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Identification of Structural Parameters Based on HHT and NExT

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ARTICLE INFO

Article history
Received: 4 December 2020
Revised: 11 December 2020
Accepted: 9 April 2021
Published Online: 16 April 2021

Keywords:
Structure
HHT
Natural frequency
Damping ratio
Mode shape

ABSTRACT

Signal processing approaches are widely used in the field of earthquake engineering, especially in the identification of structural modal parameters. Hilbert-Huang Transformation (HHT) is one new signal processing approach, which can be used to identify the modal frequency, damping ratio, mode shape, even the interlayer stiffness of the shear-type structure, incorporating with Natural Excitation Technique (NExT) method to take information from the response records of the structure. The stiffness of the structure is of great importance to judge the loss of its bearing capacity after earthquake. However, all of modal parameters are required to calculate the stiffness of the structure by use of HHT and NExT, which means that the response records shall contain all of modal information. However, it has been found that the responses of the structure recorded only contain the former order modal information; even it is excited by earthquake. Therefore, it is necessary to found a formula (formulas) to calculate the stiffness only using limited modal parameters. In this paper, the calculation formulas of the interlayer stiffness of shear-type structure are derived by using of the flexibility method, which indicate that all of interlayer stiffnesses could be worked out as long as any one set of modal parameters is obtained. After that, Taking Sheraton-Universal Hotel subjected to North Bridge earthquake in 1994 as an example, HHT and NExT are used to identify its modal parameters, the derived formulas are used to calculate the interlayer stiffnesses, and their applicability and accuracy are verified.

1. Introduction

Hilbert-Huang Transform (HHT), a signal processing method, was firstly put forward by Dr. Norden E. Huang of National Aeronautics and Space Administration (NASA) in 1998. The main innovations embodied in this method are the present of the concept of the Intrinsic Mode Function (IMF) and introduction of the method of Empirical Mode Decomposition (EMD). A signal is first decomposed into IMFs by the EMD, and then, Hilbert spectrum is obtained through Hilbert transform, marginal spectrum of the signal can also be obtained [1-3]. The Hilbert spectrum can present precise description of instantaneous frequency and amplitude, embodies higher presentation compared with Fourier spectrum and Wavelet spectrum. Thus, it is quite applicable to analyze nonlinear and non-stationary signal, and has been applied in many fields since it is raised.

In the modal parameters identification field, the researchers have put forward two methods, namely, using HHT and RDT to identify modal parameters of linear structure and using HHT and NExT to identify modal parameters of linear structure [4]. The former only can identify the modal fre-

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DOI: https://doi.org/10.30564/frae.v4i1.2746

quency and the damping ratio, while the latter can identify the mode shape and the interlayer stiffness of the shear-type structure besides the modal frequency and the damping ratio. However, all of modal parameters are required to calculate the stiffness of the structure by use of HHT and NExT, which means that the response records shall contain all of modal information. However, it has been found that the recorded responses of the structure only contain the former order modal information; even it is excited by earthquake. Therefore, it is necessary to found a formula (formulas) to calculate the stiffness only using limited modal parameters. This paper firstly introduces the principle of HHT and NExT used to identify the structural parameters and points out its limitation, then, derives the calculation formulas of the interlayer stiffnesses of shear-type structure by using of the flexibility method, which indicate that all of interlayer stiffnesses could be worked out as long as any one set of modal parameters is obtained. After that, Sheraton-Universal Hotel is firstly reduced as the four-floor shear-type structure based on the lumped mass method, then, the first three modal frequencies, damping ratios and vibration modes of this structure are identified by using of HHT and NExT, the referenced data are the seismic records obtained in North Bridge earthquake in 1994, and the interlayer stiffnesses are worked out. Finally, the earthquake responses calculated by the elastic time-history analysis based on the identified structural parameters are compared with the seismic records, and the results show that the identified structural parameters are valid.

