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Analytical Model of Nonlinear Semi-rigid Frames Using Finite Element Method

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ABSTRACT

Performance-based design for a constructional steel frame in nonlinear-plastic region requires an improvement in order to achieve a reliable structural analysis. The need to explicitly consider the nonlinear behaviour of structures makes the numerical modelling approach much more favourable than expensive and potentially dangerous experimental work. The parameters considered in the analysis are not limited to the linear change of geometry and material yielding, but also include the effect of large deformations, geometrical imperfections, load eccentricities, residual stresses, strain-unloading, and the nonlinear boundary conditions. Such analysis requires the use of accurate mathematical modelling and effective numerical procedures for solving equations of equilibrium. With that in mind, this paper presents the mathematical formulations and finite element procedures of nonlinear inelastic steel frame analysis with quasi-static semi-rigid connections. Verification and validation of the developed analytical procedures are conducted and good agreements are obtained. It is an approach that enables the structural behaviour of constructional steel frames to be traced throughout the entire range of loading until failure. It also provides information on the derivation of the structural analysis by using finite element method.

1. Introduction

In view of the advantages possessed by the semi-rigid connections in constructional steelwork, there has been a substantial volume of research studying such behaviour[1]. With the availability of computer applications and advances in numerical analysis, interest in studying semi-rigid connections has been widespread. Structural analysis coupled with nonlinearities and connection flexibility is essential for a reliable design of steel structures. The possible sources of nonlinearities when applying loads are shown in Figure 1. Seen below is the progressive collapse of a structure; a large deformation may exist and the problem is exhibited on the convergence in the finite element analysis.

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The finite element approach has been the most versatile and widely used numerical method in solving complex problems. This is usually done by subdividing the real structure into a finite number of elements. In each of the finite regions, the behaviour of the element can be described via a set of shape functions. These shape functions, with the connection behaviour already taken into account, uniquely define the state of displacements within the finite element in terms of nodal displacements which also ensures continuity and equilibrium throughout the whole system [2].

The finite element method has been used in structural analysis for static and dynamic actions towards structural system in buildings. There are many references discussing the structural analysis with the finite element method such as concrete structures [3], masonry structures [4], stainless steel [5], composite structures [6,7], and connection [1].

Performance based design for steel frames in the plastic region has been studied [8]. Stepping into a performance based design, consideration of the nonlinearities and rigidity of connection became important to maintain reliable structural behaviour. This paper discusses the numerical derivation approach in steel structures analysis, involving nonlinearities and connection flexibility. The parameters considered in the analysis are the effect of large deformations, geometrical imperfections, load eccentricities, residual stresses, strain-unloading, and the nonlinear boundary conditions. Moreover, validations have been conducted for the column behaviour and semi-rigid frame analysis. Theoretical column behaviour and a previously developed program, INSTAF [9], are used to compare with the developed analytical model where the columns exhibit nonlinearities as a result of its structural behaviour. Furthermore, load-unloading behaviour of material properties for a semi-rigid frame is applied to the analysis for the comparison between developed model and INSTAF. The developed analytical model should achieve a positive correlation in these comparisons where it can be applied in the performance based plastic design of steel structures.
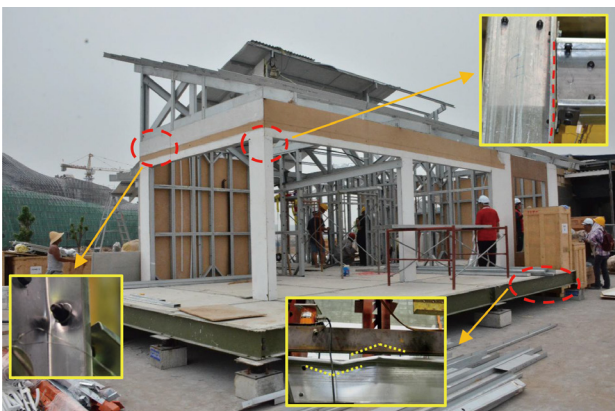

Figure 1. Possible sources of nonlinearities for a steel structure

2. Finite Element Procedures

The analytical model is developed for steel structural analysis with semi-rigid connections. Standards or codes are emphasised on elastic or amplified elastic at plastic region on sway design of moment frames that may underestimate the sway capacity with the amplified factor. When obtaining reliable structural behaviour under actual loadings, there are several features that need to be considered in the structural design. Geometrical imperfection, geometrical nonlinearity, material nonlinearity (large plastic ductile deformation) and rigidity of connection affect the P - δ and P - Δ behaviour in sway frame analysis.

