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REVIEW

Anthropic Principle Algorithm: A New Heuristic Optimization Method

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ABSTRACT

Heuristic optimization is an appealing method for solving some engineering problems, in which gradient information may not be available, or yet, when the problem presents many minima points. Thus, the goal of this paper is to present a new heuristic algorithm based on the Anthropic Principle, the Anthropic Principle Algorithm (APA). This algorithm is based on the following idea: the universe developed itself in the exact way to allow the existence of all current things, including life. This idea is very similar to the convergence in an optimization process. Arguing about the merit of the Anthropic Principle is not among the goals of this paper. This principle is treated only as an inspiration for heuristic optimization algorithms. In the end of the paper, some applications of the APA are presented. Classical problems such as Rosenbrock function minimization, system identification examples and minimization of some benchmark functions are also presented. In order to validate the APA's functionality, a comparison between the APA and the classic heuristic algorithms, Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) is made. In this comparison, the APA presented better results in the majority of tested cases, proving that it has a great potential for application in optimization problems..

1. Introduction

In the last decades, optimization problems have motivated great improvements in mathematics and engineering. Methods like Newton, steepest descendent and Levenberg-Marquardt have made possible the solution of a series of design optimization problems [1]. However, these methods require strong conditions to have their convergence proved, such as availability of gradients, convexity, and so on[2,3]. It is important to point out that in several industrial applications the designer has to deal with some peculiarities such as non-linearity, non-convexity, existence of several local minima, presence of discrete

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and continuous design variables, among others[4].

One class of optimization methods, which are potentially able to handle these characteristics, are the metaheuristic algorithms. Known advantages of these algorithms include the following: (i) they do not require gradient information and can be applied to problems in which the gradient is difficult to obtain or simply is not defined; (ii) they do not become stuck in local minima if correctly tuned; (iii) they can be applied to non-smooth or discontinuous functions; (iv) they furnish a set of sub-optimal solutions instead of a single solution, giving the designer a set of options from which to choose; and (v) they can be easily employed to solve mixed variable (discrete and continuous) optimization problems[5]. Among the most popular metaheuristic algorithms are the Genetic Algorithm (GA)[5], the Ant Colony Optimization (ACO)[6], and the Particle Swarm Optimization (PSO) [7], all of them inspired by biological principles.

Other principles have been employed for the development of metaheuristic optimization algorithms, such as the Imperialist Competitive Algorithm[8], based on the imperialist policy of extending the power and rule of a government beyond its own boundaries, Group Search Optimizer inspired by animal searching behavior[9], and the Biogeography-Based Optimization, that uses the geographical distribution of biological organisms as inspiration for algorithms in the optimization area[10].

Following the line of nature inspired algorithms, this paper proposes a new metaheuristic optimization algorithm based on the Anthropic Principle, originated in Physics. According to the Anthropic Principle, uncountable factors had to converge, in the history of the universe, to make the human existence possible [11]. Thus, the universe evolution can be seen as an optimization process whose objective function aims to minimize the effects that go against the human existence.

Details as the low eccentricity of the Earth's orbit, the relationship between the Sun's mass and distance from the Earth to the sun are examples in the set of suitable conditions for human existence[11]. If just one element in this set were different, life as it is known could probably not be developed. In other words, the constants of the universe seem to be evolved in such a manner to assume ideal values to make life possible[11]. This concept is very similar to "fine-tune" performed in a metaheuristic optimization algorithm.

This paper is organized as follows: in section 2, the Anthropic Principle is conceptually detailed. In section 3, the necessary concepts to the Anthropic Principle Algorithm (APA) are exposed and its operators are described. In section 4, the pseudo-code related to the proposed algorithm is presented. Section 5 presents some examples of applications of the proposed algorithm, and finally, the concluding remarks are in section 6.

2. Anthropic Principle

The anthropic principle was formally defined by astrophysicist Brandon Carter in 1974[12]. Following, John D. Barrow and Frank J. Tipler improved the ideas of this principle and compiled their formulations in the book, Cosmological Principle[13].

In order to achieve suitable conditions for human existence, there is a series of necessary factors. Details such as a different ratio between the electron and proton masses could derail the existence of more complex structures of matter[11]. If the electromagnetic force were changed for a small quantity, the organic molecules could not be able to group themselves [11]. If, for example, the distance between sun and earth had been changed, mankind would not have developed. If the gravitational force were minimally changed, planets orbits would not be formed, and consequently, there would be no life [14].

In fact, the reasoning of the anthropic principle starts with the premise that given the existence of mankind, only the universe stories compatible with that fact can be considered as physical models of this universe. This principle is known as the Weak Anthropic Principle (WAP). By this idea, it is understood that the characteristics of our universe and its physical laws allowed the existence of mankind. Due to the great number of coincidences necessary to create life in a universe, some physicists developed a theory in which the universe developed itself with the objective of allowing the existence of life. This concept is known as Strong Anthropic Principle (SAP) and it is very controversial in Physics[15]. According to the SAP, the universe evolved in such a way as to make possible the existence of mankind[11]. In other words, the physical and cosmological quantities evolved to promote the creation of life.

The anthropic principle in its strong version is rather controversial. There are many arguments contrary to it, as the fact that values of the fundamental constants incompatible with the development of intelligent life will never be observed[16]. In this paper, the Anthropic Principle is only considered as an inspiration for the development of a new metaheuristic optimization algorithm. The idea behind the anthropic principle seems to be promising for a meta-heuristic algorithm. In this context, we propose the Anthropic Principle Algorithm (APA), which makes use of the ideas presented above and apply them in an optimization framework. In the next section, the concepts required for this new algorithm are presented.