2. Principle of HHT and NExT Used to Identify Structural Parameters

The motion equation of multi-degree linear system is given by

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=F(t) \tag{1}$$

where $[M]$, $[C]$ and $[K]$ are the mass matrix, damping matrix and stiffness matrix, respectively; $\{x\}$ is the displacement vector of the structure; $\{\dot{x}\}$ is the velocity vector; $\{\ddot{x}\}$ is acceleration vector; $F(t)$ is the seismic excitation. If $\{\phi_1\}$, $\{\phi_2\}$,…, $\{\phi_n\}$ represent the mode shape series, they are independent due to their orthogonality. According to the linear algebra theory, the victor $\{x\}$ with n dimensions can be described by the linear combination of n independent vectors, so the displacement vector of linear structure $\{x\}$ can be expressed as

$$\{x\}=\sum_{j=1}^{n} q_j \{\varphi_j\} \tag{2}$$

where $q_j\,(j=1,2,\cdots,n)$ are the canonical coordinates of the mode shape, and $\{\phi_j\}$ is the j[th] mode shape. Substituting Equation (2) into Equation (1), suppose the mode shape is orthogonal to the damping matrix, Equation (1) can be written as

$$\ddot{q}_j+2\xi_j\omega_j\dot{q}_j+\omega_j^2 q_j=\{\phi_j\}^{\mathrm{T}}F(t)/m_j \tag{3}$$

Assuming $F(t)$ is the ideal white noise, it can not be obtained wholly, and thus, the traditional modal identification methods can not be used to identify the modal parameters. However, a good discovery is that the correlation function of the response of the linear structure under white noise excitation is similar mathematically to the impulse response function [5]; therefore, it can be substituted by the cross-correlation function between the responses of two floors. If every floor has the response record after earthquake, the response record of the i[th] floor is taken as the reference point, and the cross-correlation function between the response of the j [th] floor and that of the i[th] floor can be expressed as [6]

$$R_{ji}(\tau)=\sum_{r=1}^{n}\frac{\psi_{jr}G_{ir}}{m_r\omega_{dr}}\exp(-\xi_r\omega_r\tau)\sin(\omega_{dr}\tau+\theta_r)(j=1,2,\cdots,n) \tag{4}$$

Where ψ_{jr} is the j[th] element of the r[th] mode shape; G_{ir} is a constant only related with i and r; m_r, ξ_r, ω_r and θ_r are the modal mass, damping ratio, circular frequency and phase angle, respectively; and ω_{dr} is the damped circular frequency. $R_{ji}(\tau)$ can be decomposed into m IMFs and one trend term by use of EMD. Ideally, m is equal to n, and the n terms at the right side of Equation (4) are the IMFs obtained through EMD. But, generally, m is greater than n, so it is required to find out the IMFs with physical significance. Here, it is recommended to find out the right IMFs through observing their Hilbert spectrums.

The r[th] IMF of the j[th] cross-correlation function can be expressed as

$$R_{ji,r}=\frac{\psi_{jr}G_{ir}}{m_r\omega_{dr}}\exp(-\xi_r\omega_r\tau)\sin(\omega_{dr}\tau+\theta_r) \tag{5}$$

The amplitude $A(\tau)$ and phase $\theta(\tau)$ of $R_{ji,r}$ can be denoted as [7]

$$A(\tau)=\frac{|\psi_{jr}|G_{ir}}{m_r\omega_{dr}}\exp(-\xi_r\omega_r\tau)\ ,\quad \theta(\tau)=\omega_{dr}\tau+\theta_r \tag{6}$$

Solving the natural logarithm of $A(\tau)$ in Equation (2-6) and the derivation of $\theta(\tau)$ can obtain

DOI: https://doi.org/10.30564/frae.v4i1.2746

$$\ln A(\tau) = -\xi_r \omega_r \tau + \ln \frac{|\psi_{jr}| G_{ir}}{m_r \omega_{dr}} \tag{7}$$

$$\omega(\tau) = \frac{d\theta(\tau)}{d\tau} = \omega_{dr} \tag{8}$$

From Equations (7) and (8), $\xi_r \omega_r$ and ω_{dr} can be solved through the amplitude and phase spectrums of $R_{ji,r}$ obtained through Hilbert transform, and $\omega_{dr} = \sqrt{1-\xi_r^2}\,\omega_r$, thus ω_r and ξ_r also can be obtained. It is worth mentioning that the building may enter the nonlinear condition after strong motion, and some locations may be damaged, consequently, the dynamic characteristics of the structure will change [8]. Therefore, $\ln A(\tau)$ and $\theta(\tau)$ will not be the ideal straight lines, and they shall be divided into several segments, and then each segment can be changed into one straight line through least-square linear fitting. Hence, for a specific instant τ_0, ω_r and ξ_r can be calculated according to the above equations, while the r[th] mode shape can be obtained through the n amplitude spectrums corresponding with ω_r, and the signs in the mode shape are determined by the phase difference, which can be obtained through the n phase spectrums [6].

