptability, where traditional rule-based or statistical approaches may fall short^[23].

Early models of fuzzy grammar typically defined grammatical rules as fuzzy constraints and measured the degree to which a sentence satisfies these constraints^[24]. The application of membership functions in these models allows for nuanced representations of partial rule satisfaction—for instance, a sentence may be 0.7 acceptable due to minor syntactic disfluency. Such approaches have been especially useful in psycholinguistic studies that seek to mirror human judgments^[25].

Despite this growing interest, the role of membership functions—the core mechanism that maps linguistic input to degrees of rule adherence—has received relatively limited

scrutiny. Most works adopt standard linear functions for simplicity and interpretability^[26], often without empirical justification or comparative analysis^[27]. However, literature from adjacent fields, such as fuzzy control systems^[28] and fuzzy expert systems^[29], suggests that the choice of membership function significantly affects system behavior and output sensitivity^[30].

In NLP applications, some attempts have been made to apply non-linear membership functions such as sigmoid or Gaussian curves in sentiment analysis^[31], information retrieval^[32], and lexical categorization tasks^[33]. These studies highlight that non-linear functions can better capture boundary cases or semantic overlap, which are common in natural language^[34]. However, their application to grammaticality judgment tasks remains underexplored.

The design of membership functions has also been examined from the perspective of human interpretability. Linguistic models that aim to be interpretable by human experts—such as educational grammar tools or cognitive NLP systems—benefit from clear, explainable mappings from rules to judgments^[35,36]. Trapezoidal functions, for instance, offer intuitive plateaus that correspond to linguistic categories such as “fully grammatical,” “marginal,” or “ungrammatical”^[37,38].

Another relevant body of work concerns the normalization and aggregation of fuzzy values in linguistic modeling. When multiple fuzzy rules apply to a single sentence, their contributions must be combined into a final acceptability score. Studies have investigated various aggregation operators, including min, max, weighted average, and OWA (Ordered Weighted Averaging) operators^[39]. While these affect the final score, they are also influenced by the shapes of the membership functions involved^[40].

Recent advances in fuzzy machine learning have introduced adaptive and data-driven techniques to learn membership functions from labeled datasets^[41]. These methods offer new opportunities for optimizing function shapes based on actual linguistic judgments rather than pre-defined assumptions^[42]. Nevertheless, their computational complexity and need for large datasets may limit their applicability in grammatical modeling scenarios with small or expert-curated samples.

In educational linguistics, fuzzy logic has been used to model learner language and grammatical competence, en-

abling adaptive feedback in intelligent tutoring systems^[43]. Here again, the granularity and accuracy of feedback depend heavily on how fuzzy rules and functions are designed^[44]. In psycholinguistics, fuzzy modeling of acceptability judgments has been proposed as a means to bridge the gap between binary grammaticality and scalar intuitions observed in human raters^[45].

In summary, while fuzzy grammar models are well established, and fuzzy logic has permeated multiple areas of NLP and linguistic analysis, the design choices surrounding membership functions remain under-theorized and under-tested. Few studies conduct direct comparisons of function types in controlled experiments, especially in the context of grammaticality judgments^[46]. This paper contributes to closing this gap by evaluating the influence of four major membership function types—linear, sigmoid, Gaussian, and trapezoidal—on fuzzy grammatical acceptability scores.

By situating our study at the intersection of fuzzy modeling theory and applied linguistics, we respond to a growing call for greater transparency, reproducibility, and methodological rigor in the design of interpretable language models.

3. Materials and Methods

This section describes the experimental design used to investigate the impact of membership function choice on fuzzy grammatical acceptability. To ensure both transparency and readability, we streamlined the methodological presentation. All parameter specifications are consolidated into a single summary table, and the overall experimental procedure is presented in a compact pseudocode format. This structure avoids unnecessary textual redundancy while preserving the level of detail required for reproducibility.

3.1. Dataset

This study employs a curated dataset consisting of five English sentences, each varying in syntactic structure, word order, and grammatical complexity. The sentences are listed below^[47]:

- S1:** “The cat sat on the mat.”
S2: “On the mat sat the cat?”
S3: “The dog chased the ball in the park.”
S4: “In the park, the ball was chased by the dog.”

S5: “Despite the rain, she managed to complete the run.”

These sentences were selected to represent a diverse range of linguistic constructions, including declarative, inverted, passive, and subordinated forms. They reflect both canonical and marked grammatical patterns, allowing for a more nuanced evaluation of fuzzy acceptability under varying membership function configurations.

Each sentence is evaluated with respect to a fixed set of linguistic rules (r), particularly focusing on:

1. **Subject–Verb Agreement**
2. **Phrase Structure and Word Order**
3. **Clause Integration and Cohesion**

In this study, we distinguish between *grammaticality* and *acceptability*. Grammaticality refers to binary conformity to syntactic rules (well-formed vs. ill-formed), while acceptability captures gradient judgments influenced by syntax, semantics, discourse context, and pragmatic markers such as punctuation. For example, Sentence S2 (“On the mat sat the cat?”) was treated as syntactically marked due to inversion and interrogative punctuation, and accordingly assigned higher violation scores for phrase structure. This operationalization reflects the broader notion of acceptability that goes beyond strict grammaticality.

Rather than assigning binary grammatical labels, each rule violation is assessed using a fuzzy value in the interval^[1], reflecting the degree of violation or conformity. For example, a partial subject–verb disagreement may receive a violation score of 0.4, while a complete mismatch may be rated 0.9. The goal is to compute a global grammatical acceptability score for each sentence by aggregating its rule-wise violation degrees.

These rule violation degrees serve as the input to the fuzzy evaluation system, where membership functions transform raw scores into linguistic membership degrees, which are then aggregated to yield the final acceptability judgment.

The assigned violation scores for each sentence are heuristically defined to capture plausible linguistic deviations under the three evaluation criteria. The scoring is as follows:

violation_scores =
 “S1”: [0.1, 0.0, 0.1],
 “S2”: [0.3, 0.5, 0.2],
 “S3”: [0.0, 0.1, 0.0],

“S4”: [0.1, 0.2, 0.1],

“S5”: [0.0, 0.2, 0.3].