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3. Anthropic Principle Algorithm: APA

The APA has an universe U as its basic element. Its structure can be defined as:

$$U = \{C, L\}, \tag{1}$$

in which C represents the characteristics and L the physical laws of that universe U.

These concepts can be better defined in the following form:

(i) Characteristics: set of attributes that characterize a universe, for example, some of the characteristics which were cited in the previous section such as distance between the Earth and the Sun, ratio between the electron and proton masses, and so on. In an optimization algorithm scheme, the characteristics of the universe represent the n design variables of the optimization problem. These characteristics are grouped into the vector C:

$$C = \{c_1, c_2, \dots, c_n\}. \tag{2}$$

(ii) Physical laws: set L of equations that update the characteristics of a universe:

$$L = \{l_1, l_2, \dots, l_n\}. \tag{3}$$

The relation between C and L can be expressed in many forms, among them, there is:

$$c_i^{(k+1)} = l_i \left\{c_i^{(k)}\right\}, \tag{4}$$

in which k stands for the k^{th} iteration of the algorithm. Equation (4) proposes that the i^{th} characteristic of the universe be updated by the i^{th} physical law, using only the current characteristic value.

The "manner" in which the characteristics of a universe U are updated is called the History of U.

The History of an universe is represented by H and it expresses the evolution of this universe between its initial state $U^{(0)}$ and its current state $U^{(k)}$:

$$H : U^{(0)} \rightarrow U^{(k)}, \tag{5}$$

in other words, the History "tells" the story of the characteristics of the best universe, through its laws and operators:

$$\{C^{(0)}, L\} \rightarrow \{C^{(k)}, L\}.$$

An individual I can be generated in a universe, U, if the characteristics of this universe are favorable for the creation of this specific kind of life. The notation used for representing the generation of an individual in a universe is:

$$U \Rightarrow I.$$

An individual generated by a universe U is called product individual: $U \Rightarrow I_P$. Here, we introduce the concept of the reference individual I_R. That is, an idealized individual generated by an ideal universe U_R, whose characteristics C_R are the most favorable for some specific kind of life, thus, $U_R \Rightarrow I_R$. Therefore, we may consider that there exists a specific kind of life in the universe U, if its product individual I_P is similar to the reference individual I_R. We propose to measure this similarity with the aid of the function F(.). That is, the universe U generates this specific kind of life if its image in the function $F(I_R, I_P)$ breaks up a threshold α_v. In other words, if $F(I_R, I_P) < \alpha_v$, there is this specific kind of life in the universe U and, consequently, the universe can be now designated as an "alive universe", which is called U^V .

In the APA, the reference individual, IR is represented by the set of requisites expected to be reached by the problem's optimal solution. For instance, if the optimization problem is the minimization of the function, $f(x_1, x_2, \cdots, x_n)$, in which $f: R^n \rightarrow R$, the I_R is represented by the minimum image of the $f(x_1, x_2, \ldots, x_n)$. If the problem is the minimization of a multi-objective function, the set of desired values for the variables, involved in the optimization process, represents the restrictions that characterize the reference individual. In a system identification problem, each pair of samples (input and output signals) is considered as a restriction for the existence of the life, and the set of these restrictions represents the reference individual.

Now, the development above can be associated to the optimization algorithm presented in this paper. In the APA, the design variables are represented by the characteristics of a universe, e.g. the values of the global solution of the optimization problem are the characteristics C_R of the reference Universe U_R. The function $f(.)$ may be set as the difference between the objective function of a given product individual and:

1) the global solution provided by I_R in the case of a target optimization problem, or,

2) the best solution found up to the current iteration of the algorithm in the minimization of a given function.

The APA was developed as a multi-agent optimization algorithm, consequently, there is a set of candidate universes for life creation. In this context, the set M of universes $\{U_1, U_2, \dots, U_m\}$ is called Multiverse and it can be expressed as:

$$M=\{U_1, U_2, \dots, U_m\} \tag{6}$$

In the initialization of the algorithm, the characteristics $C=\{c_1, c_2, \dots, c_n\}$ of each universe are randomly generated, and the physical laws of each universe are randomly generated, and physical laws of each universe can have

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different kind of rules of update. For example, consider as possible laws the difference equations in the form:

$$l_i: \quad c_i^{(k+1)} = a_1 c_i^{(k)} + a_2 c_i^{(k-1)} + \cdots + a_n c_i^{(k-n-1)}, \qquad (7)$$

in which the coefficients a_1, a_2, a_n, an are randomly initialized. The initial conditions for equation (7) can be computed with the previous values of the characteristic, c_i. If these variables are unavailable, their values are assumed to be zeros.

Another example of a possible kind of physical law is presented in equation (8),

$$l_i: \begin{cases} \Delta c_i^{(k+1)} = a_1 c_i^{(k)} \Delta c_i^{(k)} + a_2 \Delta F^{(k)}(.) \\ c_i^{(k+1)} = c_i^{(k)} + \Delta c_i^{(k+1)} \end{cases} \qquad (8)$$

in which the coefficients a_1, $a_2 \in \mathbb{R}^{+}$ are also randomly initialized and $\Delta F^{(k)} = F^{(k)} - F^{(k-1)}$. In this kind of laws, the update of the characteristics is indirect, using steps Δc_i, and there is a feedback, implemented by the last image of evaluation function, F(.), aiming at choosing adequately the size of the step.