All of mode shapes obtained form the principle mode matrix $[\Phi] = \{\{\varphi_1\}\{\varphi_2\}\cdots\{\varphi_n\}\}$. And according to the theory of structural dynamics, it obeys the following three orthogonal relations

$$[\Phi]^{\mathrm{T}}[M][\Phi] = \begin{bmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & m_n \end{bmatrix} = [M^*] \tag{9}$$

$$[\Phi]^{\mathrm{T}}[K][\Phi] = \begin{bmatrix} k_1 & 0 & \cdots & 0 \\ 0 & k_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & k_n \end{bmatrix} = [K^*] \tag{10}$$

$$[\Phi]^{\mathrm{T}}[C][\Phi] = \begin{bmatrix} c_1 & 0 & \cdots & 0 \\ 0 & c_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & c_n \end{bmatrix} = [C^*] \tag{11}$$

m_r, k_r and c_r denote the r[th] elements of diagonal matrixes $[M^*]$, $[K^*]$ and $[C^*]$, respectively. Generally, $[M]$ is assumed to be constant, so, if $[\Phi]$ is determined, $[M^*]$ will be determined. And $k_r = m_r \omega_r^2$, $c_r = 2m_r \xi_r \omega_r$, so, $[K^*]$ and $[C^*]$ will also be determined. Conducting the inverse transformation to Equations (10) and (11), $[K]$ and $[C]$ will be obtained. Then, the interlayer stiffness also can be solved by $[K]$.

From the above description, it can be seen that, in order to obtain the mass matrix $[K]$, the principle mode matrix must be determined firstly, which means that all of modal parameters must be identified. However, the recorded responses of the structure under earthquake or other excitations always only contain the former order modal information, and it is impossible to take all of modal parameters. Therefore, it is necessary to found a formula (formulas) to calculate the stiffness only using limited modal parameters.

3. Derivation of Interlayer Stiffness of Shear-Type Structure

According to Equation (1), the characteristic equation of the shear-type structure can be expressed as

$$([I] - \omega^2[\delta][M])\{Y\} = \{0\} \tag{12}$$

where $[\delta]$ is the flexibility matrix. Suppose $r_i = 1/k_i$, in which k_i is the interlayer stiffness of the i[th] floor, the flexibility matrix $[\delta]$ can be written as

$$[\delta] = \begin{bmatrix} r_1 & r_1 & r_1 & \cdots & r_1 \\ r_1 & r_1+r_2 & r_1+r_2 & \cdots & r_1+r_2 \\ r_1 & r_1+r_2 & r_1+r_2+r_3 & \cdots & r_1+r_2+r_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_1 & r_1+r_2 & r_1+r_2+r_3 & \cdots & r_1+r_2+\cdots+r_n \end{bmatrix} \tag{13}$$

where $[M]$ is the mass matrix of the structure. Suppose m_i is the interlayer mass of the i[th] floor, the mass matrix $[M]$ can be written as

$$[M] = \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_n \end{bmatrix} \tag{14}$$

Suppose $\{Y\}$ is the principle mode matrix. Due to the orthogonality, the j[th] mode shape $\{Y_j\}$ satisfies the following equation

$$\begin{bmatrix} Y_{j1} \\ Y_{j2} \\ Y_{j3} \\ \vdots \\ Y_{jn} \end{bmatrix} = \begin{bmatrix} r_1 & r_1 & r_1 & \cdots & r_1 \\ r_1 & r_1+r_2 & r_1+r_2 & \cdots & r_1+r_2 \\ r_1 & r_1+r_2 & r_1+r_2+r_3 & \cdots & r_1+r_2+r_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_1 & r_1+r_2 & r_1+r_2+r_3 & \cdots & r_1+r_2+\cdots+r_n \end{bmatrix} \begin{bmatrix} \omega_j^2 m_1 Y_{j1} \\ \omega_j^2 m_2 Y_{j2} \\ \omega_j^2 m_3 Y_{j3} \\ \vdots \\ \omega_j^2 m_n Y_{jn} \end{bmatrix} \tag{15}$$