The rationale for these scores is summarized below:

- **S1** is a canonical sentence with minimal violations.
- **S2** contains word-order inversion and interrogative punctuation, resulting in higher structural violations.
- **S3** is a well-formed declarative sentence with negligible issues.
- **S4** features passive voice and fronted prepositional phrase, incurring slight penalties for structure and complexity.
- **S5** includes clausal subordination, leading to moderate violations in cohesion and integration.

This configuration ensures a controlled yet diverse test set for evaluating the effect of different membership function

designs on fuzzy grammatical acceptability computation.

Although the curated dataset of five English sentences provides a controlled testbed for our analysis, we acknowledge its limited size as a potential threat to generalizability. Future studies should incorporate larger and more diverse corpora to evaluate the robustness of membership function effects. In particular, extending the analysis with human acceptability ratings would provide valuable ground truth. Such ratings could be validated through inter-annotator agreement metrics (e.g., Krippendorff’s α) to ensure reliability and comparability with fuzzy-model outputs.

To ensure transparency and reproducibility, **Table 1** presents the rubric used to assign violation scores across linguistic dimensions. For consistency, all rules employ a three-level scale: 0 for no violation, 0.3–0.4 for minor violation, and 0.7–0.9 for major violation.

Table 1. Rubric for assigning violation scores across linguistic dimensions.

Rule Dimension	Score Range	Interpretation/Example Cases
Subject–Verb Agreement	0	Full agreement (e.g., <i>She runs</i>).
	0.3–0.4	Minor mismatch (e.g., <i>She run</i>).
	0.7–0.9	Clear disagreement (e.g., <i>He go yesterday</i>).
Phrase Structure & Word Order	0	Canonical structure (e.g., <i>The cat sat on the mat</i>).
	0.3–0.4	Marked but interpretable order (e.g., <i>On the mat the cat sat</i>).
	0.7–0.9	Disrupted or ill-formed order (e.g., <i>Mat the on sat cat</i>).
Clause Integration & Cohesion	0	Smooth clause integration (e.g., <i>She said that he left</i>).
	0.3–0.4	Moderate complexity/ambiguity (e.g., <i>Although raining, she run</i>).
	0.6–0.9	Severe incoherence or fragmentation (e.g., <i>Because raining. She run</i>).

3.2. Experimental Setup

To investigate the impact of membership function design on fuzzy grammaticality evaluation, we apply four distinct types of membership functions to the rule violation scores of each sentence.

3.2.1. Linear Membership Function

The Linear Membership Function provides a direct and uniform mapping from the input value (e.g., rule violation score) to a membership degree. It assumes a consistent rate of change in membership, where the degree of ungrammaticality increases proportionally with the violation score. In its simplest form, the linear membership function is defined as:

$$\mu(x) = x, \text{ for } x \in [0, 1].$$

Figure 1 implies that the membership degree is directly

proportional to the input value. For example, a rule violation score of 0.7 corresponds to a 70% degree of ungrammaticality. Alternatively, the linear function can be expressed in a more general parameterized form as a piecewise function:

$$\mu(x) = \begin{cases} 0 & , \quad \text{if } x \leq a \\ \frac{x-a}{b-a} & , \quad \text{if } a < x < b \\ 1 & , \quad \text{if } x \geq b \end{cases}$$

where a and b define the lower and upper bounds of the transition region, respectively. When $a = 0$ and $b = 1$, this piecewise formulation simplifies exactly to $\mu(x) = x$, reinforcing the idea of a linear and continuous progression in membership across the input space.

This function is often used as a baseline in fuzzy modeling due to its simplicity, transparency, and lack of bias toward any particular threshold. However, it does not provide enhanced sensitivity at specific ranges and treats all

deviations equally across the entire domain.

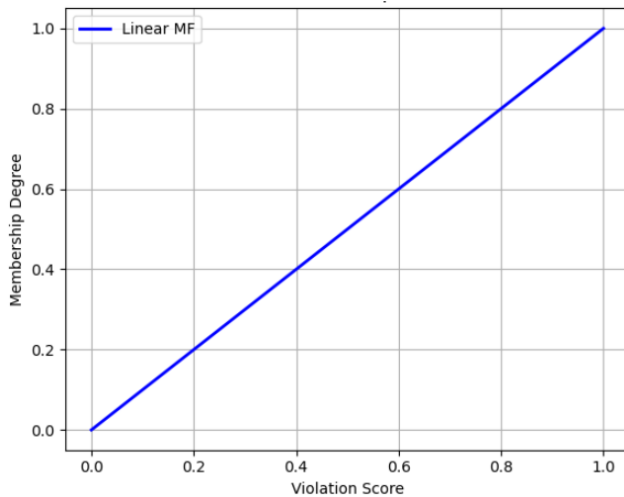


Figure 1. Linear membership function.

3.2.2. Sigmoid Membership Function

The Sigmoid Membership Function introduces a smooth and nonlinear transition in membership degree from 0 to 1. Unlike the linear function, it emphasizes changes around a central point, making it suitable for modeling gradual boundary conditions or soft thresholds in fuzzy systems. Mathematically, it is defined as:

$$\mu(x) = \frac{1}{1 + e^{-\alpha(x-c)}}$$

where x is the input variable (e.g., rule violation score), α is the steepness parameter (also called the slope factor), and c is the center or inflection point of the function.

Figure 2 yields a characteristic S-shaped curve. When $x = c$, the membership degree is exactly 0.5, indicating maximum uncertainty. The parameter α controls how sharply the function transitions:

- A large α results in a steeper curve, closely approximating a binary threshold.
- A small α produces a gentler slope, enabling broader transition zones.

The sigmoid function is widely used in fuzzy logic applications where soft, probabilistic, or gradual interpretations of rule satisfaction are desired. It provides better sensitivity near the decision boundary compared to linear or trapezoidal functions.