The coefficients a_1 and a_2 are initially all positives, but its signals are changed, as it is shown in table 1, at each iteration. This is valid if the characteristic, c_i, is positive. Otherwise the signals of a_1 and a_2, presented in this table, are inverted.

Table 1. Signals of the coefficients a_1 and a_2 related to the physical law (8)

$\Delta c^{(k)}$	$\Delta F^{(k)}$	a_1	a_2	$\Delta c^{(k+1)}$
-	-	-	-	-
-	+	-	+	+
+	-	+	-	+
+	+	-	-	+

The objective of this permutation of signals is to keep $\Delta F^{(k)}$ negative. The signals of the coefficients a_1 and a_2 are chosen depending on the signals of the two first columns of table 1, aiming to obtain the correct signal for $\Delta c^{(k+1)}$, shown in the third column of the same table.

It is important to mention that only two kinds of physical laws were presented here, but many others are possible.

In order to continue the presentation of the proposed algorithm, we define in the sequel three different classes of universes. The universe U_i of M which presents the best characteristics C, e.g. the lowest objective function value, is denoted as propagating universe U^*. It receives this name because it propagates its characteristics to some least developed universes, as it will be shown in section 3.2.

Two other classes of universes can also be defined: the promising $U^{\lozenge}$ and the stagnated $U^{\dagger}$ universes. The former are the ones which have favorable physical laws for the universe evolution. In other words, universes with physical laws that improve, in one iteration, the image of the evaluation function, $F^{(k+1)}(.) < F^{(k)}(.)$, in a minimization problem. In the opposite way, the universes with physical laws that go against the universe evolution in p iterations, i.e., $F^{(k+p)}(.) > F^{(k)}(.)$, are called stagnated $U^{\dagger}$ universes.

It is important to mention that, in APA, the reference IR, is fixed and the environment, U, is "adapted" to him, while in classic Genetic Algorithm (GA) the environment is fixed and the individuals are evolved.

In order to construct the APA, the main operators proposed for this algorithm are presented next.

3.1 Characteristic's Update

The characteristic update operator has the objective of systematically change the characteristics C of a universe U, through its physical laws L. It is done, in such a manner, that each characteristic, ci, is updated by its corresponding physical law, li, as it is illustrated in figure 1.

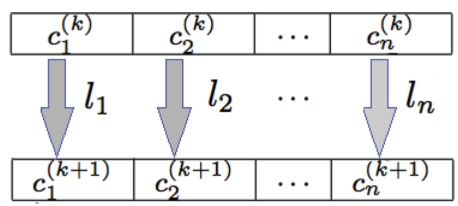

Figure 1. Update of the Universe's characteristics, $C = c_1, c_1, \cdots, c_n$, through the physical laws, $L = l_1, l^2, \cdots, l_n$

The update operator can be implemented in different ways, depending on how the structure of the laws is used. In order to exemplify this, two kinds of update are presented here:

1) The direct update: in which the law, l_i, acts directly over the characteristic, c_i, as it is shown in equation (7). In this kind of physical law, each characteristic has its own dynamic, and the algorithm have to select the laws that has the tendency to lead the evaluation function to a lower value;

2) The update by Δc: in which the law, l_i, creates some quantity Δc_i, which is summed to the characteristic c_i, as it is shown in equation (8). In this kind of physical law, the step $\Delta c^{(k+1)}$ is calculated taking into account its last value $\Delta c^{(k)}$ and the last value of the evaluation function $F(.)$. Thus, the law creates a control mechanism, a feedback, aiming to avoid or at least decrease the fast divergence of a given universe from the solution.

It should be pointed out that since the physical laws of each universe are not the same, the characteristics will be updated in different manners, sweeping the searching

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space in different ways.

It is worth mentioning that APA has no fixed kind of physical laws. Differently of Ant Colony Optimization [6] and Particle Swarm Optimization [7], and many others heuristic algorithms, in APA the kind of actualization rule of the parameters can be chosen by the user, or by the algorithm itself, depending on the kind of problem being treated.

3.2 Characteristic's Propagation Operator

The propagating universe may transfer its characteristics integrally to less suit- able universes. The characteristic propagation operator is represented by the symbol and express the propagation of all characteristics, $C = [c_1, c_2, \ldots, c_n]$, from the propagating universe, $U^{\square}$, to the universe U:

$\{C^*, L^*\} \gg \{C, L\}$,

or simply:

$U^* \gg U$.

The action of this operator can be seen in the figure 2.

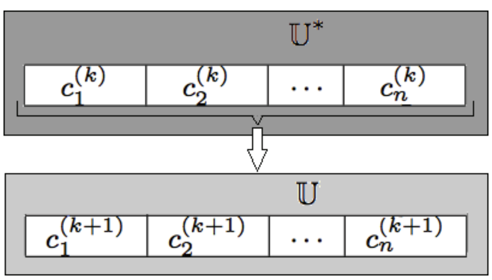

Figure 2. Characteristic Propagation Operator from propagating universe, U^*, to the universe U

It is worth to mention that this operator only acts on universe characteristics, C, not changing the physical laws, L, of any universe. In this context, this operator can also be applied from an alive universe U^V to a promising universe $U^{\Diamond}$:

$U^V \gg U^{\Diamond}$.

This procedure aims to spread all the best characteristics, C^V of the alive universes, U^V, to the promising universes, $U^{\Diamond}$, that have favorable physical laws, $L^{\Diamond}$. Thus, potential characteristics are lead to potential laws and its values can be evolved.

