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ARTICLE

Application of the Bayesian Statistical Approach to Develop a Stone Mastic Asphalt (SMA) Pavement Performance Model

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ABSTRACT

Stone mastic asphalt (SMA) has not been widely used in the pavement industry, and there are no detailed design specifications for this type of asphalt. Therefore, long-term behavior properties of this pavement type are not accessible widely, and no model has been established for SMA regarding its performance. The main purpose of this study was to incorporate expert experience (using the Markov-chain process) and data from field experiments to propose a model for SMA performance using the Bayesian approach. The implementation of these sources resulted in a well-organized method to develop a performance model for SMA pavements, which did not have a long-term data. Finally, a linear performance model was established to calculate the SMA service life. The service life of SMA can be predicted explicitly according to the developed performance model which has been validated using a new set of data.

1. Introduction

Stone Mastic Asphalt (SMA) was used in Germany in the 1960s, and its first technical instruction was published in 1984. Since 1990, it has been used prevalently throughout Europe and America and is now known as one of the most useful asphalt types, meeting technical requirements in all climates. The main purpose of this asphalt is to provide a deformation-resistant surface and overcome the rutting caused by the friction of studded tires on road surfaces [1]. SMA mainly consists of coarse aggregates, bitumen, filler, and fibers. This combination of components in SMA pavement has a high percentage of coarse aggregate, less empty space, and a high mastic of bitumen that complies with EN 13108. Fillers and fibers are usually added to provide suitable stability of bitumen [2]. Due to the high stone content and open-graded structure of SMA, heavy traffic loads are transmitted through the interlocking stone skeleton rather than through the mortar. Even though SMA is rut-resistant, due to high coarse aggregates content, SMA has inadequate performance in fatigue resistance because of fine aggregate content [1]. Per the State Department of Highways, SMA could have a longer service life (about 33–103%) than conventional dense-graded mixtures [3].

There are no detailed design specifications for SMA mixtures. The combination of the coarse aggregate and mastic composition is principally established by the selection of aggregate grading, the fraction of filler and a binder. In the Unites States, a few mixture design specifi-

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cations have been created for SMA and published by the US National Asphalt Pavement Association in Quality Improvement Publication (QIP) 122 [4]. In 1997, 86 SMA projects were studied to evaluate the SMA performance with respect to rutting, cracking, ravelling, and surface distresses. One of the most important findings from that study was that 85% of cases had high abrasion. Many projects (about 90%) reported rutting of less than 4 mm. In addition, SMA seemed to have higher resistance to cracking than dense mixtures; no evidence of ravelling was found on the SMA mixtures. Notably, fat spots appeared as the principal distress in SMA projects [5]. According to these previous studies, some of the characteristics of SMA, such as high rutting resistance or high durability, increased surface resistance to cracking and reduced noise pollution [6-10]. Regarding the life cost, Smith et al. used performance analysis to generate life-cycle models for SMA pavements. Exploiting the life-cycle models and historical-based best estimates of pay item unit costs, deterministic and probabilistic life-cycle cost analyses (LCCAs) were conducted during that research [11]. In order to reduce or eliminate expensive tests to investigate asphalt-aggregate mixtures for the design and control of flexible pavements, micromechanical modeling has been used in the field of asphalt technology. Buttlar and You used a Microfabric Discrete Element Modeling (MDEM) technique for the SMA microstructure [12]. This model was a simple extension of a conventional discrete element modeling (DEM) analysis, in which different material phases were studied with clusters of discrete and minute elements.

Qiang et al. assessed the SMA performance, such as Marshall modulus, mixture stability, residual stability, bending stiffness modulus, and dynamic stability to obtain the optimum content of materials in an asphalt mixture [13]. In another study, Al-Hadidy and Tan evaluated the behavior of the SMA made with the different levels of binder by testing their Marshall stability, tensile strength, tensile strength ratio, resistance to permanent deformation, flexural strength, and resilient modulus [14]. They reported the pavement design of SMA mixes according to the anisotropic elasticity analysis through finite-element simulation. SMA pavement smoothness is widely considered as the most substantial measure of pavement performance because it is most evident to the travelling public [15]. The international roughness index (IRI) is commonly used as an indicator of SMA pavement surface conditions. The IRI (i.e., expressed in meter per kilometer units) is a common method to show the reaction of a vehicle to roadway profile and roadway roughness. The Federal Highway Administration (FHWA) has required all states to report IRI values annually since 1990, which serves as an input to the Highway Performance Monitoring System (HPMS). Based on the specifications, greater IRI values represent rougher roads. The FHWA indicated thresholds for roadway smoothness based on rehabilitation decisions, among other pavement rehabilitation factors. There is a challenge that these tolerances often regress the smoothness of the ride as perceived by local drivers. Some states are in the process of determining IRI thresholds based on Present Serviceability Rating (PSR). The problem appears to be more severe in urban areas in which the dominant features are arterials, collectors, and local streets. Thus, the public's tolerance for pavement roughness is comparatively higher because of low vehicle speeds. The standard IRI value for good ride quality is 3.5 m/km. This IRI level is in agreement with drivers' acceptability [16].