Equation (15) can be written as

$$\begin{cases} Y_{j1} = \omega_j^2 m_1 Y_{j1} r_1 + \omega_j^2 m_2 Y_{j2} r_1 + \cdots + \omega_j^2 m_n Y_{jn} r_1 & (1) \\ Y_{j2} = \omega_j^2 m_1 Y_{j1} r_1 + \omega_j^2 m_2 Y_{j2} (r_1 + r_2) + \cdots + \omega_j^2 m_n Y_{jn} (r_1 + r_2) & (2) \\ Y_{j3} = \omega_j^2 m_1 Y_{j1} r_1 + \omega_j^2 m_2 Y_{j2} (r_1 + r_2) + \cdots + \omega_j^2 m_n Y_{jn} (r_1 + r_2 + r_3) & (3) \\ \vdots & \\ Y_{jn-1} = \omega_j^2 m_1 Y_{j1} r_1 + \omega_j^2 m_2 Y_{j2} (r_1 + r_2) + \cdots + \omega_j^2 m_n Y_{jn} (r_1 + r_2 + \cdots + r_{n-1}) & (n-1) \\ Y_{jn} = \omega_j^2 m_1 Y_{j1} r_1 + \omega_j^2 m_2 Y_{j2} (r_1 + r_2) + \cdots + \omega_j^2 m_n Y_{jn} (r_1 + r_2 + \cdots + r_n) & (n) \end{cases} \tag{16}$$

Subtracting Sub-equation (n) by Sub-equation (n-1) obtains

$$r_n = \frac{Y_{jn} - Y_{jn-1}}{Y_{jn} m_n \omega_j^2} \tag{17}$$

Then, the interlayer stiffness of the nth floor can be given by

$$k_n = \frac{Y_{jn} m_n \omega_j^2}{Y_{jn} - Y_{jn-1}} \tag{18}$$

Likewise, subtracting the adjacent two sub-equations in Equation (16), the interlayer stiffnesses of the floors except the first floor and the nth floor k_i can be given by

$$k_i = \frac{\sum_i^n Y_{ji} m_i}{Y_{ji} - Y_{ji-1}} \omega_j^2 \qquad i = 2,3 \cdots n-1 \tag{19}$$

And the interlayer stiffness of the first floor can be given by

$$k_1 = \frac{\sum_{i=1}^n Y_{ji} m_i}{Y_{j1}} \omega_j^2 \tag{20}$$

4 Identification of Structural Parameters of Sheraton-Universal Hotel

4.1 Strong Motion Observation Scheme

Sheraton-Universal Hotel is a 20-storey reinforced concrete frame structure, located in Hollywood, north of Los Angeles, CA, USA, 34.1380 degrees north latitude and 118.3590 degrees west longitude. This region is in the southeast corner of San Fernando Valley, with high earthquake frequency. Thus, Sheraton-Universal Hotel was selected as the strong earthquake observation object when it was in construction, and was set with the strong motion array with five observation plane, ten observation points and sixteen channels before the occurrence of North Bridge earthquake in 1994, and the concrete observation scheme is shown in Figure 1. The five observation planes were set on the basement, third, ninth, sixteenth and top floors, two observation points were set at each plane, one was set at the west side, and the other was set at the east side. For the third, ninth, sixteenth and top planes, the basement, the observation point in the west had two orthogonal channels, arranged vertically and horizontally, while the observation point in the east only had one channel arranged horizontally. For the basement plane, the observation point in the east had two channels arranged vertically and horizontally, and one channel perpendicular to the plane, and it was the only one with three components; while the other observation point only had one transversal channel. In the North Bridge earthquake, occurred on 17th, November, 1994, Sheraton-Universal Hotel obtained sixteen suits of strong motion records with the sampling interval of 0.02 s and each suit of record contained the displacement, velocity and acceleration information. The longitudinal acceleration records are used to identify the structural parameters, and the longitudinal acceleration recorded on the ninth floor is shown in Figure 2.