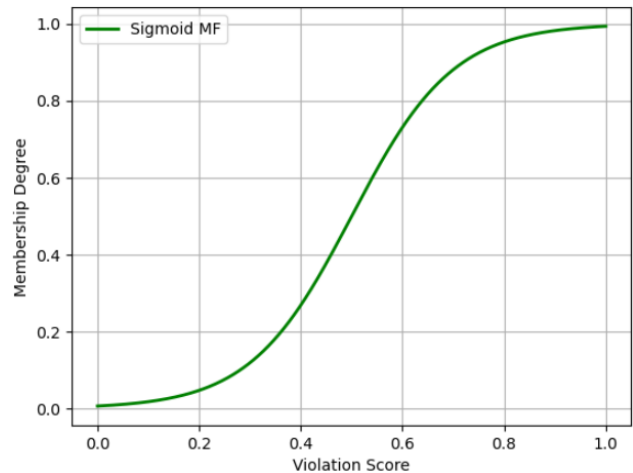


Figure 2. Sigmoid membership function.

3.2.3. Gaussian Membership Function

The Gaussian Membership Function models fuzzy membership using a symmetric, bell-shaped curve centered around a specific input value. It is particularly useful in scenarios where uncertainty is distributed normally around a central tendency, and where extreme deviations from the center are progressively penalized.

Mathematically, it is defined as:

$$\mu(x) = \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

where x is the input variable (e.g., rule violation score), μ is the center of the curve (mean of the distribution), and σ is the spread or standard deviation, which controls the width of the curve. **Figure 3** illustrates the Gaussian membership function, which provides a smooth, bell-shaped curve centered around the mean value. This function assigns high membership degrees to input values near the center and gradually decreases the membership degree as the input moves away from the mean. The smoothness of the curve makes the Gaussian membership function particularly suitable for representing uncertainty and gradual transitions between linguistic terms.

The Gaussian function achieves maximum membership of 1 when $x = \mu$, and the membership value decreases symmetrically as x moves away from μ . A smaller σ results in a narrower and sharper peak, making the function more sensitive to deviations near the center. Conversely, a larger σ produces a wider curve with a more gradual decline.

This function is advantageous when modeling natural, noise-tolerant, or probabilistic relationships in fuzzy sys-

tems. It offers smoothness and differentiability, making it suitable for integration with machine learning models and optimization algorithms.

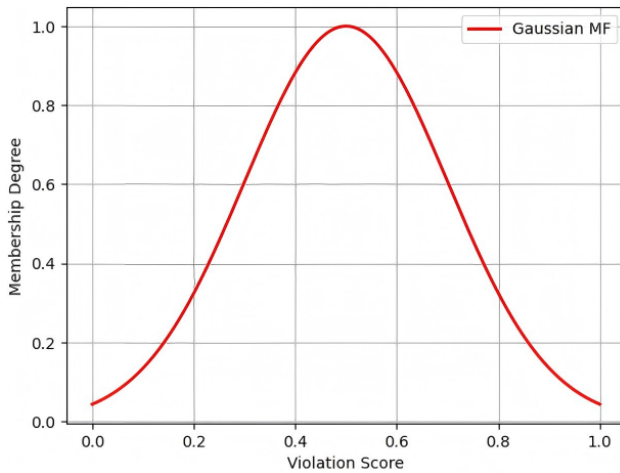


Figure 3. Gaussian membership function.

3.2.4. Trapezoidal Membership Function

The Trapezoidal Membership Function defines membership degrees using a piecewise linear function with a flat top, allowing for a region of full membership. It is particularly well-suited for representing fuzzy concepts with clearly bounded core regions and gradual transitions at the boundaries. Mathematically, the trapezoidal function is defined as:

$$\mu(x) = \begin{cases} 0 & , \quad \text{if } x \leq a \\ \frac{x-a}{b-a} & , \quad \text{if } a < x \leq b \\ 1 & , \quad \text{if } b < x < c \\ \frac{x-a}{b-a} & , \quad \text{if } c \leq x < d \\ 0 & , \quad \text{if } x \geq d \end{cases}$$

where a and d are the **feet** of the trapezoid (points where membership is 0), b and c are the **shoulders** of the trapezoid (start and end of full membership, where $\mu(x) = 1$). **Figure 4** shows the trapezoidal membership function, which is characterized by a flat region where the membership degree remains at its maximum value. This function increases linearly from zero to one, stays constant over a certain range, and then decreases linearly back to zero. The trapezoidal membership function is useful for modeling concepts with a well-defined core and gradual boundaries, offering flexibility in representing fuzzy sets with both pre-

cise and imprecise regions.

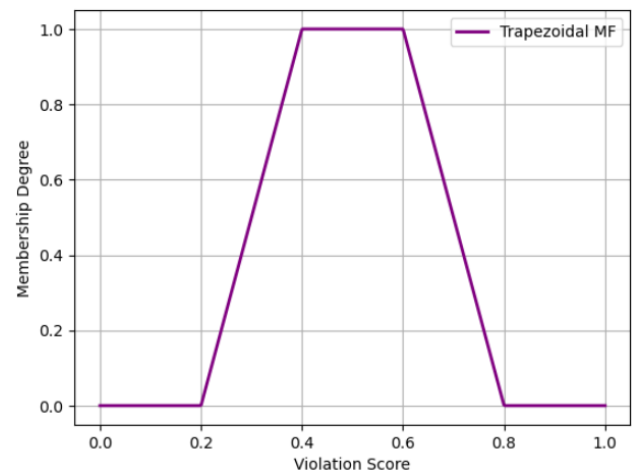


Figure 4. Trapezoidal membership function.

The function increases linearly from 0 to 1 between a and b , remains at 1 between b and c , and then decreases linearly from 1 to 0 between c and d . This shape makes it highly interpretable for linguistic terms like “medium,” “acceptable,” or “moderate,” where a core range is clearly defined and surrounded by transitional zones. Trapezoidal membership functions are computationally efficient, easy to parameterize, and widely used in rule-based fuzzy systems, especially where boundaries are known and need to be explicitly encoded.

Each membership function is applied to the same rule violation scores to ensure fair comparison across sentence evaluations.