The Pavement IRI can be evaluated during the service life. The initial smoothness (after the pavement construction) is one of the most valuable quality control criterion. It indicates the condition of construction and the roadway pavement future performance. Several aspects can influence the pavement initial smoothness, such as pavement design and construction operations. There is a wide range of variations in material properties and within pavement's structures. Initial smoothness is being used by state agencies to make sure that roadway pavement is meeting design specifications. A bad initial smoothness rating could cause the new roadway pavements to have failed quality testing that lead to a shorter service life. In addition, contractors using the initial IRI value as a control factor to identify and address issues quickly and cost-effectively [17].

Pavement performance can be stated based on distresses, such as cracking, rutting, and roughness. Moreover, subjective indicators including the PSR, which was implemented by the American Association of State Highway Officials (AASHO) during the Road Test, can be used for this purpose [18]. Many studies have developed pavement performance patterns to investigate the deterioration process thus far. These efforts were either the empirical or mechanistic method, or the two methods combined. Garcia-Diaz and Riggins established an empirical curve to predict the pavement deterioration [19]. Rahut devised a mechanistic approach to fit curves on the damage functions, such as rutting, fatigue cracking, and loss of pavement serviceability index (PSI) [20]. In another research, Paterson worked on a series of empirical performance models to analyze the Road Test data [21].

Although SMA has been extensively acknowledged in terms of laboratory performance and field performance, limited research studies have evaluated the performance of SMA in a systematic way regarding the identification of distresses over time. To evaluate the pavement performance for this purpose, long-term pavement monitoring and data collection are needed. Because SMA has not been used commonly in the past, long-term pavement performance data are often hard to find. Therefore, the Bayesian technique is offered as a method to solve this problem.

The Bayesian method has been extensively used in pavement performance modeling due to the fact that it presents a systematic method for the integration of new information with previous data to create new values for current results [22]. Thus, the Bayesian method offers the flexibility to integrate existing knowledge (experience) [23]. The main approach of using the Bayesian approach is to use data sets, such as experimental data and expert knowledge, to predict the posterior probabilities. Bayes' theorem outlines the transformation from prior probability (expert knowledge) to posterior probability (experimental data) [24].

In a theoretical research experiment, Park et al. modeled theoretical pavement distress using a sigmoidal equation with coefficients based on prior engineering knowledge in a Bayesian formulation [25]. Saliminejad and Gharaibeh combined spatial data analysis and Bayesian statistics to propose a computational technique for the imputation of missing or inaccessible Pavement Management System (PMS) data [26]. Hajek and Bradbury used the Bayesian model by combining information from field studies of existing projects with information elicited from experts. Hajek and Bradbury's model predicted the pavement deterioration in terms of a distress index, which was a function of age, the mixture, and traffic pattern [27].

Classical regression analysis is used on prior data (expert knowledge) to find the parameters coefficients. The variance between Bayesian regression and the classical regression is that the classical regression does not apply the prior data to estimating the coefficients. However, classical regression provides a good basis for use in the Bayesian regression. Bayesian regression is beneficial when a database is of low quality, or when unsatisfactory data are available. The prior data are strengthened with the experimental data. More experimental data make the posterior more conclusive. Thus, the reliability of the posterior approximations is greater than prior expert knowledge data. The final objective is to govern the posterior data of the coefficients.

The calculations used in the Bayesian regression directly parallel those for classical regression, and the subsequent linear regression equation is in the same way as the classical result [28]. The classical approach of estimation may encounter difficulties with small sample sizes or situations in which there is heavy censoring [29]. However, the Markov-chain sampling of Bayesian approach can make particular inferences with no resorting to asymptotic calculations [30]. Hence, in this article the Bayesian approach was used to estimate the parameters. The Bayesian regression approach modifies the classical regression to a general style that consists of prior information (expert knowledge), which is used by the Markov-chain, and experimental data (field investigation).