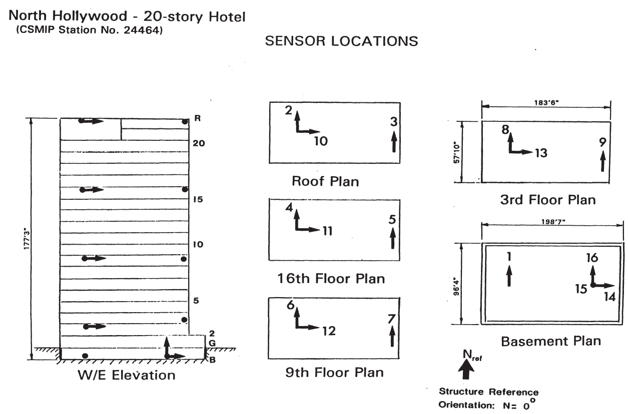

Figure 1. Sensor locations

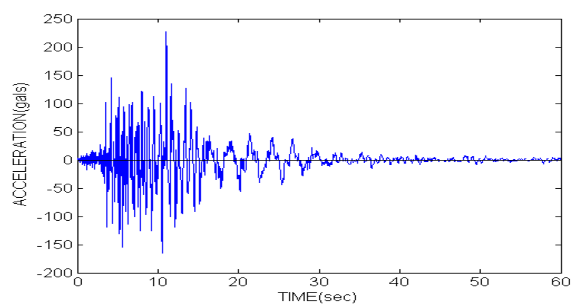

Figure 2. Longitudinal acceleration record on the ninth floor

4.2 Identification of Structural Parameters

It was reported that Sheraton-Universal Hotel was

DOI: https://doi.org/10.30564/frae.v4i1.2746

little damaged after North Bridge earthquake on 17th, November, 1994. Figure 2 shows that the vibration of the structure is gradually damped after 20 seconds, similar to free damping oscillation; therefore, it can be taken as linear structure. Suppose the base of the structure is rigid, and the mass is concentrated on the third, ninth, sixteenth and top floors, which are connected with each other by the stiffness component and damping component, as shown in Figure 3.

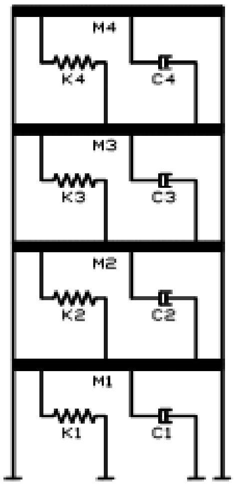

Figure 3. Structural Model

Taking the ninth floor as the reference floor, based on the longitudinal acceleration records, the former three modal parameters are identified by use of the method described above, listed in Table 1 and Table 2. According to the first modal frequency and mode shape, the interlayer stiffness of the structure after 20 s can be worked out by use of Equations (18), (19) and (20), listed in Table 3.

Table 1. Identified modal parameters

	The first order		**The second order**		**The third order**	
	f_1/**Hz**	ξ_1	f_2/**Hz**	ξ_2	f_3/**Hz**	ξ_3
Floor 3	0.359	6.5%	0.995	3.2%	1.195	4.7%
Floor 9	0.367	6.9%	1.075	4.4%	1.155	4.1%
Floor 16	0.35	5.9%	0.956	3.3%	1.215	4.6%
Roof	0.366	6.9%	0.987	3.2%	1.179	4.5%

Table 2. Mode shapes in the longitudinal direction

	The first order	**The second order**	**The third order**
Floor 3	1.000	1.000	1.000
Floor 9	0.902	0.138	-0.172
Floor 16	0.629	-1.061	1.058
Roof	0.374	-0.526	0.324