One of the most valuable applications of the finite element method is when the load-deflection response is nonlinear. There are generally three different sources of nonlinearity. The first is material nonlinearity, which arises from a nonlinear constitutive relationship (i.e. the relationship between stress and strains) and is associated with progressive material degradation. The second is geometric nonlinearity which results from significant changes in the structural configuration and is often associated with large displacements. These displacements are continually monitored and the stiffness of the structure is updated as displacements change. The third source of nonlinearity comes from the boundary conditions when the restraint changes during the loading. All of these three sources of nonlinearity are applicable and need to be considered in the current study. An elastic-perfectly plastic stress-strain curve is used to represent the material properties of the steel. As the study analyses the behaviour of the frames up to failure load, the geometric nonlinearity is of prime importance. Finally, the highly nonlinear connection behaviour justifies the need to consider nonlinear boundary conditions.

Another important point that needs to be considered is the inelastic behaviour. A structure is said to behave inelastically, if when unloaded, it may follow a different path pattern to the loading path. This could be due to the nonlinear inelastic response of the elastic-perfectly plastic material and the connection modelling as a result of cyclic loading.

In nonlinear analysis, it is not possible to directly obtain an internal stress distribution, $\{\sigma\}$, which is in equilibrium with the applied loading, $\{F\}$. This is due to the changes in stiffness from material, geometric and boundary condition effects that cause an out-of-balance vector, $\{\psi(\delta)\}$, which is a function of displacement. The objective of the non-linear analysis is therefore to calculate a situation of static equilibrium by eliminating the out of balance vector. Traditionally there are three methods

that have been used to solve nonlinear problems, namely the iterative method [10-13], the incremental method [14-16] and the incremental-iterative method [17-18]. Several modified methods have been presented and their respective advantages and disadvantages have been reviewed by Desai and Abel [17], Akryod [19], Goverdhan [20], Narayanan [21], Lee et al. [10] and Liu et. al. [22].

The incremental-iterative, (also known as the Newton-Raphson method) was adopted as the non-linear solution procedure. When using this method, the loading is applied in increments and iterative corrections are performed within each increment to restore equilibrium. The nonlinear convergence criteria are used to monitor the extent of non-equilibrium and to assess when to apply the next load increment. Hence, this paper presents the formulation of Newton-Raphson method which was used to develop the nonlinear inelastic finite element model for steel framed structures.

3. Verification of the Developed Analytical Model

3.1 Verification with Column Behaviour

Columns are one of the sources of nonlinearities under various loadings. Imperfections and material plasticity were also considered in the analysis as these features affect the behaviour of the structural column . A slender column of 15 m will also be included in the comparison to magnify the nonlinear analysis.

3.1.1 Short Column

In the validation process, the load-deflection response of pin-ended perfect and imperfect columns are considered. Two columns with the length of 3.4 m and 7.5 m respectively and an initial imperfection of L/1000 at mid-height were loaded incrementally until failure. It is difficult to obtain actual / physical experimental results for this particular load case. Therefore, an established computer program INSTAF [9] is used for validation. The lateral deflection of an isolated column with an initial geometric imperfection (in the form of half-sine wave) is given by Chen [23] as Eq. 1.

$$y = \left(\frac{P/P_E}{1 - P/P_E} \right) \delta_o \sin\left(\frac{\pi x}{L} \right) \tag{1}$$

where P is the applied load, P_E is the Euler buckling load, δ_o is the amplitude of the crookedness at the centre of the column and L is the column length. This load-deflection relationship, together with the results obtained using the developed analytical model, INSTAF and Eq.

