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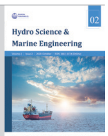

ARTICLE

Numerical Simulation of Flow over Stepped Spillways with Varying Step-Angle

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ABSTRACT

In the present study, the flow over the stepped spillway was numerically investigated by using Flow3D model. The effect of step angle on different properties of Nappe flow regime such as the water surface profile, location of free-surface aeration inception, Froude number at the spillway's toe, and pressure, flow velocity, air concentration and cavitation index were evaluated. The realizable k–ε was applied as the turbulence model, and Volume of Fluid (VOF) model was used to determine the free surface flow profiles of the spillway. The model was verified using experimental data. In order to investigate the different characteristics of Nappe flow regime, 17 numerical runs was designed, in which, four step angles, four flow discharge were considered to investigate the flow characteristics over the stepped spillway. The results indicated that the numerical model is well suited with the experimental data over the stepped spillway (RMSE = 0.147 and ARE = 6.9%). In addition, with increasing the step angles, the aeration inception point is generally moved downstream. By increasing the step angles from zero to 10 degrees, the Froude number does not change significantly, however, at the angle of 15 degrees, the Froude number decreases by about 42 percent.

1. Introduction

Stepped spillways are the direct spillways that the weir crest is connected by successive steps to the spillway's toe. These spillways have a unique structure that results in a different flow structure than that of smooth spillways [1-4]. In general, the flow pattern over a stepped spillway can be divided into nappe flow, transition flow, and skimming flow. One of the important issues in the design of these type of spillways is the energy dissipation. Stepped spillways are used for dissipating the generated energy through the spillway, which reduces the dimensions of the stilling basin. In this regards, different researchers, including Sorensen (1985).[1], Christodoulou [5], Chen et al. [6], Chinnarasri and Wongwises [7], Gonzalez et al. [8], Felder and Chanson [9], Zare and Doering [10], Otun et al. [11], Aras and Berkun [12], and Munta and Otun [13] have studied the effect of the flow pattern, step roughness, step size, and slope of a stepped spillway on the energy dissipation.

One of the important issues in the design of stepped spillways is the cavitation phenomenon. Typically, in dif-

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ferent types of spillways and weirs, as the flow velocity exceeds a critical value, the structure is subject to the cavitation, in which, the evaporation is occurred due to the reduction in the local pressure at a constant temperature. Cavitation is one of the important issues that has always been of interest to designers and researchers in the design of hydraulic structures, either in closed systems (including pumps and turbines) or in open-flow systems (such as tunnels, dams, spillways, etc.). In order to prevent the cavitation, it is necessary to identify the locations of the points, in which, by increasing the flow velocity the pressure can be reduced up to the vapor pressure. Cavitation damage is associated with many factors and parameters, including cavitation index (σ), flow velocity (V), surface strength of structural materials (S), operation period, air concentration (C). In this regard, a criterion, called the cavitation index (σ_c), is used, as follows:

$$\sigma_c = \frac{p_0 - p_v}{\rho u^2 / 2} \tag{1}$$

where, p_0 is local pressure, p_v is the vapor pressure at a given temperature, u is the flow velocity. The cavitation occurs when σ is smaller or equal to σ_c. This critical value depends largely on the flow geometry, shape, and height of the surface roughness. According to Falvey (1990)'s study [14], the cavitation will not occur over the stepped spillways until $\sigma > 1.8$. In addition, the cavitation is considered as a serious problem when the flow velocity reaches 25 m/s. According to previous studies, there is a risk of cavitation for flow rates greater than 80 m^2/s. In principle, the cavitation index in the stepped spillways is high, and if it is not protective against cavitation, its critical discharge is more than 15 m^2/s.

The complex nature of the flow over the spillways has made it difficult to conduct a comprehensive numerical analysis. Numerous researchers have numerically investigated flow over the stepped spillways. Savage and Johnson [15] used Reynolds's averaged equations for flow analysis using Flow3D software. They used the FAVOR method for rigid boundary modelling and the VOF model for the free surface. Tabara et al. [16] simulated the flow over the stepped spillways by ADINA software. In this study, two types of step sizes and four different layouts were considered. The parameters of velocity, measured pressure and free surface profile were simulated using Volume of Fluid (VOF) method. Chinnarasri and Wongwises [7] investigated the flow over a stepped spillway with sloping steps and developed some equations for minimum critical depth for formation of a continuous flow and maximum critical depth for skimming flow regime in a simple and sloping stepped spillways. Kositgittiwong et al. [17] analysed the velocity profiles over the stepped spillways using CFD simulations and large-scale laboratory experiments. They considered five different turbulence models of the Standard k-ε, the Realizable k-ε, the Renormalization group k-ε, the Standard k-ω and the shear stress transport k-ω model. The results showed that the numerical model involving any of these turbulence models can satisfactorily simulate the velocity profiles. All five turbulence models performed satisfactorily well on large-scale stepped spillways. The k-ω models may be slightly better suited in the lower region, while the realizable k-ε model provided slightly better results in the upper part of the velocity profile. Rafi et al. [18] evaluated the effects of increased reservoir conservation level on velocities, discharge capacity, and cavitation risk of the spillway. They used mathematical model to estimate the flow velocities and cavitation risk. Shahheydari et al. [19] investigated numerically the flow over the stepped spillway using Flow3D software. They used RNG k-ε model as the turbulence model, and Volume of Fluid (VOF) model to determine the free surface flow profiles. They designed 112 numerical spillway models (96 stepped spillway models and 16 smooth spillway models (i.e., WES profile)) in order to investigate the various features of skimming flow regime. They considered two step sizes, six configuration, four discharge and four profile slopes (15, 30, 45, and 60 degrees) with various relative discharges to investigate the energy dissipation and discharge coefficient. Their results indicated that the discharge coefficient and energy dissipation have inverse relationship. Also by increasing the relative discharges, the energy dissipation decreased and discharge coefficient increased. Dursun & Ozturk [20] studied the energy dissipation ratio and inception point location of stepped spillways with and without end sills using computational fluid dynamic (CFD) methods. Flow characteristics and air inception points were determined for slope angles of 30°, 40° and 50°. It was found that the length of the non-aerated flow region was closely related to energy dissipation. In addition, the flow characteristics of the stepped spillways can be reliably determined by using CFD analysis. Bai et al. [4] studied the pressure distributions of three types of stepped spillways with different horizontal face angles in fully developed skimming flow regions. They used horizontal surfaces of V-shaped steps and inverted V-shaped steps. They concluded that negative pressures on the vertical surfaces of V-shaped, inverted V-shaped and traditional stepped spillways occurred near the sidewalls, near the axial plane and along the entire cross-section, respectively. For all the stepped spillways studied. They found that the minimum pressure decreased with increasing Froude number and increasing absolute values of the horizontal face angles.