To evaluate the sensitivity of fuzzy modeling to parameterization, we systematically explored parameter grids for each membership function. **Table 2** summarizes the parameter ranges tested.

The parameter values were selected heuristically to ensure stable and interpretable mappings of violation scores to ungrammaticality values. In addition, sensitivity analyses were conducted (**Figure 8**, **Figure 9**) to demonstrate how varying parameters (e.g., sigmoid steepness, Gaussian spread) influences acceptability outcomes. While future work could optimize parameter selection against human acceptability ratings (e.g., by minimizing RMSE), the present study focuses on methodological comparison across membership function types.

Table 2. Membership function parameters used in the experiments. The table lists the specific values adopted for each function: Linear (a, b), Sigmoid (α , c), Gaussian (μ , σ), and Trapezoidal (a, b, c, d). These parameters were determined through sensitivity analysis to ensure stable and interpretable mappings of rule-violation scores to acceptability degrees.

Function	Parameters	Grid Explored	Values Used
Linear	a, b	a = 0, b = 1 (fixed)	0, 1
Sigmoid	α , c	$\alpha \in \{5, 10, 15, 20\}$, c = 0.5	$\alpha = 10$, c = 0.5
Gaussian	μ , σ	$\mu \in \{0.1, 0.2, 0.3, 0.5\}$, $\sigma = 0.2$	$\mu = 0.5$, $\sigma = 0.2$
Trapezoidal	a, b, c, d	a = 0.2, b = 0.4, c = 0.6, d = 0.8	0.2, 0.4, 0.6, 0.8

3.3. Definition of Membership Degree (μ)

In this study, the membership degree μ is explicitly defined as a measure of ungrammaticality. That is, μ increases monotonically with the severity of a rule violation. A value of $\mu = 0$ indicates full conformity to a grammatical rule, while $\mu = 1$ represents maximal violation.

For example, a mild word-order deviation may yield $\mu = 0.2$, whereas a strong subject–verb disagreement may result in $\mu = 0.8$. This convention ensures monotonic behavior across all membership functions: stronger violations always produce higher values of μ .

Accordingly, the overall acceptability of a sentence is computed as the complement of the average ungrammaticality across all rules:

$$\text{Acceptability}(s) = 1 - \frac{1}{r} \sum_{r=1}^r \mu_r(s)$$

where r is the number of rules and $\mu_r(s)$ denotes the violation degree for rule r . This explicit definition avoids hidden modeling bias and makes the interpretation of all subsequent results transparent.

In general fuzzy set theory, membership functions such as Gaussian, triangular, or trapezoidal are not necessarily monotonic, since they are often used to model categories with a central peak. However, in this study we explicitly define μ as ungrammaticality, which by definition must increase as violation severity increases. To maintain this semantic consistency, all membership functions—including Gaussian and Trapezoidal—were parameterized in a monotone-increasing form with respect to violation scores. This ensures that stronger violations always yield higher values of μ , and consequently lower acceptability scores under the aggregation formula.

Algorithm 1. Experimental pipeline for fuzzy grammatical acceptability (pseudocode)

Input:

- Sentence set $S = \{s_1, s_2, \dots, s_N\}$
- Violation scores $V[s][r] \in [0, 1]$ for each sentence $s \in S$ and rule $r \in \{\text{agreement, structure, cohesion}\}$
- Membership functions $M\mathcal{F} = \{\text{Linear, Sigmoid, Gaussian, Trapezoidal}\}$

Output:

- Acceptability scores $A[s][r]$ for each sentence ss and membership function $f \in \mathcal{F}$

Procedure

1. Initialize table A as empty
2. For each sentence s in S do
3. For each membership function f in \mathcal{F} do
4. sum_mu \leftarrow 0
5. For each rule r in {agreement, structure, cohesion} do
6. x \leftarrow V[s][r] // violation score in [0,1]
7. $\mu \leftarrow$ MF(f, x) // membership degree in [0,1]
8. sum_mu \leftarrow sum_mu + μ
9. End For
10. avg_mu \leftarrow sum_mu / 3
11. A[s][f] \leftarrow 1 - avg_mu // higher violations \rightarrow lower acceptability
12. End For
13. End For
14. Return A

In this study, we adopt the simple mean as the aggregation operator across the three rule dimensions (agreement, structure, cohesion). This choice ensures comparability across membership functions and maintains focus on their relative effects. Alternative aggregation operators (e.g., min, max, OWA) were not repeated here, as they have already been systematically analyzed in our previous work (*A Comparative Study of Fuzzy Aggregation Strategies in Modeling*

the Syntax–Pragmatics Interface).

4. Results

The results of the experiments are presented in this section. We first report the sentence-level acceptability scores obtained under each membership function. **Table 3**

Table 3. Grammatical acceptability scores across membership functions for each of the five test sentences (S1–S5). The table shows how Linear, Gaussian, Sigmoid, and Trapezoidal functions yield different magnitudes of acceptability despite applying the same rule-based violation scores.

Sentence	Linear	Gaussian	Sigmoid	Trapezoidal
S1	0.933	0.895	0.986	1.000
S2	0.667	0.356	0.778	0.500
S3	0.967	0.926	0.990	1.000
S4	0.867	0.802	0.972	1.000
S5	0.833	0.675	0.942	0.833

Figure 5 and **Figure 6** present the grammatical acceptability scores for each sentence under four different membership function configurations. **Figure 5** displays the results as bar charts, highlighting how score magnitudes differ across functions, with Gaussian generally producing lower values and Sigmoid higher ones. **Figure 6** provides a heatmap representation, which makes it easier to visually compare scoring patterns across sentences and functions at a glance.

Figure 7 illustrates rank divergence across membership functions. While Linear, Sigmoid, and Gaussian produced identical sentence rankings, Trapezoidal introduced ties for highly acceptable sentences due to its plateau effect. This visualization emphasizes that although overall ranking patterns are broadly consistent, Trapezoidal behaves differently in cases of near-perfect grammaticality.