1 are shown in Figure 2 that indicating the reliability of developed analytical procedures. The differences in the failure load between INSTAF and developed analytical procedures for the two types of column considered (3.4 and 7.5 m) are 2.7% and 2% respectively. This is considered as an acceptable result.
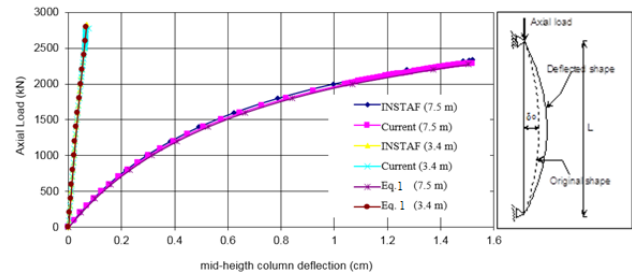

Figure 2. Comparison of the Load-deflection relationship between current study, INSTAF and Eq. 11

3.1.2 Slender Column

The next test considered a similar column but with a high slenderness ratio. Three analyses were conducted, by varying the column length from 10 m to 15 m. The corresponding load-deflection relationships for the columns are shown in Figure 3. Figure 4 shows a Southwell plot to allow the elastic critical load to be estimated by determining the gradient of the line. The theoretical elastic critical load is the Euler load. The results obtained from both the Southwell plots and the calculations done manually compare very well, as shown in Table 1.

Table 1. Comparison between computer model and theoretical elastic loads

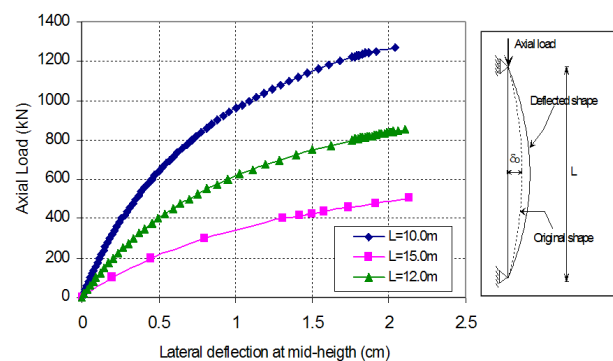|	Elastic Critical Load		
Column Length	L = 10.0m	L = 12.0m	L = 15.0m
Proposed analytical procedures	1903 kN	1321 kN	846 kN
Euler load, P_{crit}	1914 kN	1329 kN	850 kN
Percentage difference	0.6%	0.6%	0.5%

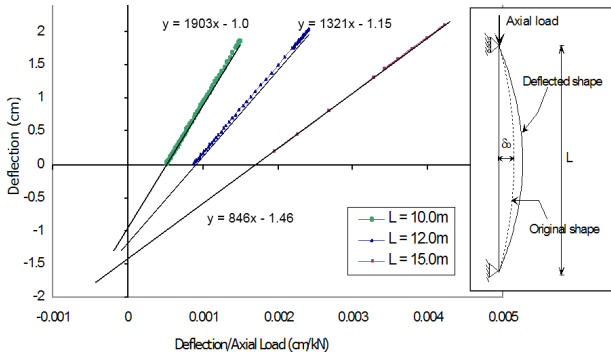**Figure 3.** Load-lateral deflection relationship for three different column length

Figure 4. Southwell plot to allow the elastic critical load to be calculated from the computer results

## 3.2 Validation of Loading and Unloading Behaviour of the Material Properties

Attempts have been made to demonstrate the response of a simple frame subjected to central point load at the centre of the beam as shown in Figure 5(a). A flange cleat connection with the moment-rotation characteristic shown in Figure 5(b) is used for the beam-to-column connections. Four types of cases are considered in the analysis. In the first case, by using the normal elastic-plastic material model, a point load is applied at the beam mid-span until the beam fails. For the second case, a similar load is applied until just before the beam fails, then the load is reduced back to zero. Subsequently, the load is increased again until the beam fails. The same loading patterns were applied for the next two cases, but used the loading-unloading material model.