Chinnarasri et al. [21] simulated numerically the flow behaviour through smooth and stepped spillways using a multiphase flow model with the realizable k-ε model. Bayon et al. [22] developed the numerical models of the flow in the non-aerated region of stepped spillways using diverse turbulence closures and discretization schemes implemented in two CFD codes of Open FOAM and FLOW-3D. They employed partial VOF (Volume of Fluid) and "True" VOF (TruVOF) approaches to simulate the free surface. The standard, RNG and Realizable k-ω, in addition to the SST k-ω model, were used for turbulence closure. They concluded that the models with turbulence closures of the k-ω family provide nearly the same predictions for the mean flow velocity with maximum differences on average smaller than 1%. Regarding discretization schemes, the first-order upwind method provides predictions for the mean flow velocity which are not significantly different (within 6%) than those obtained with second-order counterparts. However, these differences can be larger when maximum values of turbulent kinetic energy (TKE) and dissipation rate of TKE at the step edges are compared. In spite of the fact that the TruVOF (FLOW-3D®) method does not account for the tangential stresses at the air-water interface, the differences in the tracking of the free surface position among this method and the Partial VOF method (OpenFOAM) were found to be smaller than 3% along the stepped spillway. Bai et al. [4] evaluated the pressure distribution of V-shaped stepped spillway using five turbulence models. They found a good agreement with physical values, however, the - model is slightly better than other turbulence models in simulating the pressure distribution of V-shaped stepped spillway. In addition, Wan et al. [23] evaluated the application of SPH method to investigate hydrodynamics and re-aeration over the stepped spillways. In the SPH method, the entrainment of dissolved oxygen (DO) was studied using a multiphase mass transfer SPH method for re-aeration. The numerical results were compared with the hydrodynamics data from Chanson and DO data from Cheng. Their simulation results showed that the velocity distribution and the location of free-surface aeration inception agree with the experimental results. Compared with the experimental results, the distribution of DO concentration over the stepped spillway is consistent with the experimental results. Their results showed that the two-phase DO mass transfer SPH model is reliable and reasonable for simulating the hydrodynamics characteristics and re-aeration process. Bai et al. [4] studied experimentally the flow pattern in a V-shaped stepped spillway. Their results indicated that the flow structure is completely different from that of a traditional stepped spillway. They used the k-ε turbulence model to investigate the flow structure.

Many previous studies have focused on stepped spillways, most of them considered the step size and arrangement, among other factors; few studies have investigated the variations in the step shape, especially, the step angle. Therefore, in this paper, the flow pattern of a stepped spillway with different step angle was numerically studied. Furthermore, important flow characteristics such as the water surface profile, location of free-surface aeration inception, Froude number at the spillway's toe, and pressure, flow velocity, air concentration and cavitation index were investigated based on a detailed comparison with a traditional stepped spillway. The results of the present study will help the engineers encountering with the design of the stepped spillways with high efficiency.

2. Materials and Methods

The geometry of the model includes a stepped spillway with the height of 4.5 m, crest length of 1.4 m, step height of 30 cm, step length of 50 cm and the number of 15 steps. To study the step angles, four different slopes of 0, 5, 10 and 15 degrees were created (Figure 1). Figure 1 shows the structure of the stepped spillway along with the location of free-surface aeration inception.

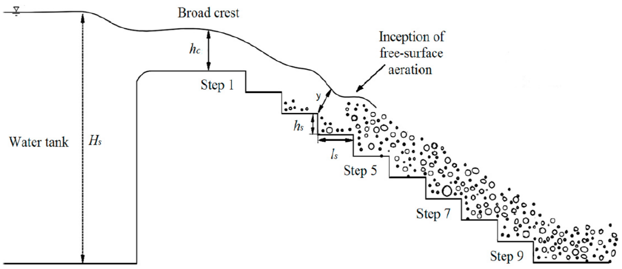

Figure 1. Structure of the stepped spillway [26]

In order to ensure the accuracy of the model results, one type of spillway made at the University of Ottawa's Lab Modeling was used and the results of a physical model were compared with a numerical model. To design the spillway, the USACE-WES design tables have been used in the Tabbara et al. (2005)'s study [16].

As an accurate and robust free-surface CFD package, FLOW-3D is an ideal tool for modeling the dam and spillway structures. FLOW-3D makes it easy to generate rating curves and detailed velocity profiles for complex spillways, including the effects of air entrainment and transport. This model also enables the quantification of structural stresses (pressure and shear forces) during normal and extreme operations, including transient effects due to gate operation. FLOW-3D is widely used to

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confirm the hydraulic performance of proposed spillway designs and to help professionals meet dam safety requirements. The realizable k–ε turbulence model developed by Shih et al. [24] was used in this study, because it is useful for simulating the flow over the stepped spillways [25-26]. In addition, the air-water interface was simulated by the volume of fluid (VOF) method.

3. Verification Using Experimental Data

After obtaining the outputs of the numerical model for the flow discharge and water surface profile over the spillway, the numerical results were compared with the experimental data of Tabara et al. (2005) [16]. In this study, the water surface profile was determined using the Volume of Fluid method (VOF).

The values of the root mean squared error and average relative error were calculated using the Eqs. 2 and 3. Accordingly, the values of RMSE and MARE were equal to 0.147 and 6.9%, respectively, indicating that Flow-3D model has an appropriate ability to estimate the hydraulic parameters of the flow over the stepped spillways.

$$\left(RMSE = \sqrt{(1/n)\sum_{i=1}^{n}\left(y_{i(\exp)} - y_{i(cal)}\right)^2} \right) \tag{2}$$

$$\left(\%MARE = 100/n\sum_{i=1}^{n}\left|\left(y_{i(\exp)} - y_{i(cal)}\right)/y_{i(\exp)}\right| \right) \tag{3}$$

4. Results and Discussion

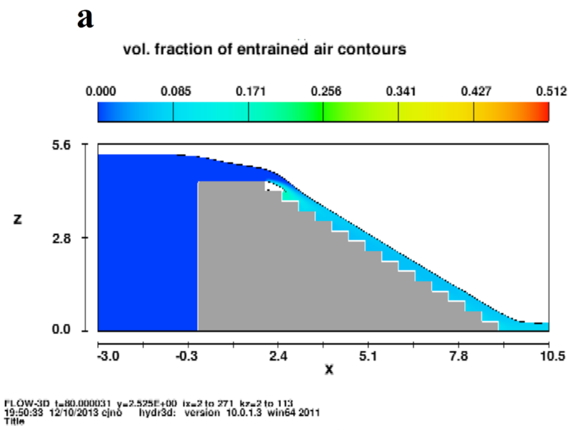
In order to investigate the effect of flow discharge, the simulations of this study were carried out with flow discharges (in unit width) of 1.157, 1.542, 1.928 and 2.313 m^2/s. Also, to ensure occurring the cavitation, the simulations were performed for two discharges of 6 and 12 m^2/s. In this study, in order to compare the results, the flow regime has been considered to be the same in all simulations. Table 1 shows the results of these analyses. According to the Essery and Horner (1978)'s [26] criteria, $(y_c/h > (y_c/h)_{onest}$, the flow regime is considered as Nappe flow in all the simulations.