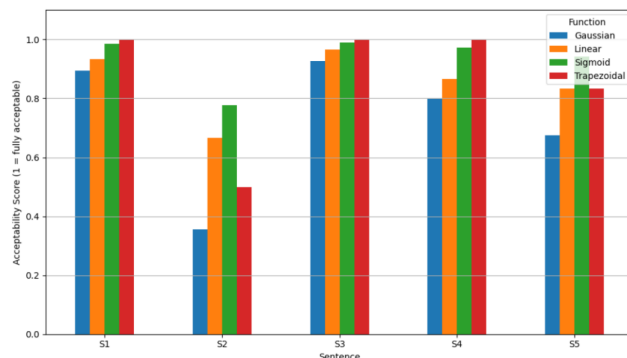


Figure 5. Grammatical acceptability scores by membership function, displayed as a bar chart. The figure illustrates variation across functions, with Gaussian generally producing lower scores, Sigmoid yielding higher scores, and Trapezoidal creating plateau effects.

ble 3 summarizes the grammatical acceptability scores across the four functions—Linear, Gaussian, Sigmoid, and Trapezoidal—for the five test sentences (S1–S5). This provides a baseline comparison of how different function shapes influence the magnitude of acceptability scores despite being applied to the same set of rule-based violation inputs.

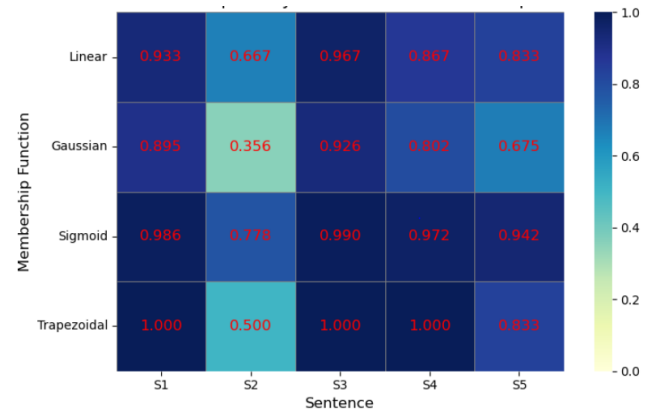


Figure 6. Heatmap of grammatical acceptability scores by membership function. Each row represents a membership function and each column a sentence, enabling visual comparison of scoring patterns across functions.

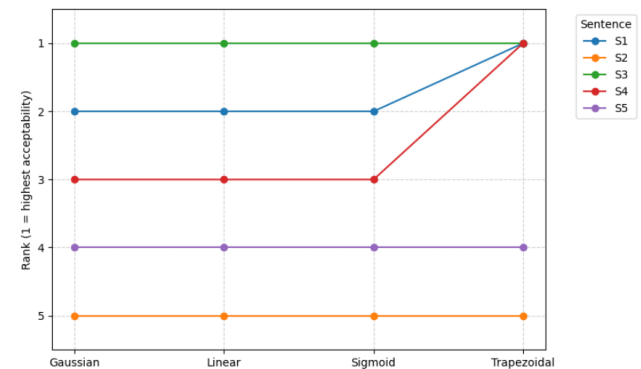


Figure 7. Rank divergence across membership functions. The figure highlights consistency in sentence ranking among Linear, Sigmoid, and Gaussian, while Trapezoidal introduces ties for highly acceptable sentences due to its plateau.

4.1. Function-Specific Behavior and Score Dispersion

Each membership function exhibits a distinct scoring profile, as seen in **Figure 5** (bar chart) and further quantified in the heatmap of **Figure 6**. The Gaussian function consistently yields lower acceptability scores relative to the others, suggesting that it imposes stricter penalties on moderate violations due to its bell-shaped curve centered at mid-range values. In contrast, the Trapezoidal function displays polarizing behavior—it either assigns full acceptability (1.000) or a substantially lower value (e.g., 0.500 for S2), reflecting its step-like plateaus and sharp transitions. Linear and Sigmoid functions produce more graded responses, with Sigmoid showing higher sensitivity around the midpoint, and Linear offering proportional changes.

4.2. Sentence S2 as a Diagnostic Case

Sentence S2 (“On the mat sat the cat?”), which features an inverted structure and interrogative punctuation, serves as a stress test for the fuzzy evaluation system. Its score sharply diverges across membership functions, as clearly shown in **Table 3** and **Figure 6**. It receives the lowest acceptability score overall under Gaussian (0.356) and the second-lowest under Trapezoidal (0.500), while Sigmoid and Linear offer more moderate evaluations (0.778 and 0.667, respectively). This divergence highlights that S2 lies near the decision boundary, and that membership functions differ significantly in how sharply they penalize intermediate violations.

4.3. Robustness on Canonical Forms

Sentences S1 (“The cat sat on the mat.”) and S3 (“The dog chased the ball in the park.”) are syntactically canonical. All membership functions assign them high scores (between 0.895 and 1.000), as evident in **Figure 5** and **Figure 6**, demonstrating that fuzzy models agree when violations are minimal. Trapezoidal outputs perfect scores (1.000) for both, confirming its insensitivity to minor infractions due to its full membership plateau. This agreement across functions in the high-acceptability regime suggests that function choice is less critical when evaluating unambiguously grammatical sentences.

4.4. Sensitivity to Gradual Complexity

Sentences S4 and S5 introduce more syntactic and discourse complexity—passive construction in S4 and clausal subordination in S5. Their scores, visible in **Figure 6**, reveal how different functions react to moderate linguistic deviations. While Trapezoidal still assigns a perfect score to S4 (1.000), it lowers the score for S5 to 0.833. In contrast, Gaussian and Sigmoid provide more differentiated gradience (e.g., S5: Gaussian = 0.675, Sigmoid = 0.942), as seen also in **Figure 5**, supporting the claim that they better capture syntactic complexity through smooth penalization.

4.5. Relative Ranking Preservation with Absolute Shifts

Despite score differences, the rank ordering of sentence acceptability remains broadly consistent across all functions: $S3 \geq S1 \geq S4 \geq S5 > S2$. This consistency is clearly visualized in **Figure 7**, which maps sentence ranks across functions. It suggests that the relative severity of violations dominates the ranking, even though the magnitude of differences varies significantly. For instance, under Gaussian, the gap between S3 and S2 is about 0.57, while under Sigmoid it narrows to 0.21, highlighting how some functions amplify contrast while others compress distinctions.