In the Markov-chain method, the pavement future conditions are predicted from the current pavement conditions [31]. A transition probability matrix (TPM) shows the level of probability that pavements in a present condition will move to future conditions. The weakness of the Markov-chain process is the reason why it is necessary to develop the TPM for each group (the combination of factors might affect the pavement performance). Pavement performance background might not be considered in the Markov-chain process due to the fact that the future conditions of SMA only depend on the present conditions. Thus, experimental field data have also been used in this research to develop a performance model. The result is the combination of the Markov-chain process and experimental data that are recognized as a well-organized method for forming a performance model applying the Bayesian approach for a SMA pavement. In a similar study, Han et al. compared various pavement materials in terms of a performance-oriented property management plan [32]. This research contrasted the life expectancy and uncertainties of SMA, Polymer Modified Asphalt (PMA), Rut-resistant Asphalt (RRA), Porous Asphalt (PA), and conventional Hot-Mix Asphalt (HMA). They employed the Markov mixture model with the hierarchical Bayesian estimation due to short time period and lack of time-series performance data. By having pavement performance data for each section, the Bayesian posterior probability method can be used to update the established TPMs [33]. This kind of modeling has been used by many researchers to improve deterioration forecasts for infrastructure, such as pavements [33-38].

2. Objective and Scope

Due to the very low implementation of SMA in the past and the consequent lack of enough information about this type of asphalt, the primary purpose of this study is to use the hybrid method (combination of Markov-chain and Bayesian) to present a performance model. Therefore, expert understanding (the Markov-chain process) and experimental data (SMA field investigation data) were integrated to propose a performance model for SMA by using the Bayesian approach. In this research, a survey

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was conducted on the Tehran-North Freeway to collect expert knowledge for creating a TPM. The four-lane Tehran-North Freeway has a length of 121 km and is intended to create a rapid connection between the central area of Iran. Volumes and quantities of earth works total 40,000,000 m^3. Also, tunnels and bridges are in total 44 km and 13 km, respectively. At the end of 2017, the construction of the first section of the Tehran-North Freeway project, connecting Tehran to the city of Chalus in the northern Mazandaran province, has progressed significantly and is now 83% complete. The preliminary estimate showed that designing and constructing the Tehran-North Freeway requires a 2.2-billion-dollar investment. Main input parameters for the design of this freeway were the Annual Average Daily Traffic (AADT) and the cumulative loading over the road design life (20 years), that is the vehicles number passing a specific point in either directions/ day taking into account the variation in the traffic flow throughout the year and the total number of axles for the same traffic volume. The AADT for the first year of traffic was 4,500 with 15% truck and 6% annual growth rate.

Field investigations have been done to control SMA pavement surface distresses. To measure the IRI, a Road Surface Profilometer (RSP), made by Dynatest Company, was utilized. This device collected data from the road surface, such as features of the route, curvature radius, longitudinal slope, geographic coordinates, roughness, skid resistance and ride comfort values. To increase the accuracy of the data collection, the road was separated into 10-meter intervals. According to the conducted studies, the IRI ranged from 1.52 to 1.83 mm/m. The IRI of SMA pavement has been described as a condition index. This condition index with the mean of expert responses has been employed to do the regression analysis and propose a performance model using the Bayesian regression approach. To verify the proposed model, SMA sections in the Tehran-Qom Freeway (also known as Persian Gulf Highway) were chosen to be compared with the sections from the North Freeway. The Persian Gulf Highway had the same conditions and predicted traffic patterns as that of the North Freeway. This highway accommodates a high level of intercity traffic of vehicles. Consequently, the data was received from the Traffic and Transportation Organization of Iran. The minimum length of each section was 400 m, and the IRI ranged from 1.43 to 3.91 mm/m. The provided model can be more comprehensive than other approaches considering some parameters affecting IRI such as climate impact and traffic spectrum, although they are not considered due to the lack of enough information.

The motivation of this study is to provide a performance model to enable users to predict the future conditions of pavements by using the present pavement condition. The benefits of this prediction are not only to estimate the remaining service life of SMA pavements, but also to have an organized PMS for rehabilitation and maintenance programs. Therefore, contractors can keep the quality of pavements in a suitable condition anytime in the service life by determining the number of years until rehabilitation is required. The results of study will be a serviceable and safe pavement condition in a cost-effective manner. The classical regression outcome (the data result) is not contrasted with the Bayesian results. If no further data was available, the Bayesian approach is preferable (prior information will be used in estimations). The principal purpose was to implement the observed data in the TPMs calibration using the Bayesian technique.

