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SHORT COMMUNICATION

Geometric Study of Two-Dimension Stellated Reentrant Auxetic Structures to Transformable Architecture

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ABSTRACT

Transformable architecture is totally linked to the study and knowledge of geometry. There are some materials in nature, whose geometric invariants establish equivalent structural behavior regarding the scalar transformations, developing different spatial typologies according to dimensional variation. Auxetic materials are characterized by their negative Poisson's ratio. They can change their geometric configuration from a line to a surface, and from a surface to a volume or spatial framework. This paper is based on establishing and comparing those stellated reentrant auxetic geometries to be able to build new spaces defined by their capacity for architectural transformation, studying analytically geometric properties of stellated reentrant auxetic structures that, from the molecular to the macroscopic level, can be part of the architecture construction. In this investigation, a comparative study by means of CAD of stellated reentrant auxetic patterns has been realized. A Computer-Aided Design study of stellated reentrant auxetic structures will be realized to use them in architecture. The geometric behavior of the different stellated reentrant auxetic patterns is analyzed from the developed study to generate a systematic comparison, evaluating properties of these forms, such as their maximum achievable area reductions in relation to the total length of bars of the structure, in order to obtain a growth factor.

Keywords: Transformable; Architecture; Geometry; Auxetic; Stellated; Reentrant; CAD; Growth factor

1. Introduction

When a material is stretched in one direction, it normally loses section in the perpendicular direction. The Poisson ratio, $\nu = - d\varepsilon_{trans}/d\varepsilon_{axial}$, ε_{trans} and ε_{axial}, can express mathematically the change of dimensions. They are the cause of axial and transversal elongations or contractions when the material contracts

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or expands in the perpendicular way. Colloquially speaking, v_{ij} is the Poisson ratio that expresses an elongation in the 'j' axe when a flattening is applied in the 'i' axe. Classical structures conserve their volume because they have a positive Poisson's ratio. If a structure thickens in all directions when pulled it and the same structure narrows when compressed it, it is an auxetic structure. A negative Poisson's ratio (NPR) is the characteristic of these types of structures [1-4]. It has been studied the auxetic properties of artificial materials (Gore-Tex®, foams, polymeric foams) and natural materials (skins, some minerals ...), developing molecular auxetic patterns to obtain auxetic structures [5].

Novel manufacturers have been designed by the utilization of auxetic patterns. Minimally invasive implantable devices, morphological structures of the forms and smart expandable actuators are the main areas of application [6]. Devices utilized for the development of drop-down satellite antennas like shape memory auxetics alloys (SMA) also exist [7]. They are based on auxetic patterns and they are utilized as intelligent actuators. Several post-processing steps were initiated in some research on the behavior of polyurethane foams with shape memory and auxetic properties [8]. The characteristics of some auxetic patterns of a new development of expandable stents in the field of medical appliances have been tested [9]. But, at the moment, the utility of these materials and their geometric behavior in the architecture size are unknown. Auxetic patterns can generate new models with novel transformable geometries. They will produce useful properties in order to generate novel deployable geometries by designing tasks. These geometries will be used for the development of novel transformable architectures.

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Some auxetic and potentially auxetic patterns, normally classified as "reentrant" [10], "chiral" and "rotating" have been described in anterior investigations and researches. Sometimes, at the molecular level only a scheme of the pliability process of auxetic patterns is presented. This study occurs when the structure is submitted to uniaxial efforts and it is not enough [11]. Information at the architectural scale is only provided for rigid knots in the re-entrant hexatruss structures. So, it is important to study about other attributes of some auxetic geometries, realizing a comparative and systematic analysis. It would be interesting to generate novel deployable structures applicable in architecture [12].

In this research a comparison of CAD separate and 2D geometries of stellated reentrant auxetic patterns is shown. The methodology of the realized research pay attention on develop some stellated reentrant auxetic patterns by means of CAD of three unity geometries and three 2D models, from the knowledge acquires in anterior investigations [13], patents [14-16] and conferences [12,13], and own researches.

The nature patterns and the models that have been generated by other authors in sizes different to architecture have rigid knots. The present research works with the behavior when these patterns have articulated knots. The research is a theoretical investigation applicable to understand the aptitude of pliability in structures suitable for architecture. The results are theoretical too, and materials are not taken into account.

When the Computer Aided Design approach is developed, the geometrical qualities of some stellated reentrant auxetic patterns were tested and compared, focusing on the particular behaviors of those patterns apt for the development of transformable structures for architecture [17]. A growth factor will be obtained from the maximum area reductions of the structures (the relation between the additions of the total length of all the bars of the structure and their areas).