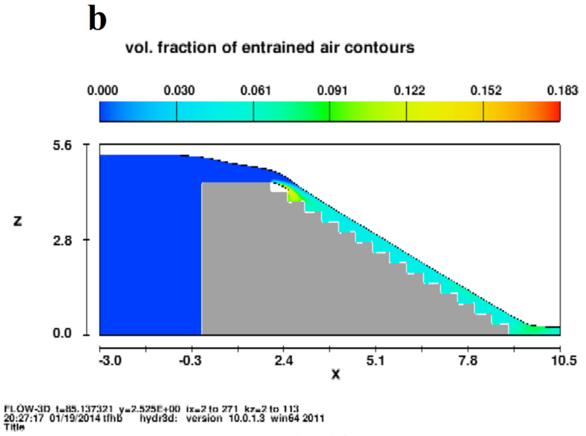
$$\left(\frac{y_c}{h}\right)_{onest} = 1.57 - 0.465\frac{h}{l} \tag{4}$$

Figures 2 and 3 show the results of simulations performed on the stepped spillway with varying step angles of 0, 5, 10, and 15 degrees and different flow discharges. In these figures the fraction of entrained air are observed over the stepped spillway.

Table 1. The results of the analysis of the flow regime over the stepped spillway in this study

Run No.	Θ (degree)	Q (m^2/s)	y_{cp} (m)	H (m)	L (m)	(y_c/H)	(y_c/H)c
1	0	1.16	0.51	0.30	0.50	1.72	1.29
2		1.54	0.62	0.30		2.08	1.29
3		1.93	0.72	0.30		2.41	1.29
4		2.31	0.82	0.30		2.72	1.29
5		6.00	1.54	0.30		5.14	1.29
6	5	1.16	0.51	0.34		1.50	1.25
7		1.54	0.62	0.34		1.81	1.25
8		1.93	0.72	0.34		2.11	1.25
9		2.31	0.82	0.34		2.38	1.25
10	10	1.16	0.51	0.39		1.33	1.21
11		1.54	0.62	0.39		1.61	1.21
12		1.93	0.72	0.39		1.86	1.21
13		2.31	0.82	0.39		2.11	1.21
14	15	1.16	0.51	0.43		1.19	1.17
15		1.54	0.62	0.43		1.44	1.17
16		1.93	0.72	0.43		1.67	1.17
17		2.31	0.82	0.43		1.88	1.17





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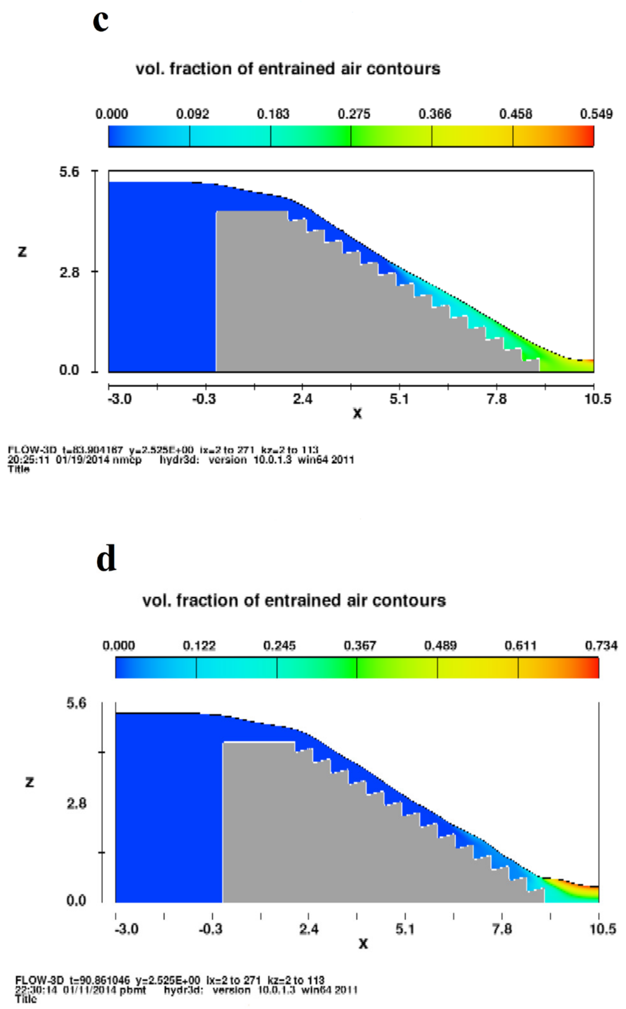
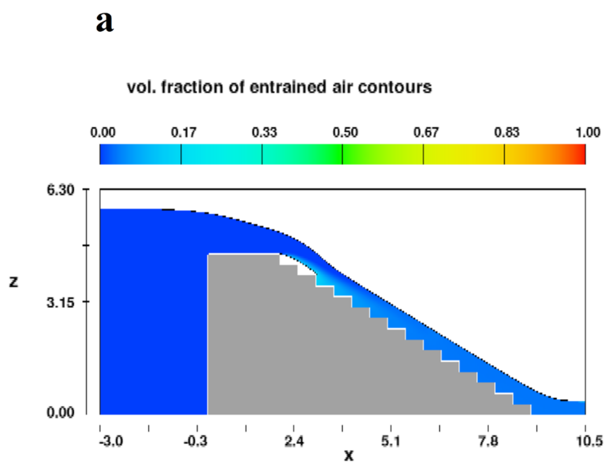


Figure 2. The results of numerical simulations for flow discharge of 1.157 m^2/s (a) θ = 0 (degree) (b) θ= 5 (degree) (c) θ = 10 (degree) (d) θ = 15 (degree)