3. Methodology

This section explains how the methods of Bayesian and Markov-chain are integrated. The basic theoretical foundation that links a Markov-chain process and the Bayesian posterior probability method has been summarized in previous studies [39]. It is assumed that the prior distribution function of the TPMs belongs to a family of distributions that is closed under consecutive sampling [33]. The simple form of research methodology is shown in Eq. 1:

$$\left\{\begin{array}{c} Field\ Data\ (Experimental) \\ Expert\ Knowledge \xrightarrow{Markov\ chain} Prior\ Data \end{array}\right\} \xrightarrow{Bayesian} Posterior\ Result \qquad (1)$$

Based on the Bayesian method, there were two initial data sets: data obtained from experts' knowledge surveys and data obtained through experimental collection. These two sets of data were combined using the Bayesian technique, which was followed by the posterior result. The experimental data were used directly, which means that no operation was processed to utilize it. This data included the age of the pavement and the IRI in relation to that age. However, data obtained from experts' knowledge should not be used directly and needs a special operation to carry out experimental analysis. This process included the Markov-chain method. The combination of these two approaches took place in this phase. The data that came from experts included the initial probability condition vector and the TPM. Based on these data, the next state for prior data was determined. The basic data were required to start developing the model. The preliminary data were collected based on expert opinions and questionnaires for developing the TPMs. Before distributing the questionnaire, a training session was held. Then, the questionnaire was distributed among 18 experts, and the average of the expert ratings was considered as the associated elements of TPM. The ex-

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perts were chosen from 11 senior pavement engineers and 7 pavement field technicians who work in projects under the supervision of Ministry of Roads and Transportation. In the questionnaire, experts were asked about using IRI as an indicator of the quality of road surfaces of this type of pavement. The only criterion for rating IRI as an indicator of pavement was the age, regardless of the other factors due to the limitations of the statistical methods used in this study. In this study, 11 SMA sections in the Tehran-North Freeway were monitored, and their distresses were recorded. The IRI, showing the overall distress condition of SMA, was calculated for all sections. To apply the experts' knowledge and the Markov-chain approach, the collected data were analyzed by applying the Bayesian procedure. The Tehran-Qom Freeway (from 3rd km to 5th km) was selected to verify the model of IRI prediction of SMA pavements, which had the analogous conditions and traffic patterns. This case study zone was started after the junction with Road 51. It should be noted that ambient conditions and number of lanes were similar which could help the validation be suitable.

The study involved the roughness and distress analysis from pavement sections. Results were integrated with the software package for each section and inserted in the PMS database. All the data including pavement distress and roughness were accumulated yearly from 2007. Measurement methods have certainly potential errors that frequently increase the overall modeling error. Although it is not likely to generate a model with no error, the purpose of this study was to produce it with the least error.

Before using experts' opinions, the IRI was classified in different states. These states were described to the experts. Then, experts were asked to rate SMA pavement, which was recently established. By the experts' opinions, the probability of being in each state was specified. After this stage, the initial probability matrix was built. Moreover, the experts were asked what percentage was likely to remain in the state and what percentage is likely to go to the next state (worse). Finally, the TPM was built. The TPM matrix could be used with an initial probability condition vector to predict the next year probability vector, as presented in Eq. 2. Similarly, the probability vector was calculated for 5 years:

$$IRI(t) = IRI(0) \times \prod_{t=1}^{T} TPM_t \quad (2)$$

Based on the defined states and probability vector for each year, an indicator IRI was defined, specifically. This value was expressed by the expected value (EV), as shown in Eq. 3:

$$EV_{IRI}(t) = E_{IRI} \times IRI(t) \quad (3)$$

where $EV_{IRI}(t)$ is the expected value in the numeric index of stage t, E_{IRI} is the vector of IRI average in each state's interval, and IRI(t) is the probability vector in stage t. Then, two categories of data processing were ready to take the conclusive results in accordance with the Bayesian approach.

4. Description of Equations Parameters

The key objective of applying the Bayesian technique was to predict the parameters' regression coefficient in the performance models. To evaluate pavement performance, long-term pavement monitoring and data collection are necessary. Since this case study had less long-term pavement performance data, the Bayesian method was used to solve this problem. Based on the regression equation, IRI might be influenced by many factors, including age, traffic, and ambient conditions. But in this study, only the effect of age on the IRI index was evaluated. As a result, the classical regression in a matrix form was used. Based on the linear regression, there is a linear relationship between dependent and independent variables in terms of coefficients, as shown in Eq. 4. The prior equation is presented as the classical regression form:

$$Y = b_0 + b_1 X_1 + b_2 X_2 + \cdots + b_k X_k + e \rightarrow Y = b_0 + b_1 X_1 + e \quad (4)$$

where k is the independent variables number, Y is the distress indicator (dependent variable), X_1 is the pavement age (regression variable), e is a random error term, and both b_0 and b_1 are the regression coefficients. The ultimate purpose of this procedure is to obtain regression coefficients for the posterior model. The b0 coefficient shows the extent of the distress indicator in the first year (Stage 0) and the b1 coefficient represents the slope of the distress indicator versus the pavement age.

To obtain the regression coefficients for both data sets, Eq. 5 and Eq. 6 were used. For each data set, a precision matrix was defined to be used at later ages. The precision matrix for both prior and experimental data were displayed with A and H, respectively. The first phase in Bayesian regression is to calculate prior information, which has the similar format as the classical regression, given by Eq. 4.