3.3 Big Bang Operator

From Physics, one may understand that our universe was generated by a "Big Bang"[11]. In the proposed algorithm, this idea can be useful. In the APA, the physical laws can be difference equations, system of equations, among others. As the initialization of these equations is a random process, then some physical laws, l_i, could lead the characteristics, c_i, of the universe U to instability. In this scenario, the Big Bang Operator, B(.) can discard these universes and create new ones, and the process go on.

Another possible application of the "Big Bang" Operator is randomly generating new universes and replaces the stagnated ones. In other words, the operator discards non-promising regions of the design domain and restarts the search randomly in a new location of the searching space.

In order to avoid the random search caused by the "Big Bang Operator", the occurrence rate of this operator should be inferior to 15% of the universes in the multiverse, at each iteration.

3.4 Armageddon Operator

Similarly to a natural catastrophe, the Armageddon operator, A(.), disturbs some characteristics of a universe. The Armageddon operator perturbs the characteristics of a universe by having small constants, Δc, added to their current values, as can be seen in figure 3.

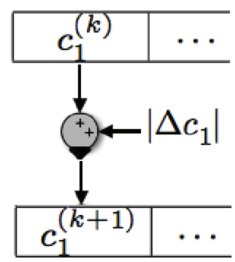

Figure 3. Armageddon Operator, A(.), acting over characteristic c_1

The effect of this operator can be tuned by the Armageddon operator rate, allowing the algorithm to perform local searches around the current characteristics of the universes.

4. Application Example

In this section, some examples of the use of APA for optimization problems will be presented. The experiments were carried out on a Macbook with 2.2 GHz Intel Processor and 4.0 GB RAM. All the codes were written and executed in Matlab R2013. The operating system was Mac OS Lion.

4.1 Rosenbrock Function Example

In order to exemplify the solution of an optimization problem using the APA, consider the Rosenbrock function:

$$f(x) = (a - x_1)^2 + b\,(x_2 - x_1^2)^2, \qquad (9)$$

in which $f: R^2 \rightarrow R$ is a non-convex function, with $a = 1$ and $b = 100$, used in tests for global optimization. The objective of the optimization is to find the vector $x^* = [x_1, x_2]$ that minimizes $f(x)$:

$$x^* = argmin\ \{ f(x) \} \qquad (10)$$

the global optimum of this function is located at $x^{\square} = [1, 1]$, the objective function is $f^{\square} = 0$.

In this example, each universe has 2 characteristics, $[c_1,$

c_2], which represent the values [x_1, x_2] of the Rosenbrock function, and two laws, [l_1, l_2] responsible for updating these characteristics. It is worth to mention that the physical laws, L, used in this example, were a class of equations with the structure presented in (8). In addition, the APA evaluation function is $f(x)$.

In order to illustrate the progress of the algorithm, the Rosenbrock function was minimized using the APA. In all runs of this section, the APA uses 200 universes, characteristics propagation rate starting with 50% and ending with 30%, the Big-Bang rate starting with 5% and ending with 10% and the Armageddon rate starting with 5% and ending with 15%, varying linearly.

For illustration purposes, the algorithm was run only for 10 iterations. The best solution found for each iteration is plotted in figure 4 and detailed in table 2.

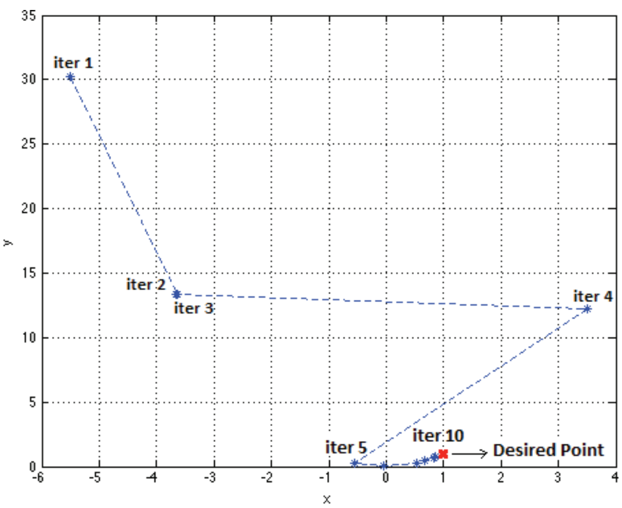

Figure 4. History of a Multiverse. The evolution of the characteristics of an universe during the APA's convergence

Table 2. Characteristics for the best Universe in each iteration

Iter.	c_1	c_2	Δc_1	Δc_2	ΔF
1	-5.5024	30.2659	NA	NA	NA
2	-3.6278	13.3869	-39.2095	-7.2623	-1.5511×10^{8}
3	-3.6582	13.3894	0.0305	0.0025	-72.4581
4	3.4940	12.2212	-7.1522	-1.1682	-8.0588×10^{7}
5	-0.5403	0.2111	-0.4129	-2.9837	-523.4355
6	-0.0485	0.0877	-0.9072	-2.3368	-230.3496
7	0.5232	0.2301	-0.6340	-0.3256	-1.9760
8	0.6736	0.4385	0.0023	0.2083	-1.9823×10^{4}
9	0.8378	0.7159	0.1642	0.1693	-1.3187×10^{4}
10	0.8448	0.7276	0.0070	0.0117	-1.2819×10^{5}

From table 2, one may see that the APA did not reach the optimal point [1, 1] in only 10 iterations, however the convergence process of the proposed algorithm can be clearly observed. In order to further investigate this process, we present the laws (represented by the coefficients a_1 and a_2) of the best universe at each iteration in table 3.