Complete load-deflection responses for all the cases using the two models are shown in Figure 6. The load-deflection response is also obtained using another reliable program, INSTAF. However, because of the limitation of INSTAF, only the elastic-plastic model is utilised. A correlation can be observed between the results obtained from the two sources with the elastic-plastic model as indicated in Figure 6(a). This indicates the reliability of the program SR-FENAP using the elastic-plastic material model.
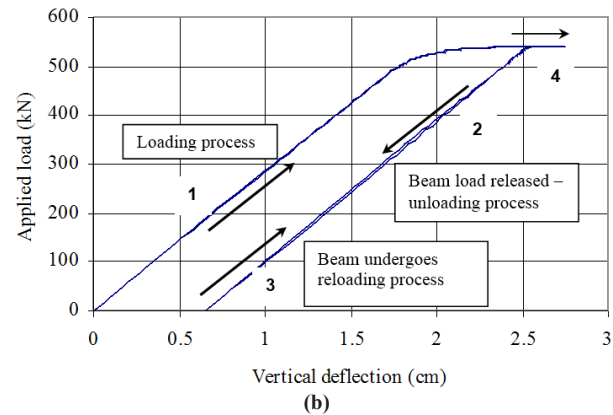

(a)
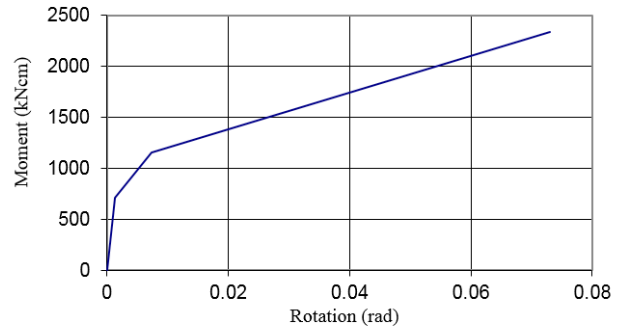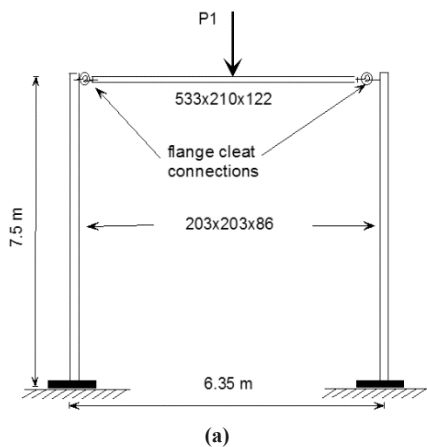
(b)

Figure 5. Simple frame considered for comparison of the response of the developed analytical model with that of INSTAF, where (a) is the frame configuration and (b) is the moment-rotation behaviour of the flange cleat connection
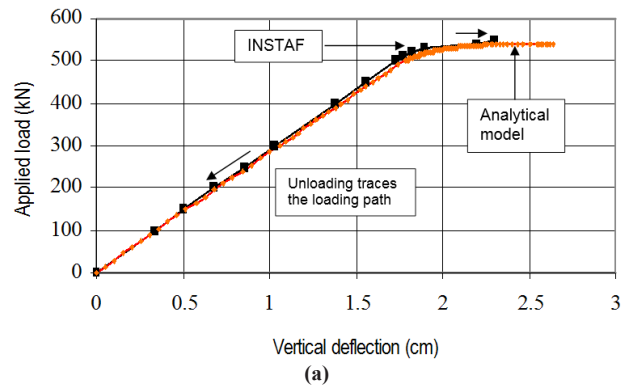
(a)

(b)

Figure 6. Load-deflection response at the beam mid-span for (a) case 1 and 2, and (b) 3 and 4 using the elastic-plastic material model

As mentioned above, for the loading-unloading material model no experimental data was available for comparison. However, the response in Figure 6(b) complies with what actually happens physically. Once the midspan of the beam yielded and the loading is released to zero, the displacements do not return to the previous loading path but rather drop, following the initial stiff-

ness value. A permanent distortion of about one centimetre was noted when the loading is released to zero. When the load is reapplied, the displacement characteristic returns along the unloading path and back to the original path until failure.