Two forms of priors can be used: N-prior and G-prior. The main difference between the N-prior and the G-prior is the way that they present the prior precision matrix. The N-prior needs a variance-covariance matrix to show the prior precision matrix; however, the G-prior independent variable data are a set of independent variables (expert knowledge), similar to the data used to derive the classical

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regression and the prior precision matrix. The G-prior is generated based on the Gaussian linear models in the context of Bayesian factor analysis. In this study, the G-prior is chosen for the sake of straightforwardness as the prior type. To establish the prior precision matrix, the G-prior independent variables data and G-prior factors were needed.

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} \quad X_G = \begin{bmatrix} 1 & X_{11G} \\ 1 & X_{21G} \\ \vdots & \vdots \\ 1 & X_{n1G} \end{bmatrix}$$

$$Y = b_{pr0} + b_{pr1}X_1 + e_{pr}$$

Prior Data (5)

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} \quad X = \begin{bmatrix} 1 & X_{11} \\ 1 & X_{21} \\ \vdots & \vdots \\ 1 & X_{n1} \end{bmatrix}$$

$$Y = b_0 + b_1X_1 + e_{pr}$$

Experimental Data (6)

where Y is a vector of the dependent variable, X is the matrix of independent variables and n is the pavement age. Also, b_{pr} is the regression coefficient associated with the related variable and e_{pr} is the random error part for the prior. After obtaining the prior data, the ordinary least-squares regression was used to approximate the average of the coefficients, as shown in Eq. 7:

$$b = (X^tX)^{-1}X^tY \tag{7}$$

where X^t and X^{-1} represent the transpose and inverse of matrix X, respectively. By using Eq. 6, b as the vector of regression coefficient means is calculated, as shown in Eq. 8:

$$b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{bmatrix} \tag{8}$$

The G-prior factors are positive numbers that were exploited as weights in the prior precision matrix calculation to modify the effects of the prior precision matrix in the posterior computation. The G-prior factor is termed g, which had the value of 1. The prior precision matrix was determined using Eq. 9:

$$A = g(X_G^tX_G) \tag{9}$$

where A is the prior precision matrix, X_G is the G-prior independent variable matrix and X^t_G is the transpose of X_G. The regression coefficients, b_{pr}, is calculated using Eq. 10:

$$b_{pr}=A^{-1} (X^t Y) \tag{10}$$

where b_{pr} is the prior regression coefficients, X^t is the transpose of independent variables and Y is the dependent variable.

The next phase is evaluating the experimental data that is like the classical regression. In order to compute the precision matrix for the experimental data, H, Eq. 11 was used. The regression coefficients are calculated using Eq. 12:

H=g(X t X) (11)

b=H -1(X t Y) (12)

where b presents the regression coefficients, H is the precision matrix of experimental data, X^t is the transpose of independent variables, and Y is the dependent variable. Finally, by using the precision matrix A and H (prior and experimental data), the posterior precision matrix for the posterior mode is determined by the Eq. 13:

$$M=A+H \tag{13}$$

This equation is used in order to estimate the posterior results by combining the prior with experimental data. Thus, the posterior precision is given by the sum of the prior precision (A) and the experimental data precision (H), and the posterior mean is given by the sum of the prior data mean and the experimental data value, each weighted by their relative precisions. The posterior precision is the same as inverse variance weighting where the weights sum to one [23,40-42]. The posterior regression coefficients for the posterior mode were calculated using Eq. 14. Finally, given the regression coefficients, the final posterior model was developed:

$$b_{pos}=M^{-1} (Ab_{pr}+Hb) \tag{14}$$

where b_{pos} is the posterior regression coefficients, b_{pr} is the prior regression coefficients, and b is the experimental data regression coefficients.

5. Data analysis

5.1. Prior Data (Markov-Chain Method)

The classifications of states and desired ranges were defined in accordance with experts' knowledge. The TPMs for forming Markov-chain models were set up. The Bayesian regression method was used to indicate

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the needed prior information. The IRI was usually limited to a range of zero to 10 m/km. However, the IRI range was defined as 1.5 to 2.5. Because the modeling was designed for five years, the quality of SMA pavement, the experts' opinion and the performance of Markov were decisive in choosing this range for IRI. Another issue was how to divide the states. Since the stage was considered one year, the classification should be such that the possibility of loss of IRI in all the stage existed to produce the TPM with optimal performance. The defined states in accordance with experts' knowledge are presented in Table 1.