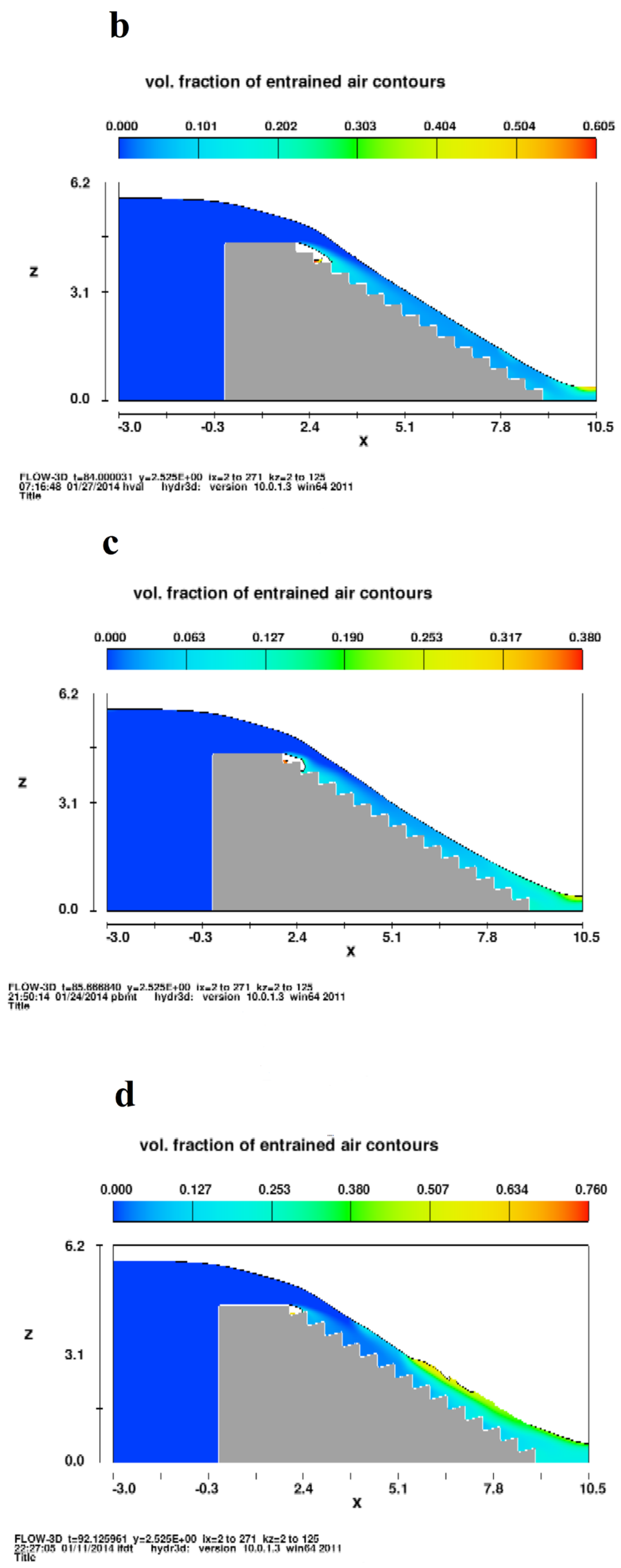


Figure 3. The results of numerical simulations for flow discharge of 2.313 m2/s (a) θ = 0 (degree) (b) θ= 5 (degree) (c) θ = 10 (degree) (d) θ = 15 (degree)

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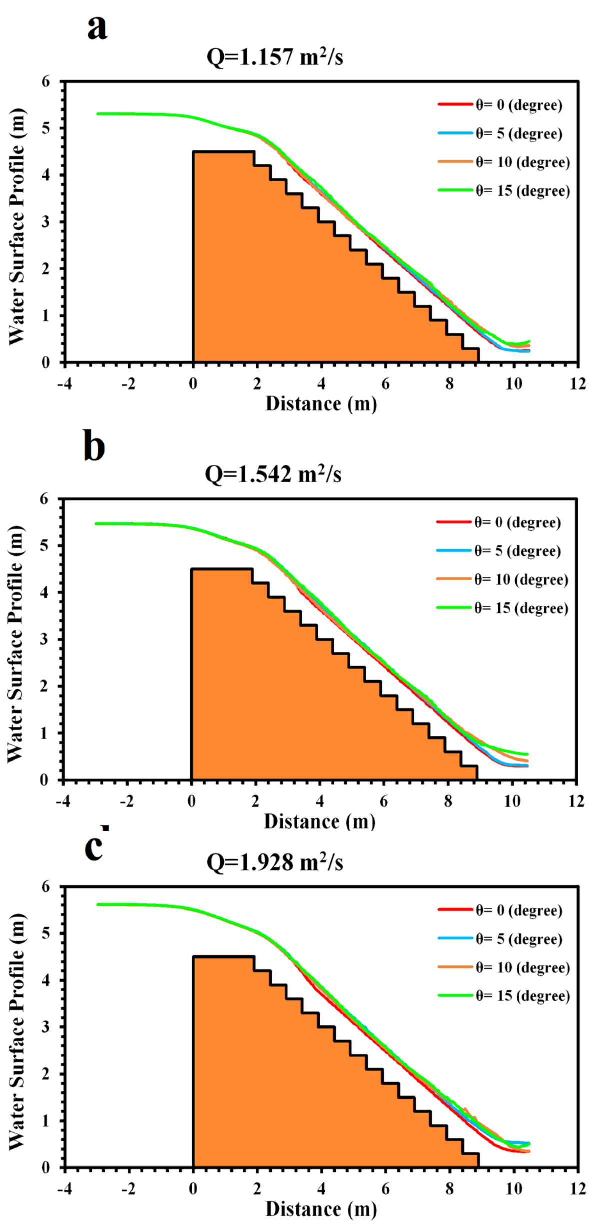
Using the simulation results, the effect of step angle are evaluated on such characteristics of the flow as the water surface profile, location of free-surface aeration inception, Froude number at the spillway's toe, and pressure, flow velocity, air concentration and cavitation index.

4.1 Water Surface Profile over the Stepped Spillway

The free surface of the flow are presented for the different step angles of 0, 5, 10, and 15 degrees in figure 4a and the flow discharge of 1.157 m^2/s (step angle of 15 degree) in figure 4b. As can be seen, the change in the step angles have a significant effect on the water surface profile in the middle steps and aerated region of the flow over the spillway. Increasing the step angles may increase the pseudo-bottom of the stepped spillway, and accordingly, this bottom will be rougher than the zero-degree step (Figure 4a-d).