Table 3. The laws for the best Universe in each iteration

	l_1		l_2	
Iter.	a_1	a_2	a_1	a_2
1	0.0556	2.0082×10^{-8}	0.6309	6.5930×10^{-8}
2	0.3287	9.9367×10^{-8}	0.1148	2.3732×10^{-8}
3	0.7401	1.6827×10^{-8}	0.3792	2.0312×10^{-8}
4	0.3889	9.225×10^{-8}	0.4464	1.5073×10^{-8}
5	0.2505	5.4600×10^{-8}	0.6141	2.7179×10^{-8}
6	0.4815	1.5400×10^{-8}	0.2903	7.9859×10^{-8}
7	0.5318	5.4355×10^{-8}	4.1028	7.9859×10^{-8}
8	0.1386	7.8980×10^{-8}	0.4280	7.8184×10^{-8}
9	0.4242	3.9651×10^{-8}	0.2491	5.8548×10^{-8}
10	0.4428	7.6638×10^{-8}	0.4401	7.2683×10^{-8}

Most of the universes presented in table 2 are not totally connected in an evolutionary way. In other words, they evolved from other universes, in the multiverse, which were not the best in the previous iteration of the algorithm.

In table 2, in 4 iterations the best universe evolved from itself: iterations 2 to 3, 3 to 4, 8 to 9 and 9 to 10. In these cases, it is possible to calculate manually the evolution of these universes using the values presented in tables 2 and 3, since the physical laws were responsible for the characteristics update that resulted in the decrease of the objective function. In the other iterations, the best universe was reached by the action of the APA operators. In the transition from iteration 4 to 5, 6 to 7 and 7 to 8 the best universe was reached by propagation of the best characteristics to universes with physical laws that allowed decreasing the evaluation function. In the transition from iteration 1 to iteration 2, the best universe was reached by the Armageddon Operator. In this example, the Big Bang operator did not generate best universes.

Now, we run the APA for 50 iterations, and then compare its results to two well-known metaheuristic algorithms: GA and PSO. In this situation, the proposed algorithm reached as best solution x□ = [1.0001, 1.0002]. As it is known that the optimum of the Rosenbrock function is [1, 1] [17], one can consider that it is a close result, reaching an evaluation function equal to 1.1125 x 10^{-8}. The APA's convergence curve for this run is illustrated in figure 5.

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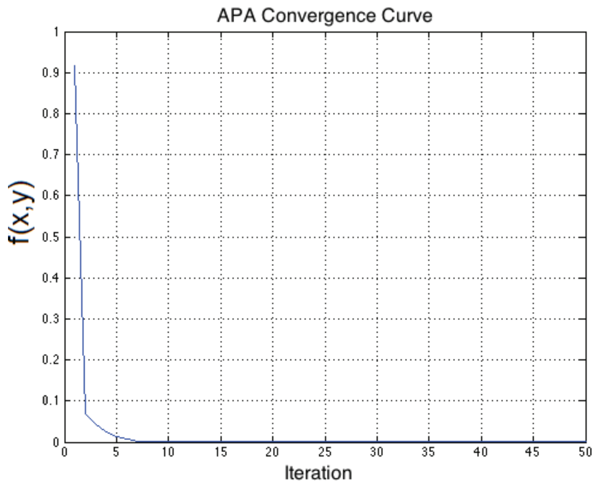

Figure 5. Convergence curve of the Anthropic Principle Algorithm

In order to compare the APA to a GA and a PSO, table 4 shows the objective function reached by these algorithms in a single optimization run. The stopping criterion for all the algorithms was 50 iterations. The APA and GA results were generated by the authors and the PSO result was taken from the literature [18].

Table 4. Minimization of the Rosenbrock Function Results

Method.	APA	GA	PSO
Function value	1.1125×10^{-8}	0.0018	0.001341

In the GA, 200 chromosomes were used, with crossover rate starting in 80% and ending with 60%. The mutation rate starting in 5% and ending in 10%, varying linearly. For the PSO, were used 200 particles with social adjustment equal to the self adjustment (1.49).

One can see in table 4 that the lowest value of the Rosenbrock function was obtained with the APA, outperforming in this case, the other metaheuristic algorithms.

4.2 System Identification Example

System Identification deals with the problem of building approximated mathematical models of dynamic systems based on observed (experimental) data [19]. In this section, we briefly present the problem to be solved, thus the interest reader is referred to [20] and [21] for further details. In this section, the APA is applied to a system identification problem. In this context, consider the following stable, linear and time invariant system, with input $u(k)$ and output $y(k)$:

$$H(z) = Z\left\{\frac{y(k)}{u(K)}\right\} = \frac{Y(z)}{X(z)} = \frac{2z-1}{z^2-0.2z+0.26} \quad (11)$$

in which Z represents the Z transform and z is the complex variable associated to this transform. Consider also that one can seek an approximated model for such a system using linear combinations of Laguerre functions, as expressed in (12):

$$M(z) = d_1L_1(z) + d_2L_2(z) + \ldots + d_rL_r(z) = \Sigma_{i=1}^{r} d_i\, L_i(z), \quad (12)$$

in which $M(z)$ is the approximated model of the system $H(z)$ based on Laguerre functions, given by:

$$\mathrm{L_i(z)} = \frac{\sqrt{1-p^2}}{z-p}\left(\frac{1-pz}{z-p}\right)^{i-1}, (13)$$

in which $p \epsilon R$ is the pole of the Laguerre functions.