Table 1. Defined intervals for IRI to describe the SMA pavements conditions

State	1	2	3	4	5	6	7	8	9	10
IRI	1.5-1.6	1.6-1.7	1.7-1.8	1.8-1.9	1.9-2.0	2.0-2.1	2.1-2.2	2.2-2.3	2.3-2.4	2.4-10

The survey was distributed to 18 experts, and the average of the answers was reported as an array of TPMs. Experts responded to another question by proposing the IRI index for pavement in the first year of operation and prior. The respondents were not asked to insert probability values in the TPM directly because that putting the probability values in TPMs with no clarifications would result in an error due to being subjective. Thus, an illustrative method was used to inform respondents about states. Authors provided pictures for each state according to the current performance. Then, experts were asked to express their opinions about what would be the state of the given pavement after each year. The probability condition vector was calculated by the authors. By summing up the experts' responses, the second step of the Markov method, the initial probability vector production was completed as shown in Table 2. This means that there are 60%, 25%, 10% and 5% chances that a new section immediately after installation will be in condition 1, 2, 3 and 4, respectively.

Table 2. The initial probability vector of IRI

State	1	2	3	4	5	6	7	8	9	10
Initial Probability Vector	60	25	10	5	0	0	0	0	0	0

The most important part of this process was producing the TPM matrix. This matrix must indicate the actual pavement performance, which was defined based on the number of states. The matrix was a 10×10 square matrix. Each element of this matrix was gathered from experts' knowledge and experience. Elements on each row of the matrix represent the probability of remaining in the same state or switching to another state. The most likely change in the pavement condition was switching from the state (i) to state (i + 1). In addition, the possibility of changing the status for two steps was considered; however, as can be seen in Table 3, switching for three steps or more was unlikely, and no possibility was considered. Another notable point of the TPM is the triangular shape of the matrix due to neglecting repairs, maintenance, and improvement of the pavement condition. In comparison, matrix elements increased in rows line by line. This increase reflects the fact that the high IRI and the loss of pavement quality decreased the rate of distresses. Therefore, this matrix results in the probability of remaining in the current condition for pavement increase.

Table 3. Transition probability matrix based on expert knowledge

states	1	2	3	4	5	6	7	8	9	10
1	0.2	0.7	0.1	-	-	-	-	-	-	-
2	-	0.2	0.7	0.1	-	-	-	-	-	-
3	-	-	0.25	0.65	0.1	-	-	-	-	-
4	-	-	-	0.25	0.65	0.1	-	-	-	-
5	-	-	-	-	0.3	0.65	0.05	-	-	-
6	-	-	-	-	-	0.3	0.65	0.05	-	-
7	-	-	-	-	-	-	0.35	0.65	0	-
8	-	-	-	-	-	-	-	0.4	0.6	-
9	-	-	-	-	-	-	-	-	0.5	0.5
10	-	-	-	-	-	-	-	-	-	1

With the production of TPM, the probability vectors of each stage were calculated easily. Thus, the initial probability vector was multiplied by the matrix TPM to generate the probability vector of stage 1. By repeating this process, four other probability vectors were obtained as shown in Table 4.

Table 4. Probability Vectors in Each Stage

	stage 0	stage 1	stage 2	stage 3	stage 4	stage 5	IRI
state 1	60	12	2.4	0.48	0.09	0.019	1.55
state 2	25	47	17.8	5.24	1.38	0.344	1.65
state 3	10	26	40.6	22.85	9.42	3.336	1.75
state 4	5	10.25	24.16	34.21	23.92	12.24	1.85
state 5	0	4.25	10.54	22.93	31.4	25.91	1.95
state 6	0	0.5	3.94	10.45	21.45	29.24	2.05
state 7	0	0	0.54	3.27	9.083	18.69	2.15
state 8	0	0	0.02	0.55	2.873	8.12	2.25
state 9	0	0	0	0.02	0.341	1.89	2.35
state 10	0	0	0	0	0.008	0.17	2.45
IRI	1.61	1.69	1.78	1.87	1.95	2.02	-

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In Table 4, one row and one column are characterized as IRI. The IRI column represents each state that is calculated by taking the average of the upper and lower bounds. The IRI row was calculated by multiplication of IRI in each column to the probability vector of each stage. Each element represented the IRI for the same stage. These are the final results obtained from the Markov method, which are predicted IRI values for 5 years based on experts' opinions and inputs for the Bayesian method.

5.2. Explaining Prior Data (Expert Knowledge)

Using Bayesian methods, two data sets obtained from experts were combined. In the beginning, the data from the experts was considered as the prior data, and the regression parameters were calculated (Eq. 15). The G-prior independent variable (X_G) is a set of data similar to data used for the regression analysis. The average (expected value) of IRI of SMA pavement was calculated using Eq. 1, as shown in Table 5.