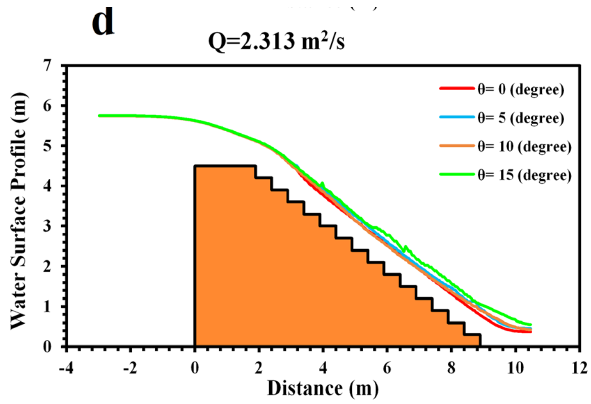
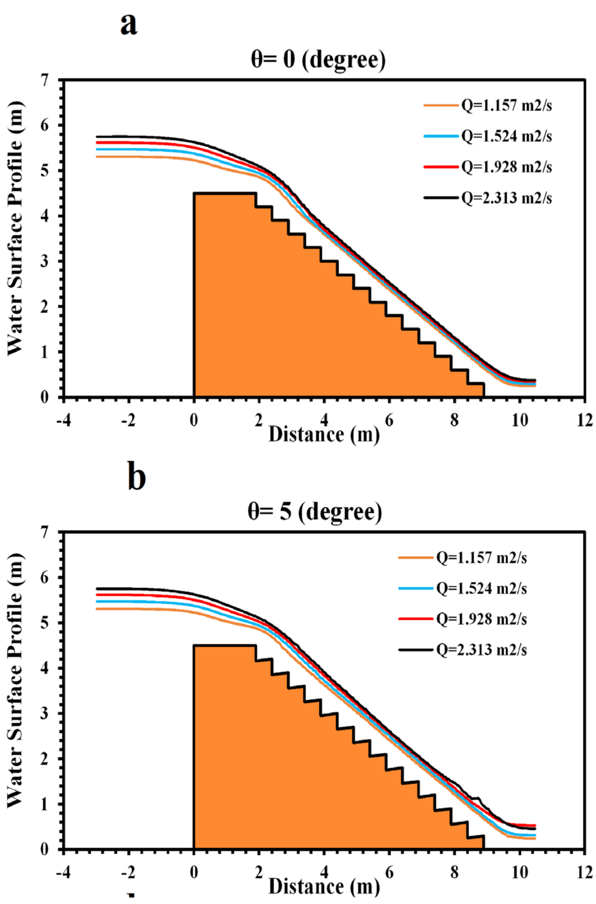


Figure 4. The effect of step angles on the water surface profile over the stepped spillway (a) Q= 1.157 m^2/s (b) Q= 1.542 m^2/s (c) Q= 1.928 m^2/s and (d) Q= 2.313 m^2/s

In addition, Figure 5 shows the simulated water surface profiles over the stepped spillway for a given step angle and different flow discharges.



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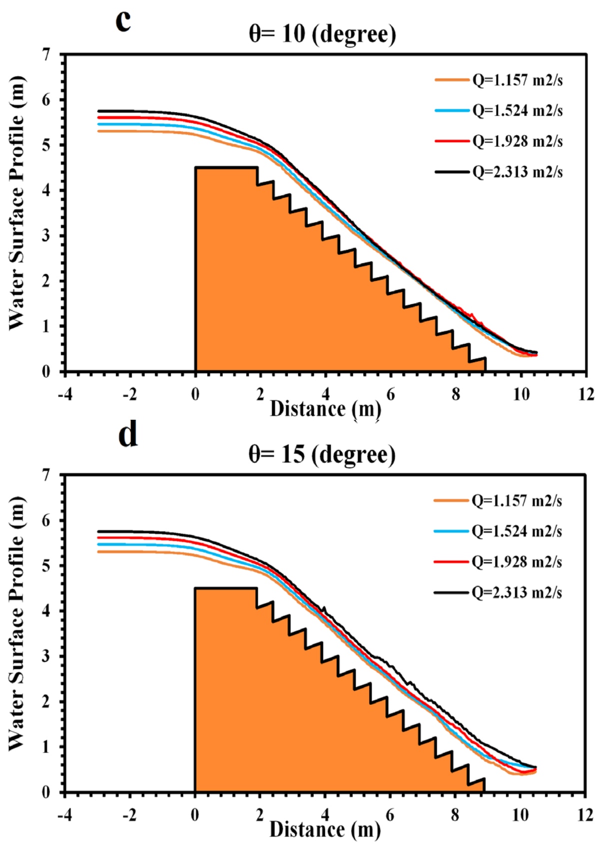

Figure 5. The effect of flow discharges on the water surface profile over the stepped spillway (a) θ = 0°, (b) θ = 5°, (c) θ = 10°, and (d) θ = 15°

4.2 Location of Free-Surface Aeration Inception

In this study, the location of free-surface aeration inception was recorded and determined by measuring the water surface level at the starting point of aeration along the centerline of the stepped spillway. The results showed that with the increase of the step angles, the aeration inception point is generally moved downstream. The reason for these changes is that by increasing the aeration inception, the effect of the steps is exacerbated as roughness elements and aeration process is intensified.

4.3 Froude Number at the Spillway's Toe

Given that the Froude number is an important factor in determining the dimensions of the stilling basin at the downstream of the spillway, the values of Fr_{dw} are plotted against the step angle for different flow discharges in Figure 6. As shown in this figure, by increasing the step angles from zero to 10 degrees, the Froude number does not change significantly, however, at the angle of 15 degrees, the Froude number decreases by about 42 percent. It can be stated that the angle, in which the lowest Froude number is observed, is about 15 degree for stepped spillway. In addition, it can be concluded that increasing the step angle may increase the dissipated energy at the downstream of the stepped spillways.

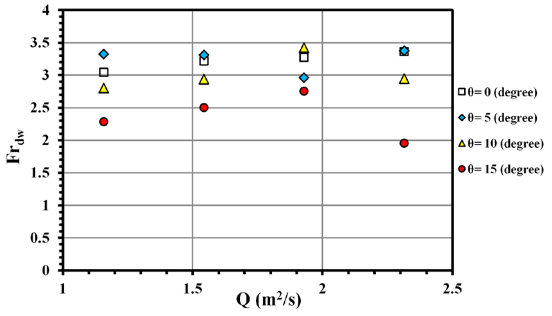

Figure 6. The values of Froude number at the toe of the stepped spillway for different flow discharges and step angles

4.4 Pressure Distribution

In order to investigate the effect of the step angle on the pressure variations on the spillway, the values of pressure over the spillway are plotted in Figure 7. As can be seen, the value of pressure on the initial steps is high, however, with increasing distance from the beginning of the spillway, the pressure decreases and reaches a constant value. In addition, increasing the flow discharge does not affect these changes, significantly.