Figures 7 and 8 illustrate the moment history at the beam mid-span and at the connection respectively. From Figure 7, it can be clearly seen that as soon as the mid-span beam yielded, the moment did not increase even with a rise in the deflection. At failure, a moment of 845.5 kNm is achieved and this value is very close to the moment capacity of the beam, 848 kNm. In Figure 8, it is observed that the connection moments behave exactly as expected and correspond to the input data, once again indicating the reliability of the program.
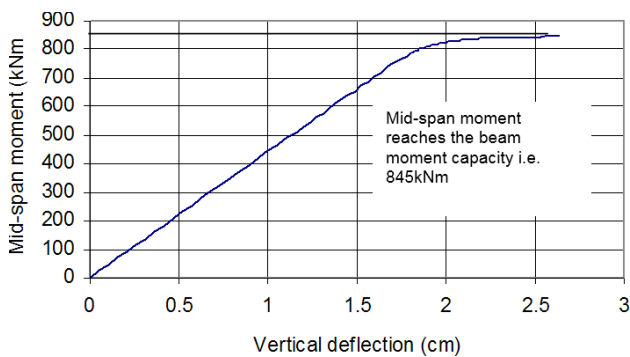

Figure 7. Moment-deflection response at the beam mid-span for case 1 using the elastic-plastic material model
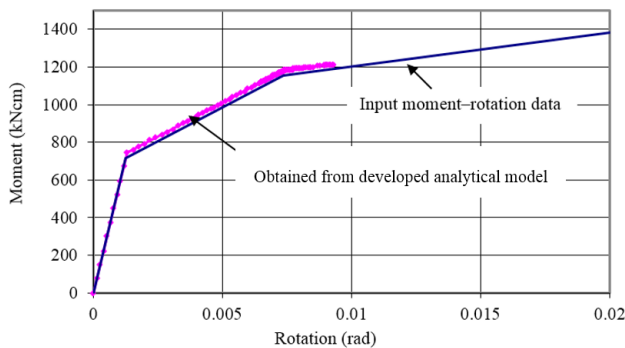

Figure 8. Moment-rotation response at the beam-to-column connection for case 1 using the elastic-plastic material model

Another method to verify the validity of the model is to trace the calculated stress-strain relationship of each of the sub-elements across the cross-section of the beam. A typical relationship of the sub-elements does in fact follow the input value of the stress-strain material model, further proving the reliability of the program. This is shown in Figure 9.
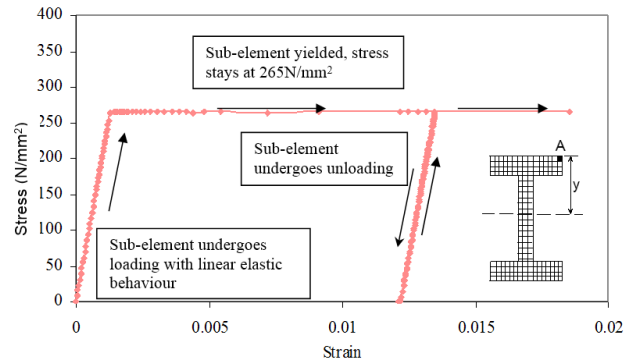

Figure 9. A stress-strain relationship of the sub-element, A at the beam mid-span under load case 4 using the loading-unloading material model

4. Conclusions

This paper discusses the analytical model for steel structural design with semi-rigid connections using a finite element approach parallel to the concept of a performance based plastic design. The derivation of the element stiffness matrix and the solution to the nonlinear finite element analysis were developed in consideration of nonlinearities and connection flexibility. These fundamentals are required in the development of nonlinear steel structural analysis.

Verification of the proposed numerical model has been made in terms of the column's behaviour, where the developed analytical model was compared to theoretical pin-ended perfect and imperfect columns. Differences less than one percent were obtained from the comparison and both the numerical and analytical models correlated. Nonetheless, validation of the material behaviour of loading and unloading was conducted with a semi-rigid frame. The moment-rotation of the connection were compared with the input data and insignificant differences were found with the developed analytical model. This analytical model can be applied in the performance based design of steel structures to obtain more reliable analysis that considering real situations of nonlinearities and connection flexibility.

However, the proposed numerical models are limited to steel structures which can be extended to concrete and composite structures. More validations should be carried out for other parameters in order to increase the reliability of the designs of constructional steel frame structures.