Table 5. SMA sections IRI

Age	IRI
0	1.61
1	1.699
2	1.787
3	1.871
4	1.952
5	2.029

$$Y=b_{pr0}+b_{pr1}X_1 \tag{15}$$

$$X_G=\begin{bmatrix}1&1\\1&2\\1&3\\1&4\\1&5\end{bmatrix} \qquad Y=\begin{bmatrix}1.699\\1.786\\1.871\\1.952\\2.02\end{bmatrix}$$

Then, the precision matrix A for expert data was calculated using Eq. 9:

$$A=\begin{bmatrix}5&15\\15&55\end{bmatrix} \qquad A^{-1}=\begin{bmatrix}1.1&-0.3\\-0.3&0.1\end{bmatrix}$$

Now the regression coefficients of prior data were estimated by incorporating both SMA sections' features and their IRI, as presented in Eq. 10:

$$b_{pr}=\begin{bmatrix}1.6196\\0.0828\end{bmatrix} \tag{10}$$

To consider the variability and reduce the uncertainty of data, the best distribution function that fits the data was selected based on the Chi-squared test as depicted in Figure 1 and used to carry out the calculations.

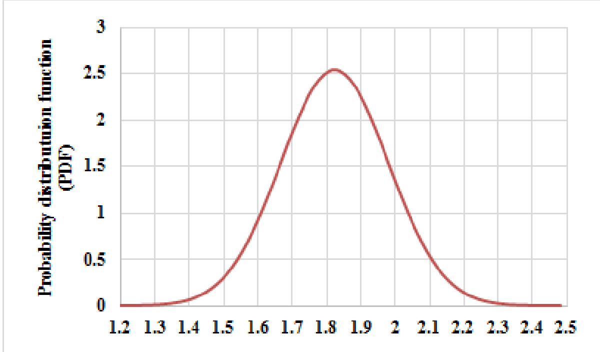

Figure 1. Prior data probability distribution function

The regression coefficients were obtained by using information from experts and the experimental information gathered was evaluated to develop a model.

5.3 Experimental Data

According to the information provided from 11 sites in the Tehran-North Freeway, the experimental data are shown in Table 6, which indicates the pavement conditions from the perspective of the IRI. For each site, one test was done every 500 m, and the IRI was recorded. By using Table 6, the matrices x and y were formed. Then, the coefficient matrix for the section was generated.

Table 6. Experimental data

Age	0	0.9	1.5	2.1	2.65	3.05	3.5	3.8	4	4.3	4.5
IRI	1.52	1.56	1.59	1.65	1.69	1.72	1.73	1.76	1.78	1.81	1.83

The X matrix is a dataset in form of the data used for the regression analysis.

$$cX_G=\begin{bmatrix}1&0\\1&0.9\\1&1.5\\1&2.1\\1&2.65\\1&3.05\\1&3.5\\1&3.8\\1&4\\1&4.3\\1&4.5\end{bmatrix} \qquad Y=\begin{bmatrix}1.52\\1.56\\1.59\\1.65\\1.69\\1.72\\1.73\\1.76\\1.78\\1.81\\1.83\end{bmatrix}$$

The X is needed to calculate the H. The precision matrix H was computed using Eq. 11 as follows:

$$H=\begin{bmatrix}11&30.3\\30.3&105.2\end{bmatrix}$$

The coefficient of an independent variable together with an intercept is given as Eq. 12:

$$\bar{b}=\begin{bmatrix}1.503\\0.0696\end{bmatrix}$$

The experimental data, as prior data, is summarized by plotting the probability distribution function. Figure 2 shows the probability distribution function for both prior data and experimental data in which a Weibull Distribution Function fitted to the data. The best distribution functions fitted to each data set was chosen based on the chi-square technique. This method is one of the best methods in matchmaking between the distribution functions and raw data. This technique compares the similarity between the frequency of data in the same data range from the raw data and distribution functions [43]. A distribution function with the highest similarity was selected as the best-fitted function.

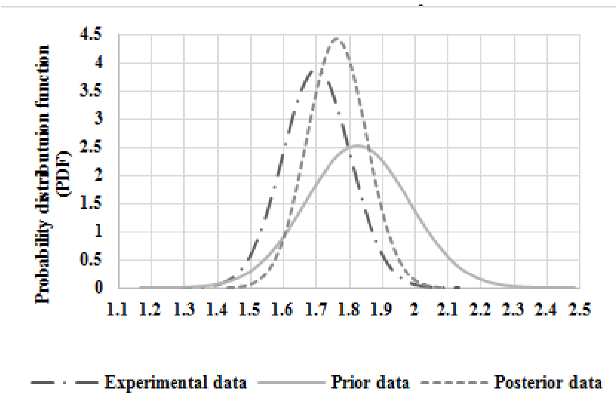

Figure 2. Prior and experimental data distribution functions

As can be seen in Figure 2, the experimental data had a smaller value than the prior data. This result indicates that the experimental data showed smaller degradation rates for IRI in SMA pavements rather than prior data. The probability distribution function for experimental data was tighter than prior data, which indicates that the results deviation of experimental data was less than that of prior data.