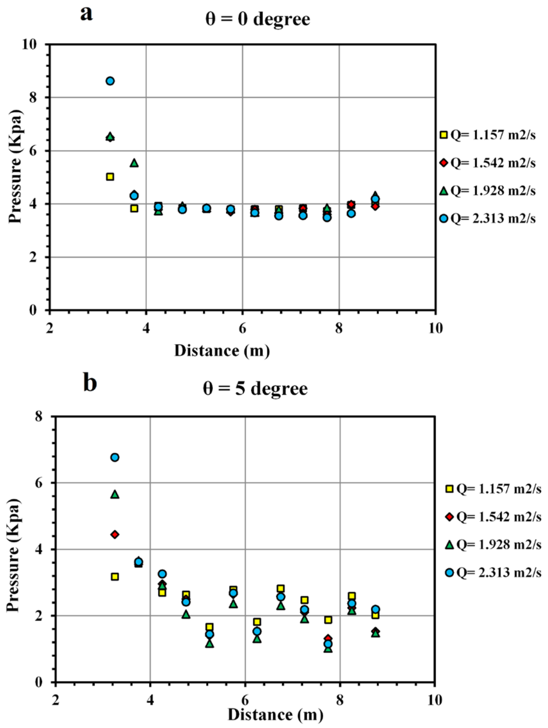

Figure 7. Pressure changes through the stepped spillway (a) θ = 0 (degree) (b) θ = 5 (degree)

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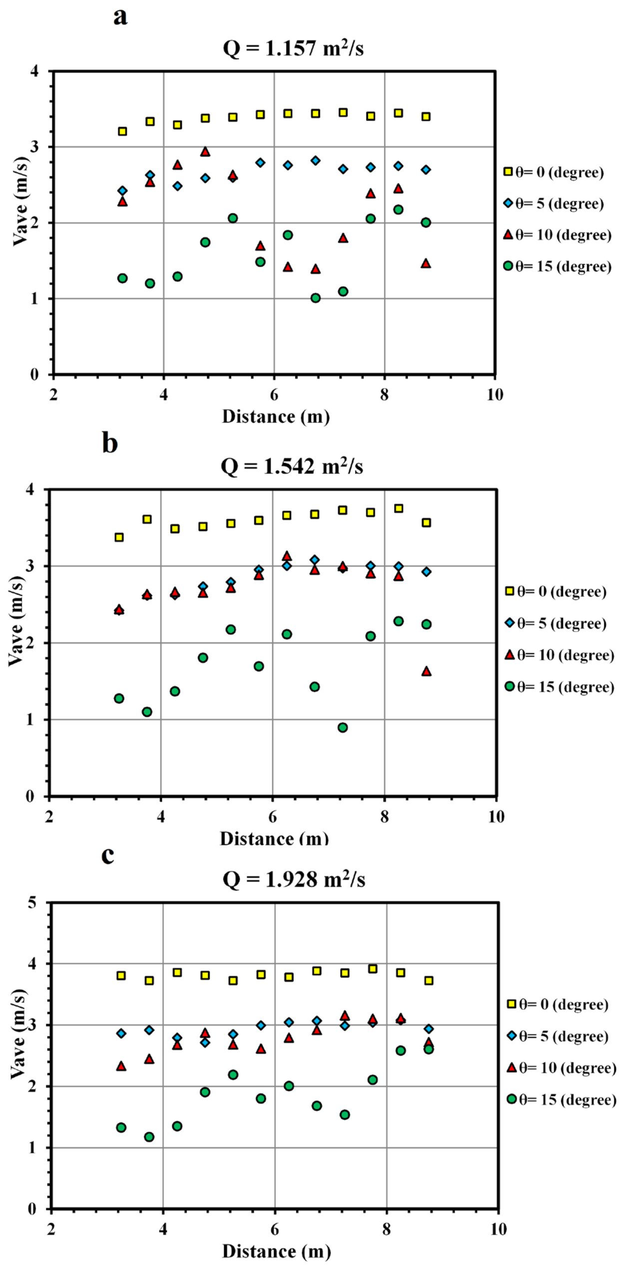
4.5 Flow Velocity

Figure 8 shows the variations of mean velocity against the distance from the beginning of the spillway by increasing the step angle for two flow discharges of 1.157 to 2.313 m^2/s. It can be seen that with increasing the step angle, the average flow velocity for a given flow discharge will reduce the velocity will be disrupted and non-uniform. It can be concluded that increasing the step angle may lead to an increase in the height of the bed roughness, which will cause a loss and thus reduce the flow velocity. It is also observed in this figure that with increasing the step angle increase the bed roughness of the pseudo-bottom of the stepped spillway, and therefore, increase the flow resistance compared with the zero-degree stepped spillways.



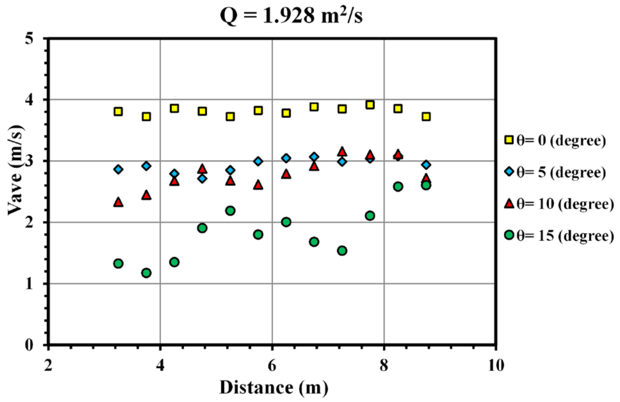
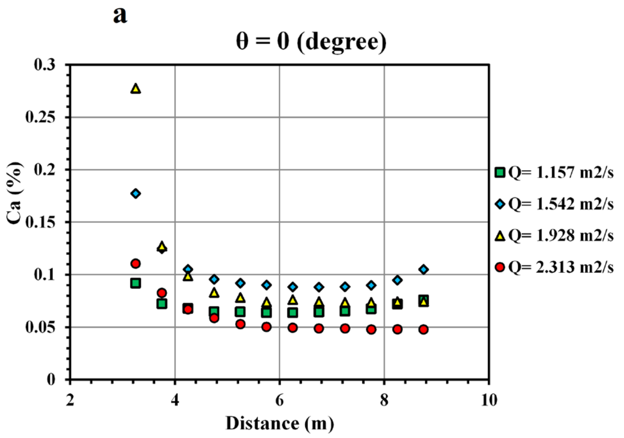