References

[1] Díaz C., Martí P., Victoria M., Querin O. M. Review on the modelling of joint behaviour in steel frames. Journal of Constructional Steel Research, 2011, 67(5): 741-758.

[2] Bathe K.J. Finite Element Procedures. Unite State of America: Prentice Hall, 2006.

[3] Bangash M.Y.H. Concrete and concrete structures: Numerical modelling and applications. London: Elsevier Applied Science, 1989.

[4] Lourenço P.B., Milani G., Tralli A., Zucchini A. Analysis of masonry structures: review of and recent trends in homogenization techniques. Canadian Journal of Civil Engineering, 2007, 34(11): 1443-1457.

[5] Ashraf M. Gardner L., Nethercot D. A. Finite element modelling of structural stainless steel cross-sections. Thin-Walled Structures, 2007, 44(10): 1048-1062.

[6] Spacone E., El-Tawil S. Nonlinear analysis of steel-concrete composite structures: State of the art. Journal of Structural Engineering, 2004, 130(2): 159-168.

[7] Bakis C. E., Bank L. C., Brown V. L., Cosenza E., Davalos J. F., Lesko J. J., Machida A., Rizkalla S. H., Triantafillou. Fiber-reinforced polymer composites for construction - state-of-the-art review. Journal of Composites for Construction, 2002, 6(2): 73-87.

[8] Banihashemi M. R., Mirzagoltabar A. R., Tavakoli H. R. Development of the performance based plastic design for steel moment resistant frame. International Journal of Steel Structures, 2015, 15(1): 51-62.

[9] El-Zanaty M.H., Murray D.W., Bjorhovde R. Inelastic Behaviour of Multi-storey Steel Frames. In: Structural Engineering Report No. 83. Alberta, Canada: Department of Civil Engineering, University of Alberta, 1980.

[10] Lee S.L., Basu P.K. Secant Method for nonlinear semi-rigid frames. Journal of Constructional Steel Research, 1989, 14(4): 273-299.

[11] Majid, K.I. Non-linear structure. New York: Wiley Interscience, 1972.

[12] Goto Y., Chen W.F. Second-order elastic plastic stability of in frame design. Journal of Structural Division ASCE, 1987, 113(7): 1501-1519.

[13] Wen Y., Zeng Q. Y. A novel approach to elasto-plastic finite element analysis of beam structures using the concept of incremental secant stiffness. Finite Elements in Analysis and Design, 2010, 46(11): 982-991.

[14] Oran C. Tangent stiffness in plane frames. Journal of Structural Division ASCE, 1973, 99(ST6): 973-985.

[15] McNamee B.M., Lu L.W. Inelastic multistory frame buckling. Journal of Structural Division ASCE, 1972, 88(ST7): 1613-1631.

[16] Robert T.M. Behaviour of non-linear structures. PhD Thesis, University College, Cardiff, UK, 1970.

[17] Desai C.S., Abel J.F. Introduction to finite element method: A numerical method for engineering analysis. New Delhi: CBS Publishers & Distributors Pvt. Ltd, 2005.

[18] Rodrigue, F. C., Saldanha A. C., Pfeil M. S. Non-linear analysis of steel plane frames with semi-rigid connections. Journal of Constructional Steel Research, 1998, 46(1-3): 94-97.

[19] Akroyd, M.H. Nonlinear inelastic stability of flexibly-connected plane steel frame. PhD Thesis, University of Colorado, Boulder, CO, 1979.

[20] Goverdhan A.V. A collection of experimental moment-rotation curves and evaluation of prediction equations for semi-rigid connections. Master Thesis, Vanderbilt University, Nashville, TN, USA, 1983.

[21] Narayanan R. Steel Frame Structures, Stability and Strength. New York: Elsevier Applied Science, 1985.

[22] Liu X.-L., Lam Y. C. Acceleration of iterative procedures for the elasto-plastic finite element method. Finite Elements in Analysis and Design, 1994, 18(1-3): 41-49.

[23] Chen, W.F., Lui, E.M. Structural Stability - Theory and Implementation. New York: Elsevier Science Publication Co., Inc, 1987.