4.6. Implications for Model Interpretability

The variation in scores across membership functions (**Figure 5**, **Figure 6**) and the shift in sentence rankings (**Figure 7**) underscore that membership function choice is not a neutral modeling decision. Linear and Sigmoid offer more interpretable gradience but may underestimate sharp syntactic anomalies. Gaussian introduces stricter evaluation, beneficial for applications that simulate human-like sensitivity to core grammatical norms. Meanwhile, Trapezoidal—though intuitive—may oversimplify acceptability judgments and introduce threshold effects that obscure subtle linguistic variation. Therefore, membership function selection should be guided by the desired interpretability and application goals of the fuzzy grammar system.

To further substantiate these observations, we conducted additional quantitative analyses to examine the robustness and sensitivity of the results. Specifically, we re-

port the per-sentence acceptability scores across membership functions, rank consistency measured by Spearman's ρ and Kendall's τ , and pairwise non-parametric statistical tests (Friedman test, Wilcoxon signed-rank with Holm correction, and Cliff's δ effect sizes). In addition, sensitivity analyses

were performed by varying the steepness parameter α in the Sigmoid function and the spread parameter σ in the Gaussian function to assess how parameterization influences acceptability outcomes. The results of these extended analyses are summarized in **Table 4**, **Table 5** and **Figure 8**, **Figure 9**.

Table 4. Rank consistency among membership functions measured by Spearman's ρ and Kendall's τ . Results show perfect consistency among Linear, Sigmoid, and Gaussian, with slightly lower correlations involving Trapezoidal.

MF1	MF2	Spearman's ρ	Kendall's τ
Gaussian	Linear	1.000	1.000
Gaussian	Sigmoid	1.000	1.000
Gaussian	Trapezoidal	0.894	0.837
Linear	Sigmoid	1.000	1.000
Linear	Trapezoidal	0.894	0.837
Sigmoid	Trapezoidal	0.894	0.837

Table 4 reports the rank consistency among membership functions based on acceptability scores across the five sentences. Spearman's ρ and Kendall's τ correlations between Linear, Sigmoid, and Gaussian reached 1.000, indi-

cating identical rankings. In contrast, correlations involving Trapezoidal were slightly lower ($\rho = 0.894$; $\tau = 0.837$), reflecting the plateau effect of the Trapezoidal function, which produced ties among highly grammatical sentences.

Table 5. Pairwise Wilcoxon signed-rank tests with Holm correction and Cliff's δ effect sizes across membership functions. While no comparisons reached statistical significance at $\alpha = 0.05$, moderate-to-large effect sizes indicate practical differences, particularly between Sigmoid and Gaussian.

MF_A	MF_B	W Statistic	p (Raw)	p (Holm)	Significant ($\alpha = 0.05$)	Cliff's δ (A vs. B)
Gaussian	Linear	0.0	0.0625	0.375	No	-0.36
Gaussian	Sigmoid	0.0	0.0625	0.375	No	-0.76
Gaussian	Trapezoidal	0.0	0.0625	0.375	No	-0.52
Linear	Sigmoid	0.0	0.0625	0.375	No	-0.60
Linear	Trapezoidal	4.0	0.7150	1.000	No	-0.32
Sigmoid	Trapezoidal	6.0	0.8125	1.000	No	-0.28

Table 5 presents the results of non-parametric Wilcoxon signed-rank tests comparing acceptability scores across membership functions, with Holm correction applied to control for multiple testing. While no pairwise differences reached statistical significance at the conventional threshold of $\alpha = 0.05$, this outcome is primarily attributable to the small dataset size (five sentences), which limits statistical power. Nevertheless, effect size estimates using Cliff's δ reveal moderate to large practical differences across several membership function pairs. In particular, Sigmoid yielded consistently higher acceptability scores than Gaussian ($\delta = -0.76$), and Linear also tended to exceed Gaussian ($\delta = -0.36$). These findings suggest that although statistical significance was not achieved, the choice of membership function exerts a meaningful influence on acceptability outcomes, especially when

practical rather than purely inferential criteria are considered.

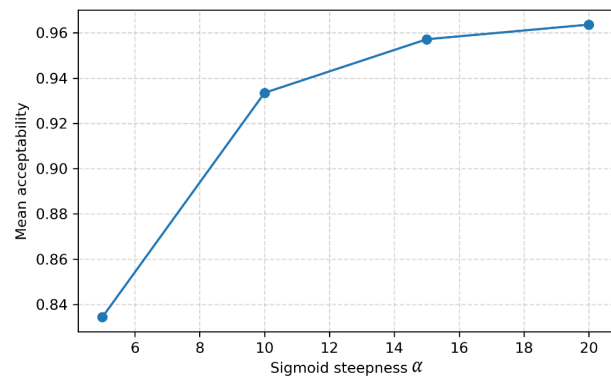


Figure 8. Sensitivity analysis of the Sigmoid membership function across different steepness values ($\alpha = 5, 10, 15, 20$). Increasing k steepens the curve, leading to higher mean acceptability scores as the function approaches a threshold-like behavior.

This table shows how varying the steepness parameter α in the Sigmoid function affects mean acceptability scores. Increasing α sharpens the transition around the midpoint ($c = 0.5$), which consistently raises acceptability scores (from 0.834 at $\alpha = 5$ to 0.964 at $\alpha = 20$). This demonstrates that steeper Sigmoid curves behave more like threshold functions, exhibiting higher tolerance toward mild violations.