Thus, in order to identify the system $H(z)$, expressed in (11), it is necessary to find the pole, p and the coefficients d_i that minimize F(.), as expressed in equation (14),

$$x^* = argmin\{F(I_R, I_p(x))\} \quad (14)$$

in which $x = [d_1, \ldots, d_r, p]$ is the design vector comprised by the r coefficients d_i and the pole p; $F(I_R,I_P)$ is the evaluation function of the algorithm, expressed by the mean square error (MSE), of the approximated model output with respect to the observed (measured) system output. It is important to mention here that, in the identification problem, the image I_R of the reference universe U_R is known from experimental data. Hence, the evaluation function can be constructed as detailed above.

In order to identify the system $H(z)$, one can resort to an input $u(k)$, PRBS type, Pseudo Random Binary Signal. For the identification step, it was used 128 input/output system samples. Another 128 samples were reserved for the validation step. In this example, we constructed the approximated model using three Laguerre functions, i.e.,r=3 in (12). Thus, the resulting optimization problem has 4 design variables: 3 coefficients d_i and the pole p.

In order to solve such a problem a multiverse with 200 universes was em- ployed for 50 iterations. The algorithm ran with characteristic propagation rate starting with 50% and ending with 30%, the Big-Bang rate starting with 5% and ending with 10% and the Armageddon rate starting with 5% and ending with 10%, varying linearly. In addition, it is worth to mention that the physical laws, L, used in this example, were a class of difference equations of first order, in which a_1 and $a_2 \epsilon$ R are generated by random values in each law of each universe.

The general structure of these laws is:

$$l_i: c_i^{(k+1)} = a_1c_i^{(k)} + a_2 \quad (15)$$

in which $c_i^{(k+1)}$ +is the i-th characteristic of a universe, updated by equation (15), in the iteration $(k + 1)$ of the algorithm.

The best characteristics obtained by the APA are given in (16), which resulted in a MSE = 6.011×10^{-4}.

$$d = [1.4350 \;\; 2.1601 \; - \; 0.1512],$$
$$p = 0.1868. \quad (16)$$

Now, using the information provided by APA, through the pole p, one can generate the Laguerre function basis, $[L_1(z), L_2(z), L_3(z)]$, used in the model (12):

$$L_1(z) = \frac{0.982}{z-0.187}, \tag{17}$$

$$L_2(z) = \frac{-0.184z + 0.982}{z^2 - 0.374z + 0.035} \tag{18}$$

$$L_3(z) = \frac{0.034z^2 - 0.367z + 0.982}{z^3 - 0.560z^2 + 0.105z - 0.0065} \tag{19}$$

and finally construct the approximation model $M(z) \approx \frac{Y(z)}{X(z)}$ as shown in (20).

$$M(z) = 1.4350L_1(z)+2.1601L_2(z)-0.1512L_3(z) \tag{20}$$

the pole, p, that has parameterized $L_1(z)$, $L_2(z)$ and $L_3(z)$ was 0.1868, as can be seen in (16).

The measured output of the system and the output obtained by the approximated model $M(z)$ are illustrated in figure 6. From this figure, we may see that the approximation model represents well the experimental data, thus successfully solving the identification problem.

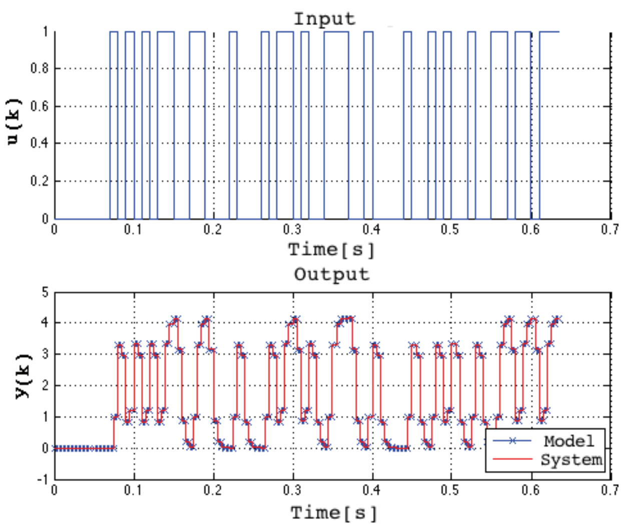

Figure 6. Input signal u(k) applied to the system H(z) and its output y(k)

The convergence curve of APA for this problem is shown in figure 7.

In figure 7, one can see that the APA convergence process is more intense in the first iterations and, then, evolves slowly to the end value.

In order to make a comparison between the performance of the APA and a classic GA, this identification problem was solved by both algorithms using the same

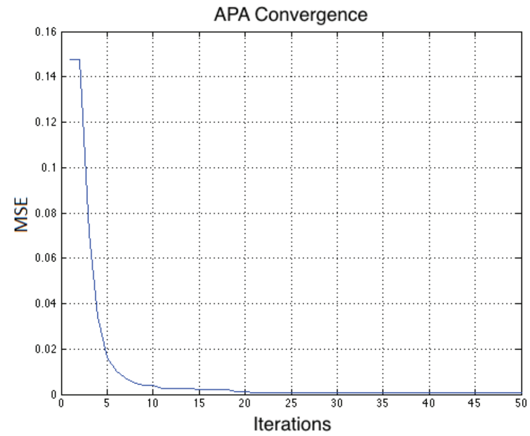

Figure 7. Anthropic Principle Algorithm Convergence

stopping criterion: 100 iterations. In the GA, the crossover rate starting in 80% and ending with 60%, the mutation rate starting in 5% and ending in 10%. The resulting MSE of the approximated model provided by the APA and GA are given in table 5.