Table 3. Structural parameters

	Floor 3	**Floor 9**	**Floor 16**	**Roof**
lumped mass(kg)	410999	1325571	2291849	19298820
Interlayer stiffness ($\times 10^8$)	6.825	9.46	7.754	9.586

4.3 Verification of Identified Results

The mass matrix, stiffness matrix and damping matrix of the structure are obtained in Section 4.2, the acceleration records of the basement after 20 s can be taken as the excitation of the structure, and the displacement, velocity and acceleration records at 19.98 s can be taken as the initial conditions, thus the seismic responses of the third, ninth, sixteenth and top floors can be obtained through elastic time-history analysis method. The identified parameters can be verified through comparing the calculated seismic responses with the seismic records, which are given in Figure 4~7. The solid lines denote the calculated results while the dotted lines denote the seismic records. It is indicated that, the calculated displacement results agree well with the records, so, the identified results by use of the method described in Section 3 are good. There exists big difference between the calculated displacement results on the third floor and the displacement records; the main reason is that the third floor mainly reflects the vibration characteristics of the base.

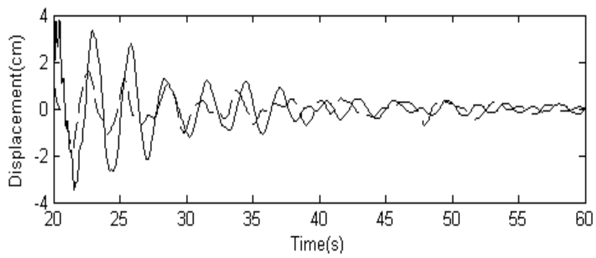

Figure 4. Comparison between seismic records and calculated results of the third floor

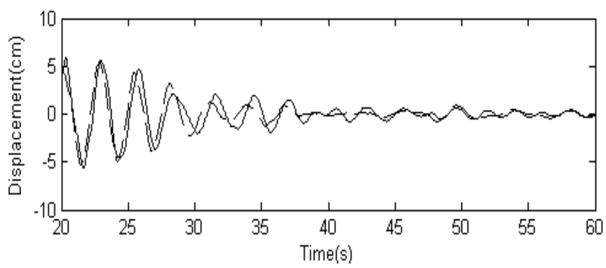

Figure 5. Comparison between seismic records and calculated results of the ninth floor

 DOI: https://doi.org/10.30564/frae.v4i1.2746

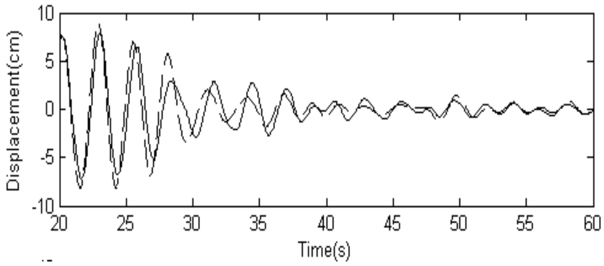

Figure 6. Comparison between seismic records and calculated results of the sixteenth floor

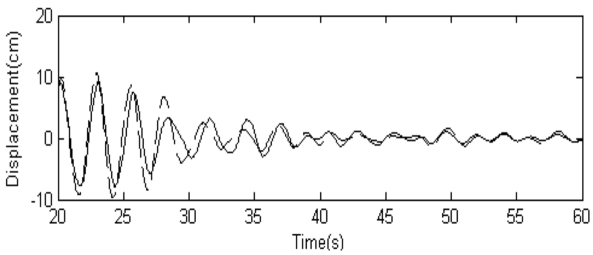

Figure 7. Comparison between seismic records and calculated results of the top floor

5 Conclusions

This paper points out the limitation of HHT and NExT used to identify the structural parameters of the structure, which is that all of modal parameters shall be identified firstly, and derives the calculation formulas of the interlayer stiffnesses of shear-type structure by using of the flexibility method which indicate that all of interlayer stiffnesses could be worked out as long as any one set of modal parameters is obtained. Therefore, these formulas obtained can solve this limitation. And the example of Sheraton-Universal Hotel shows that these formulas are quite applicable to identify structural parameters together with HHT and NExT.

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DOI: https://doi.org/10.30564/frae.v4i1.2746