Figure 8. Changes in the mean flow velocity over the stepped spillway by increasing the step angles (a) Q = 1.157 m^2/s (b) Q = 1.542 m^2/s (c) Q = 1.928 m^2/s (d) Q = 2.313 m^2/s

4.6 Air Concentration

In order to investigate the effect of the step angles on the average air concentration over the stepped spillway, the air concentrations are plotted against the distance from the beginning of the spillway in figure 9. As can be seen, as the step angle increases, the air concentration in the initial steps decreases and for the end steps increases. The results showed that the average air concentrations increase significantly with the increase of distance from the beginning of the stepped spillway. So that the air concentration on the tenth step is about 4 to 8 percent and is 99 percent more than that of on the first step. This increase in air concentrations is associated with a reduction in the local pressure. It is also observed that the average air concentrations on the tenth step is about 36%, which increases by about 78% of the mean air concentration, compared to a concentration of 7.5% on the step with zero angle.



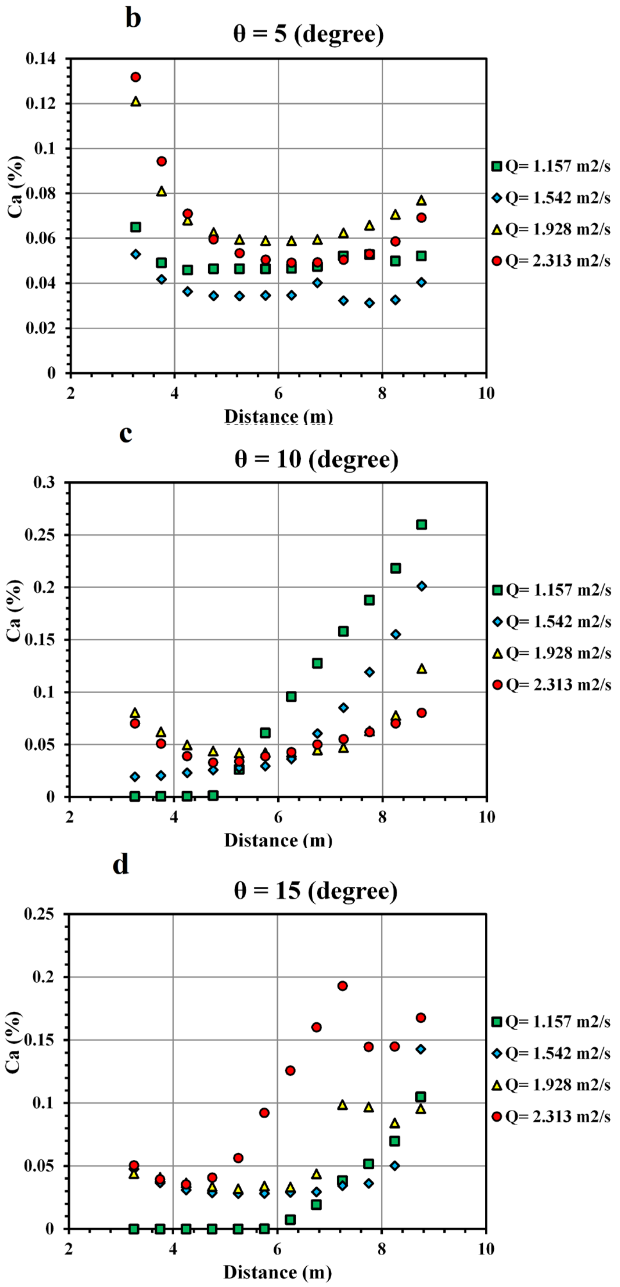


Figure 9. Variations of average air concentration over the stepped spillway by increasing step angles and flow discharges (a) θ= 0 degree (b) θ= 5 degree (c) θ= 10 degree (d) θ= 15 degree

4.7 Cavitation Index

In this study, the Falvey (1990)'s [14] criterion is used to investigate the occurrence of the cavitation phenomenon. Thus, the cavitation index was calculated using pressure and velocity values at different points in the stepped spillway. According to the Falvey (1990)'s [14] study, the cavitation will occur if the cavitation index is less than 1.8.

Figure 10 shows the values of the cavitation index against the distance from the beginning of the stepped spillway. As can be seen, the cavitation phenomenon does not occur in all the discharges and step angles, because this index is greater than 1.8. In general, with the changes in the index, it is possible to consider the occurrence of the cavitation phenomenon.

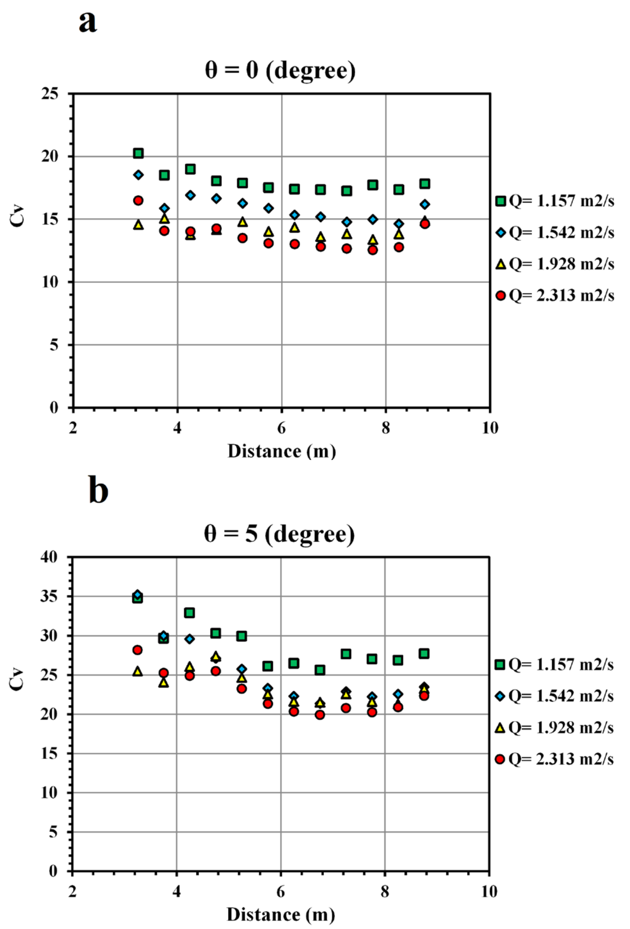


Figure 10. Variations of cavitation index over the stepped spillway

In the above figures, it is observed that as the flow discharge increases, the cavitation index decreases, therefore, the occurrence of the cavitation phenomenon can be increased. It can be concluded that by increasing the flow discharge and, as a result, the velocity, the amount of pressure decreases. Therefore, the value of cavitation index decreases.