Table 5. Results of the minimization of MSE in the identification system problem with Laguerre functions

Method.	APA	GA
MSE	6.011×10^{-4}	7.110×10^{-4}

As it can be seen in table 5, the APA also provided a slightly better solution than the GA for this example.

It is import to mention that the APA was developed focusing in system identification problems, because the restrictions that characterize the IR are achieved directly from the sampled input/output data. However, is possible to infer that the APA algorithm can be applied for general optimization problems as can be seen in the next sections.

5. Statistical Analysis

In order to pursue a fair comparison among the APA and other metaheuristic algorithms, not only the best design found using each method should be com- pared, but also the statistics of each algorithm due to their stochastic nature. Thus, here, each optimization problem is run independently several times and the statistics of these runs are computed, such as the optimum mean value and coefficient of variation (C.O.V.) of the optimum objective function. The number of independent runs (NIR) for all the problems is taken as 50. Also, the stopping criterion used in all the problems is the number of iterations, which is set to 100.

The seven classical test functions shown in table 6 are analyzed. Functions f_1, f_2 and f_3 are unimodal functions, with dimension equal to 30; f3 is a random function; functions f_4, f_5 and f_7 are multimodal, with 30 dimensions and

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many local minima, while function f_6 is a polynomial type with 2 dimensions [9].

The results of the comparison between the APA and the others algorithms is presented in table 7. The GA and PSO results were taken from the literature, in He et. al., (2009) [9].

Table 6. Seven Benchmark functions, in which n is the dimension of the function, S represents the function domain and f_{min} is the global minimum value of the function

Test function	n	S	f_{min}
$f_1(x) = \sum_{i=1}^{n} x_i^2,$	30	$[-100, 100]^n$	0
$f_2(x) = \sum_{i=1}^{n} (\sum_{j=1}^{i} x_j)^2,$	30	$[-100, 100]^n$	0
$f_3(x) = \sum_{i=1}^{n} ix_i^4 + rand[0, 1),$	30	$[-1.28, 1.28]^n$	0
$f_4(x) = \sum_{i=1}^{n} (x_i^2 - 10cos(2\pi x_i) + 10)^2,$	30	$[-5.12, 5.12]^n$	0
$f_5(x) = \frac{1}{4000} \sum_{i=1}^{30} (x_i - 100)^2 - \prod_{i=1}^{n} cos(\frac{x_i - 100}{\sqrt{i}}) + 1,$	30	$[-600, 600]^n$	0
$f_6(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4,$	2	$[-5, 5]^n$	-1.0316
$f_7(x) = 20exp\left(0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - exp\left(\frac{1}{n}\sum_{i=1}^{n} cos2\pi x_i\right) + 20 + e,$	30	$[-32, 32]^n$	0

It is worth to mention that the domain and correct minima of the functions are shown in table 6 on the third and fourth column. In the APA, the initialization of Universe's characteristics has followed the range expressed in these domains. From table 7, one can see that the APA reached better values than GA on functions f_1, f_2, f_5, f_6 and f_7. In comparison with the PSO, the APA results were better on functions f_2, f_5, f_6 and f_7. GA and PSO algorithms presented better results for the cases expressed by the functions f_3 and f_4. The stochastic nature of the function f_3 can explain why the APA did not return the best result for this case. Concerning function f_4, since it is a multimodal function, the conclusion is that APA got stuck in a local minimum, even though its result is a value near to the one presented by the PSO.

Table 7. Results for the seven-benchmark functions expressed in table 6

Func.	Algorit.	Mean	Std.	C.O.V.
f_1	GA PSO APA	3.1711 3.6927×10^{-37} 1.5859×10^{-10}	1.6621 2.4598×10^{-36} 1.6380×10^{-10}	0.5241 6.6612 1.0329
f_2	GA PSO APA	9749.9145 1.1979×10^{-3} 7.4209×10^{-6}	2594.9593 2.1109×10^{-3} 1.3054×10^{-5}	0.2661 1.7622 1.7591
f_3	GA PSO APA	0.1045 9.9024×10^{-3} 8.9354	3.6217×10^{-2} 3.5380×10^{-2} 0.5094	0.3466 3.5729 5.7009×10^{-2}
f_4	GA PSO APA	0.6509 20.7863 34.6328	0.3594 5.9400 36.8309	0.5521 0.2858 1.0635
f_5	GA PSO APA	1.0038 0.2323 2.1888×10^{-4}	6.7545×10^{-2} 0.4434 2.4709×10^{-4}	6.7289×10^{-2} 1.9087 1.5857
f_6	GA PSO APA	-1.0298 -1.0160 -1.0316	3.1314×10^{-3} 1.2786×10^{-2} 3.8328×10^{-16}	NA NA NA
f_7	GA PSO APA	12.9804 2.2500 0.0219	0.5979 4.5895 0.1256	0.0461 2.0398 5.7322

In summary, the APA reached reasonable results for the majority of tested cases, overcoming the others metaheuristic algorithms in many cases.

6. Conclusion

In this paper, the concepts related to the Anthropic Principle were presented, as well as a new metaheuristic algorithm based on it. This algorithm is conceptually simple and easy to implement. Regarding the study of the effect of the control parameters, the authors found that the APA is not so sensitive to most of parameters with the exception of the coefficients of the physical laws. With respect to the applicability of the algorithm, one can see that it is appealing for solving real world problems, since it is applicable to a variety of optimization problems.