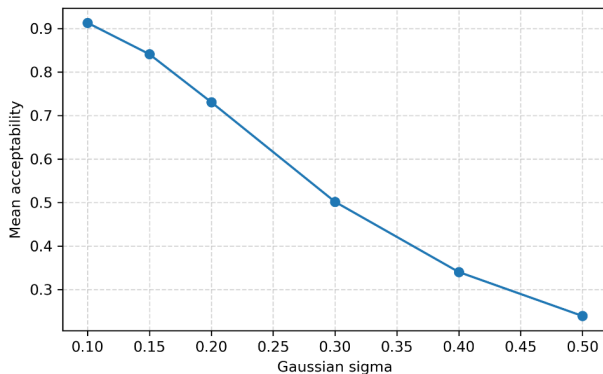


Figure 9. Sensitivity analysis of the Gaussian membership function across different spread values ($\sigma = 0.10$ – 0.50). Larger σ values flatten the curve and reduce mean acceptability scores, showing that Gaussian becomes more conservative as σ increases.

This table reports the sensitivity of acceptability scores to changes in the spread parameter σ of the Gaussian function. Smaller σ values ($\sigma = 0.10$) result in higher acceptability (0.913), as penalties are concentrated near the central point ($\mu = 0.5$). Conversely, larger σ values ($\sigma = 0.50$) broaden the curve and substantially lower acceptability (0.239). Thus, Gaussian functions become more conservative as σ increases, penalizing violations across a wider input range.

5. Discussion

The results of this study confirm that the design of the membership function plays a significant and non-trivial role in shaping fuzzy grammatical acceptability judgments. Although all four membership functions—Linear, Sigmoid, Gaussian, and Trapezoidal—operate over the same input violation scores and utilize a common aggregation mechanism, they yield notably different outcomes in terms of both absolute scores and sensitivity patterns. This section discusses the implications of these findings in terms of model behavior, linguistic interpretability, and the broader use of fuzzy logic in natural language modeling.

5.1. Function Shape and Judgment Sensitivity

The first major insight is that the shape of the membership function directly governs the sensitivity of the system to mid-level violations. The Sigmoid function, with its characteristic “S” curve, shows heightened responsiveness near the decision boundary (around 0.5), allowing it to magnify differences between mildly acceptable and mildly unacceptable structures. This is useful in scenarios where nuanced gradience is critical—such as second language learning assessments or acceptability rating studies in psycholinguistics.

In contrast, the Gaussian function, due to its central peak and symmetrical decay, disproportionately penalizes moderate violations. As observed in the score drop for Sentence S2 (0.356), as visualized in **Figure 6**, the Gaussian function behaves conservatively, which may be advantageous for systems that need to prioritize precision and penalize structural ambiguity. However, its rigidity may also suppress subtle acceptability nuances, making it less suitable for exploratory linguistic modeling. The Linear function offers proportionality and simplicity. It responds predictably to increasing violation severity and provides moderate differentiation without amplifying boundary effects. This makes it appropriate for general-purpose fuzzy grammar applications where interpretability and smooth behavior are prioritized over expressiveness.

The Trapezoidal function, by contrast, introduces hard transitions between full membership and rejection zones, which leads to score plateaus (e.g., perfect scores for S1, S3, and S4 regardless of slight violations). This behavior, shown in **Figure 5**, is appealing for rule-based systems that enforce linguistic thresholds (e.g., diagnostic tools or automated feedback systems), but it may mask gradual linguistic deterioration and underestimate variability in real-world language use.

5.2. Linguistic Interpretability and Model Transparency

From an interpretability standpoint, each function implicitly encodes a linguistic philosophy. Gaussian and Sigmoid align with continuous, scalar interpretations of acceptability, consistent with findings from gradient grammaticality research. Linear preserves a neutral, assumption-free mapping, while Trapezoidal mimics categorical grammar models

that treat acceptability in quasi-discrete bands. Thus, the selection of a membership function should be guided by the intended linguistic framework. For example, if a model aims to reflect native speaker intuition in acceptability rating tasks, sigmoid or linear functions may be more appropriate. Conversely, educational tools that must offer clear, rule-based feedback might benefit from trapezoidal definitions.

Furthermore, this study reveals that membership functions not only affect final outputs but also shape the pedagogical narrative a system presents to its users. For instance, under a Trapezoidal function, learners might receive consistent positive feedback on borderline sentences, whereas a Gaussian-based system might flag the same sentence as marginally acceptable. Such discrepancies—highlighted by color-coded score differences in **Figure 6**—can influence learner perception, feedback quality, and downstream behavior, highlighting the epistemological impact of seemingly technical modeling decisions.

5.3. Model Robustness and Consistency

Despite functional differences, the consistency in sentence ranking across all functions— $S3 > S1 > S4 > S5 > S2$ —is visualized clearly in **Figure 7**. This consistency suggests that the underlying violation profile dominates the general ordering. It implies that membership functions mostly affect the magnitude rather than the direction of judgments.

Nevertheless, the magnitude of variation is critical when fuzzy outputs are interpreted quantitatively (e.g., for thresholding, grading, or clustering). For instance, Sentence S2's acceptability score ranges from 0.356 (Gaussian) to 0.778 (Sigmoid), as seen in both **Figure 5** and **Figure 6**—a difference of over 0.4, which could materially affect downstream decision-making. This further reinforces the need for function-aware calibration in fuzzy NLP systems.

5.4. General Implications for Fuzzy Grammar Modeling

This study contributes to the broader discourse on transparency and methodological rigor in fuzzy linguistic modeling. The frequent adoption of default or linear membership functions in prior research often overlooks the functional assumptions and behavioral consequences these choices en-

tail. By demonstrating that different function shapes yield systematically different outputs—even under fixed input and aggregation—this work challenges the notion of membership function neutrality and calls for greater methodological reflectiveness.

In practical terms, researchers and developers should consider integrating membership function selection as a tunable component during model development, rather than treating it as a static design choice. Comparative experiments, such as the one presented here, should become standard practice, especially in applications where model outputs inform high-stakes decisions or user feedback. **Figure 5**, **Figure 6**, and **Figure 7** together illustrate how the choice of membership function can introduce significant divergence in scores, heatmap patterns, and ranking structures—even when all other components of the fuzzy system remain unchanged.

These findings have practical implications for grammar-checking systems, where membership function choice may influence the granularity of error feedback. In second language assessment and tutoring, fuzzy acceptability models can provide more nuanced evaluations than binary judgments, helping learners understand partial correctness. Similarly, in NLP system calibration, carefully chosen membership functions can improve the interpretability of probabilistic predictions by aligning them with human-like gradient judgments.