5.4 Posterior Data Calculations

The precision matrix for both parts of prior data and field data were calculated. The regression coefficient matrix was calculated from the two matrices, as shown in Eq. 16:

$$M = A + H = \begin{bmatrix} 5 & 15 \\ 15 & 55 \end{bmatrix} + \begin{bmatrix} 11 & 30.3 \\ 30.3 & 105.2 \end{bmatrix} = \begin{bmatrix} 16 & 45.3 \\ 45.3 & 160.27 \end{bmatrix}$$

$$b_{pos} = M^{-1} (Ab_{pr} + Hb) \quad (16)$$

$$b_{pos} = \begin{bmatrix} 1.529 \\ 0.078 \end{bmatrix}$$

Figure 2 shows that the probability distribution for the posterior estimate of b was tighter than both the prior and the experimental data. This was likely realistic, because the prior and experimental data endorsed each another with a similar estimate of the mean of b. Figure 2 corroborates the advantage of using the Bayesian regression method in which enough prior data was presented. By adding long-term performance data, the posterior continued to become definitive (i.e., more confidence in its estimate of b) [28]. Thus, the justification for the use of Bayesian regression can now be addressed. The contrast between classical regression and Bayesian regression was in taking advantages of prior information for estimating the parameter b. If no additional data were available, using the Bayesian approach will be recommendable.

6. Conclusion

Due to lack of sufficient information and performance data about SMA pavement, whether statistical information or in the context of the experts' knowledge and experiences in the field, proposing a distress model for pavement performance would not be reasonable. Since the Bayesian method has been developed for such cases, this method was used by combining two models based on the field data and prior data to develop a new model, as shown in Eq. 11:

$$IRI=1.529+0.078\times Age$$

With a steady increase in the rate of IRI index, the prior data were generated by using the TPM. It is necessary to assess performance models by using prior, experimental, and posterior data. Figure 3 displays how the prior data model had the largest deterioration rate, while the experimental data model had the smallest deterioration rate. The posterior model was in the middle. As displayed in Figure 3, the posterior line represents the IRI rate increase based on the new model posterior.

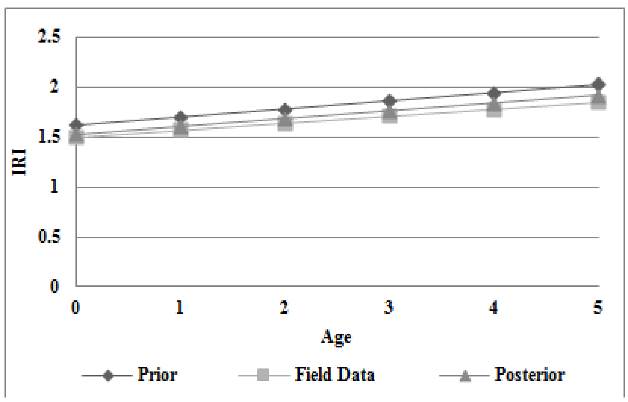

Figure 3. Performance models for prior, field, and posterior data

To validate this model, IRI data from SMA sections in the Tehran-Qom Freeway were deployed to verify

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the model in this study. Figure 4 shows the relationship between the actual and predicted IRI for this freeway. From this figure, it can be noticed that the IRI never surpassed 2 in the first 5 years that represents the segments of the road had an acceptable IRI. The coefficient of multiple determinations (R^2) was 98.3%, which reflects sufficient evidence of dependency relationship between the measured IRI and predicted values. Although Figure 4 shows that the IRI value increases by increasing pavement age due to the impact of deterioration elements, it shows a reasonable distribution of data points around the correlation line. Moreover, it can be concluded that the combination of the Bayesian and Markov-chain methods is appropriate for cases that require modeling with incomplete data.

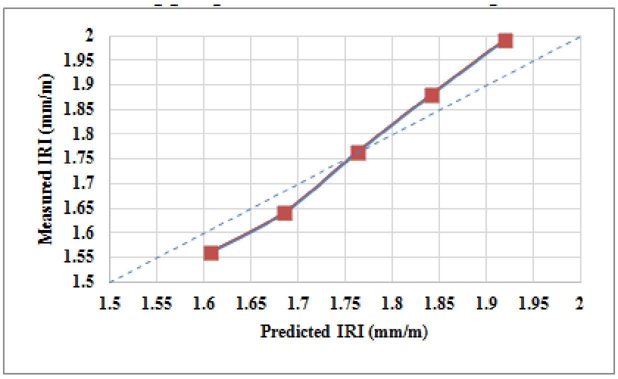

Figure 4. Predicted and measured IRI relationship

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