In order to test APA capability to find the minimum of a non-convex function, the Rosenbrock function was analyzed and compared, in a single run, with GA and PSO. From this comparison, one can conclude that the APA reached a value close to the known minimum, with better results than GA and PSO.

In order to test the algorithm in a practical engineering application, the APA was applied to a system identification problem. In other words, the APA was used to find the parameters in a Laguerre model for a dynamic system. In the comparison between the algorithms, the APA outperformed the GA. By the lower MSE reached and from figure 6, one can see that the approximated model is very close to the experimental data. Therefore, one can conclude that the approximated model provided by the APA is representative.

In figures 5 and 7, one can see that, typically, the APA convergence process is more intense in the firsts iterations and then, evolves slowly, but almost continuously, as seen in these figures.

In order to compare the statistics of the APA, GA and PSO, a set of seven benchmark functions were employed. 50 runs of each algorithm were pursued for each function. From table 7, one can see that the APA reached better values than GA on functions f_1, f_2, f_5 and f_6. In comparison with the PSO, the APA results were better on functions f_2, f_5 and f_6.

Considering that the APA is in its initial development, it is reasonable to assume that many improvements are still possible in the algorithm. In future work, it is possible to use the gradient-based methods as physical laws for the APA, mixing the deterministic and metaheuristic optimization.

Is important to mention that, in APA, the reference individual, IR, is fixed and the environment, U, is "adapted" to him, while in GA, the environment is fixed and the individuals are evolved. It also is noteworthy that the APA differs from the GA in the issue of defining its search di-

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rections. The spread of the characteristics of propagating universe to other universes generates a range of "direction searches". As the physical laws are random, it is likely that different universes have different physical laws, and the search space is swept in different manners.

In the comparison between the results obtained with application of Anthropic Principle Algorithm and other heuristic algorithms, one can conclude that the APA presented reasonable results in all studied cases, and reached the best values and statistics in the majority of the tested cases. Moreover, it is expected that APA can be used for general heuristic optimization problems.

References

[1] Nocedal, Jorge and Stephen, Wright. Numerical Optimization. Springer Publisher, 1st edition,1999.

[2] Boyd, Stephen Vandenberghe L. Convex Optimization. Cambridge University Press, 4th edition, UK, 2004.

[3] Rao, S. S. Engineering Optimization: Theory and Practice. Wiley Press: New York, NY, 1996.

[4] Mockus, Jonas; Eddy, William and Reklaitis, Gintaras. Bayesian Heuristic Approach to Discrete and Global Optimization: Algorithms, Visualization, Software and Applications. Kluwer Academic Publishers, 1st edition, 1997.

[5] Koza, John. Genetic Programming: On the Programming of Computers by Means of Natural Selection. MIT Press, 6th edition, 1998.

[6] Solnon, Christine. Ant Colony Optimization and Constraint Programming. Wiley Press, 1st edition, 2010.

[7] Olsson, Andrea E. Particle Swarm: Theory, Techniques and Applications. Nova Science Publishers, 1st edition, 2011.

[8] Atashpaz, Esmaeil and Lucas, Caro. Imperialist Competitive Algorithm: An Algorithm for Optimization Inspired by Imperialistic Competition. IEEE Congress on Evolutionary Computation, IEEE, 2007, 4661-4667.

[9] He, S. and Wu, K. H. and Saunders, J. R. Group Search Optimizer: An optimization Algorithm inspired by animal Searching Behavior. IEEE Transactions on Evolutionary Computation, 2009.

[10] Simon, Dan. Biogeography-Based Optimization. IEEE Transactions on Evolutionary Computation, 2008.

[11] Hawking, Stephen and Mlodinow, Leonard, The Great Project, Nova Fronteira Print, 1st edition, 2012.

[12] Carter, B. Confrontation of Cosmological Theories with Observational Data. IAU Symposium, Krakow, 1973.

[13] Barrow, J. D. and Tipler, F. J. The Anthropic Cosmological Principle. Oxford Univ. Press. Oxford. 1986.

[14] Comitti, V. S., Princípio Antrópico Cosmológico, Revista Brasileira de Ensino da Física, 2011.

[15] Kallosh, Renata and Linde, Andrei. M theory, Cosmological Constant and Anthropic Principle. Physical Review D 67, 023510, 2003

[16] Starkman, Glenn D. and Trotta, Roberto. Why Anthropic Principle Cannot Predict α. Physical Review Letters,2006.

[17] Rosenbrock, H. H. An automatic method for finding the greatest or least value of a function. The Computer Journal 3, 1960.

[18] Raval, Falguni and Makwana, Jagruti. Optimization of Resonance Frequency of Circular Patch Antenna at 5GHz Using Particle Swarm Opti- mization. International Journal of Advances in Engineering and Technology, 2011.

[19] Ljung, Lennart. System Identification, Theory for the User. 2nd edition. Prentice Hall. 1999.

[20] Oroski, E. and Holdorf R. and Bauchspiess, A. Nonlinear Buck Circuit Identification Using Orthonormal Functions with Heuristic Optimization. XX Congresso Brasileiro de Automática, 2014.

[21] Rosa, Alex and Campello, Ricardo and Amaral, Wagner. Exact Search Directions for Optimizations of Linear and Nonlinear Models Based on Generalized Orthonormal Functions. IEEE Transations on Automatic Control, 2009.

  DOI: https://doi.org/10.30564/jaeser.v1i1.353