5.5. Threats to Validity

The present study is limited by the use of a small, curated dataset of five sentences, which may restrict generalizability. While this setup allows for controlled comparison of membership function behaviors, it does not capture the full diversity of natural language usage. This design choice reflects the methodological scope of the paper, which aims to isolate the impact of membership function design under a controlled testbed rather than to provide a large-scale corpus study.

To partially mitigate this limitation, we introduced a transparent violation scoring rubric (**Table 1**, Section 3.1), which specifies how fuzzy values were assigned to rule violations. This rubric ensures reproducibility of the inputs and provides a clear rationale behind the violation scores. Nevertheless, the absence of human judgment data prevents direct validation of model outputs against ground-truth ac-

ceptability ratings. We recommend future work to address this limitation by collecting larger annotated corpora and evaluating inter-annotator reliability (e.g., Krippendorff's α).

Finally, while our operationalization of acceptability includes syntactic, structural, and pragmatic factors, there remains a risk of construct overlap with grammaticality that should be further refined in subsequent studies. Despite these limitations, we conducted a series of robustness checks and sensitivity analyses to ensure that our findings remain interpretable and practically meaningful. These complementary evaluations are reported in the following subsection.

5.6. Robustness Checks and Parameter Sensitivity

To complement the descriptive findings and address potential concerns regarding the robustness of our results, we conducted additional statistical analyses. Rank consistency measures confirmed that sentence orderings were largely stable across membership functions, with only minor deviations observed for the Trapezoidal function due to its plateau effect. Non-parametric statistical tests (Friedman and Wilcoxon signed-rank with Holm correction) did not yield significant differences at $\alpha = 0.05$, primarily due to the limited sample size. However, effect size estimates (Cliff's δ) indicated moderate to large practical differences, particularly between Sigmoid and Gaussian functions. In addition, sensitivity analyses demonstrated how parameterization—specifically the steepness parameter α in the Sigmoid function and the spread parameter σ in the Gaussian function—directly influences acceptability scores, underscoring the importance of careful function tuning. These robustness checks highlight that, even under a small dataset, the impact of membership function design on fuzzy grammatical modeling is both systematic and practically meaningful. Overall, these robustness checks confirm that the observed differences across membership functions are systematic and practically relevant, even under a limited dataset. Nevertheless, future studies should extend these analyses to larger corpora and incorporate human acceptability ratings to empirically validate the statistical patterns reported here. Such extensions would provide stronger evidence for the generalizability of the findings and establish a more comprehensive framework for modeling fuzzy grammatical acceptability.

6. Conclusions

This study investigated the impact of membership function design on fuzzy grammatical acceptability judgments by systematically comparing four widely used membership functions: Linear, Sigmoid, Gaussian, and Trapezoidal. Using a controlled dataset of five linguistically varied sentences and a fixed set of rule-based violation scores, we demonstrated that the choice of membership function significantly affects the magnitude and behavior of acceptability scores, despite a shared aggregation strategy. Our findings highlight that membership function design is not a neutral modeling choice. Instead, it shapes model sensitivity, interpretability, and feedback behavior. Sigmoid and Gaussian functions emphasize mid-range differences, making them suitable for capturing gradient intuitions, while Linear offers stable proportionality. Trapezoidal, though useful for rule-like cut-offs, may obscure nuanced grammatical variations. These functional differences carry practical implications for linguistic modeling, second language assessment, and fuzzy-based grammar-checking systems. Importantly, while sentence ranking remained relatively consistent across functions, score divergence was substantial, underscoring the need for careful selection, justification, and possibly empirical calibration of membership functions in fuzzy NLP applications. We therefore advocate for the explicit treatment of membership function selection as a core methodological decision in fuzzy linguistic modeling, rather than a secondary or default parameter. Nevertheless, the study has limitations. It relies on a small, curated dataset of five sentences, which, although controlled, cannot fully represent the diversity of natural language. Future work should incorporate larger and more varied corpora to enhance robustness. Moreover, integrating human acceptability ratings as ground truth would provide a critical validation step, with inter-annotator agreement metrics (e.g., Krippendorff's α) employed to assess reliability. Such extensions would improve the empirical grounding of fuzzy grammar models and help clarify the construct boundary between grammaticality and acceptability in real-world linguistic contexts. Beyond larger corpora and human annotations, future research should also explore adaptive or data-driven approaches to membership function learning, which can optimize function shapes based on empirical linguistic judgments. Another promising direction is the adoption of Type-2 fuzzy sets and generalized bell

membership functions, which provide richer representational flexibility and can better capture uncertainty in gradient acceptability judgments. Incorporating these advanced designs would not only strengthen the theoretical grounding of fuzzy grammar models but also enhance their applicability in diverse natural language processing scenarios. Ultimately, this research contributes to the growing movement for greater transparency, rigor, and explainability in computational models of language.

Author Contributions

Conceptualization, R.; methodology, R.; software, R.; validation, R.; formal analysis, R.; investigation, R.; resources, T.R.; data curation, T.R. and R.; writing—original draft preparation, R.; writing—review and editing, R.; visualization, R.; supervision, R.; project administration, R. All authors have read and agreed to the published version of the manuscript.

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Institutional Review Board Statement

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Informed Consent Statement

Not applicable. This study did not involve humans, and therefore informed consent was not required.

Data Availability Statement

The dataset used in this study was obtained from a previously published article: Mohammad, S., Yogeesh, N., Al-Daoud, K.I., et al., 2025. A Mathematical Fuzzy Model for Syntax-Pragmatics Interface. *Forum for Linguistic Studies*, 7(6), 26–41. DOI: <https://doi.org/10.30564/fls.v7i6.9618>. To facilitate reproducibility, we provide a public repository containing a Python notebook that regenerates all tables and figures reported in this study. The repository is accessible at <https://github.com/rustam-itb/fuzzy-grammar-mf>.

Conflicts of Interest

The authors declare no conflict of interest.

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