In addition, figure 11 shows the variation of the cavitation index against the distance from the beginning of the stepped spillway by increasing the step angle. It can be seen that with increasing the step angle, the cavitation index increases. Accordingly, it can be concluded that increasing the step angle reduces the occur-

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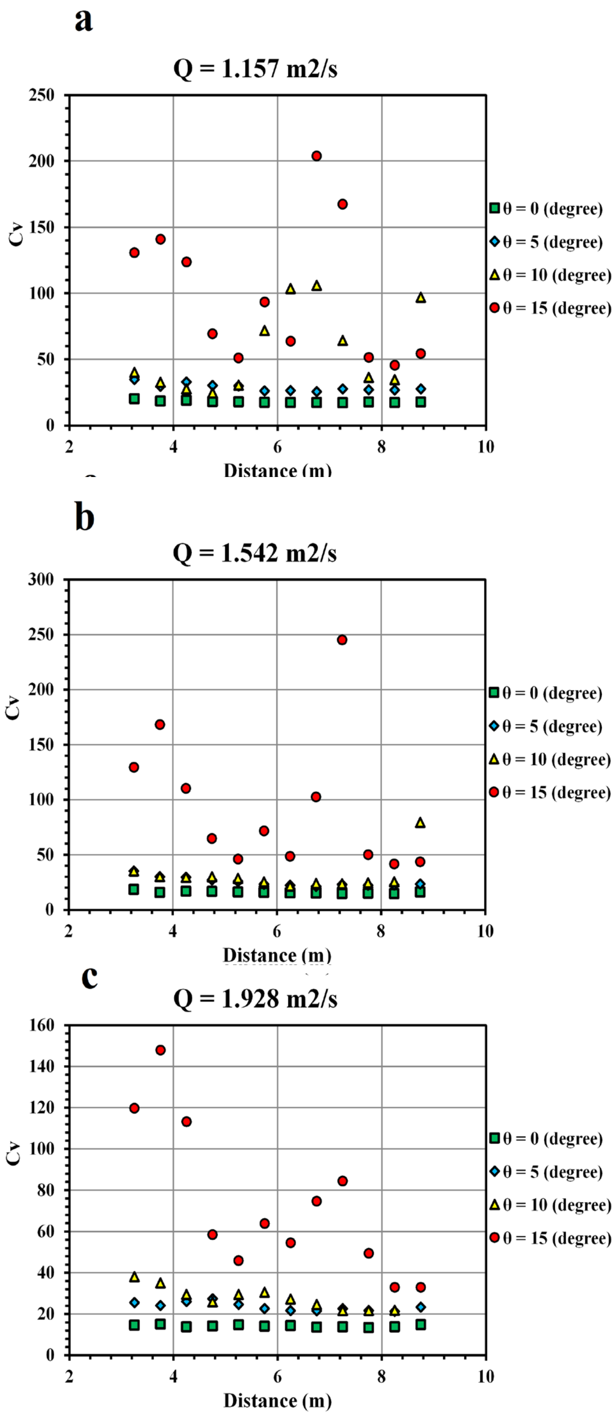
rence of the cavitation phenomenon. It can be said that with increasing the step angle, the flow velocity over the stepped spillway decreases and the flow pressure increases. These changes increase the amount of cavitation index.



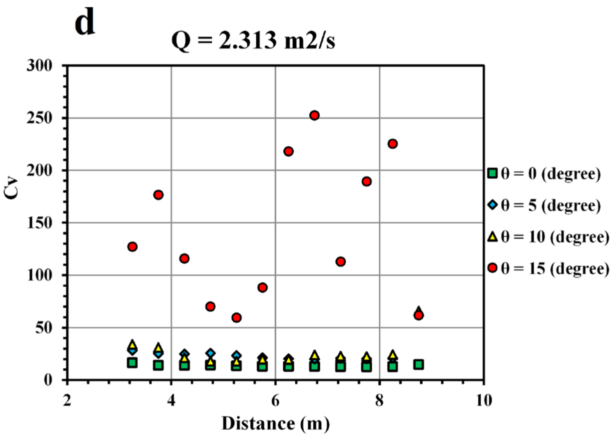


Figure 11. Changes in the cavitation index during the spillway by increasing the step angles (a) Q = 1.157 m^2/s (b) Q = 1.542 m^2/s (c) Q = 1.928 m^2/s (d) Q = 2.313 m^2/s

5. Conclusion

In this numerical study, the effect of step angle on the characteristics of the flow over the stepped spillways, consisting of water surface profile, location of free-surface aeration inception, Froude number at the spillway's toe, and pressure, flow velocity, air concentration and cavitation index, has been investigated. The major results of this study are:

(1) Validation of the results showed that the Flow3D model is very good in estimating hydraulic parameters, so that there was observed up to 7% error in the simulated water surface profile over the stepped spillway.

(2) The changes in the step angles have a significant effect on the water surface profile in the middle steps and aerated region of the flow over the stepped spillway.

(3) With the increase of the step angles, the aeration inception point is generally moved downstream. The reason for these changes is that by increasing the aeration inception, the effect of the steps is exacerbated as roughness elements and aeration process is intensified.

(4) By increasing the step angles from zero to 10 degrees, the Froude number does not change significantly, however, at the angle of 15 degrees, the Froude number decreases by about 42 percent.

(5) The value of pressure on the initial steps is high, however, with increasing distance from the beginning of the spillway, the pressure decreases and reaches a constant value. In addition, increasing flow discharge does not affect these changes, significantly.

(6) Increasing the step angle may lead to an increase in the height of the bed roughness, which will cause a loss and thus reduce the flow velocity.

(7) With increasing the step angles, the average air con-

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centration over the stepped spillway decreases.

(8) The cavitation phenomenon does not occur in all the discharges and step angles, because this index is greater than 1.8.

(9) As the flow discharge increases, the cavitation index decreases, therefore, the occurrence of the cavitation phenomenon can be increased. It can be concluded that by increasing the flow discharge and, as a result, the velocity, the amount of pressure decreases. Therefore, the value of cavitation index decreases.

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