This research is the only investigation of comparison of stellated reentrant auxetic patterns in architecture done so far. A lot of patterns could be verified changing the dimension of the length of each bar of the separate structure. So, the geometric studies on these patterns could be infinite. The comparison of geometric changes for the same structure will not be realized. A comparison between regular unity patterns and regular 2D patterns will be carried out.

The research is supported by geometrical basic criteria.

2. Methodology and objectives

According to Yanping Liu & Hong Hu [4], auxetic structures are classified in different groups depending on its geometric configurations: Reentrant structures, chirals, rigid or semi-rigid rotating units, laminated angular layers, microporous polymers and crystalline liquid polymers. For architectonic structural elements usage, the proposed ones are: Reentrant, chirals, rotating units and microporous polymers.

In this article, the stellated reentrant structure, in its different variations, has been modeled in CAD software, firstly as individual pattern, in order to combine with itself and check the geometrical pliability conditions according to the chosen group configurations. The auxetic existing models with rigid knots have been studied. These auxetic models with rigid knots have been drawn in the CAD software like articulated structures that can be deployed. The work by flexion of the bars of existing models of two dimension stellated reentrant auxetic structures is transformed in a simple work of traction and compression. These patterns include the repetition of the unity pattern in order to form a planar structure (**Figure 1**). Depending on the repeated geometry this lattice will generate forms that can grow of quadrangular or circular form.

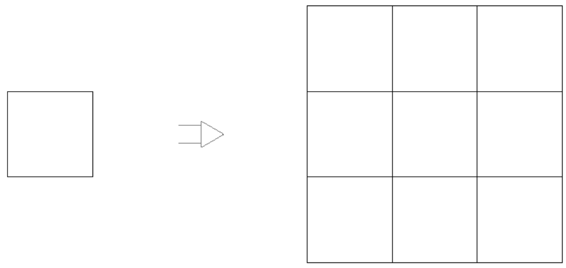

Figure 1. Generation of 2D pattern from the unity pattern, own elaboration.

The target is checking in a geometrical form the structural behavior of these geometries through the developed of a CAD library. The characteristics of some stellated reentrant auxetic patterns will be tested and a systematic comparison will be established. It will be connected the singular characteristics of those patterns useful to the development of transformable structures applies in architecture. A growth factor will be obtained from the maximum area reductions of the structures (the relation between the additions of the total length of all the bars of the structure and their areas).

3. Geometric study of stellated reentrant auxetic structures

The stellated reentrant auxetic structures have been developed with the computer-aided design program. These structures have been drawn in different positions to understand their geometrical behavior. As stated by Darcy Thomson [18] in his book "The growth of form", all our form concepts must be linked to magnitude and direction. The reason is that the object's shape can only be defined by the knowledge of its magnitude, real or relative, in some directions; and growth implies the same concepts of magnitude and direction, in relation to one more dimensional concept: Time.

In similar geometries the surface increases with the square, and the volume with the cube, of the linear dimensions. Having calculated that a small wading bird, the stilt, weighed 120 g. and that its legs measured 20 cm, it was supposed that a flamingo, which weighs more than 2 kg, should have legs 3 m long, to keep the same proportion. But it is obvious that, with the weights of both birds in the proportion of 1:15, the legs length (or any other linear dimension) will vary as the cube root of these numbers, approximately to the proportion 1:2.5. And according to this scale, the flamingo's legs should be as they really are, about 50 cm in length.

Although length growth and volume growth (which are usually equivalent to mass or weight) are parts of the same phenomenon or process, growth in the first case is remarkable. For example, a fish doubling its length multiplies its weight by at least eight; to double its weight, it is enough to go from 10 cm to 12 cm in length. Secondly, the correlation between length and weight in some animals is understood. In other words, determining the value of k in the formula W = kL3, allows us to relate one magnitude to the other at any time.

The scaling effect does not depend on itself. It depends on its relationship with its entire environment. One of the most common scaling effects is caused by some physical efforts acting directly on the surface of a body.

There is a principle of dynamic similarity whereby the "dimensions" remain the same in equilibrium equations, but the relative values are altered with the scale. Numerous architectural projects have been based on principles of nature [19].

Possible positions in space of some auxetic stellated reentrant patterns have been developed from they are fully folded until their maximum opening is achieved, achieving a variation of their surface or volume with the same mass. The target is to know the opening and closing conditions of the angles generated in nodes, to determine if these turns are superficial or spatial and in which directions they are generated according to the geometric conditions of the imposed structure.

The relationship between the amounts of material used and the surface obtained is also analyzed, by the 2 dimensions structure analysis. For this purpose, the total length of all bars used in the design of each pattern will be counted, as an analogy to the quantity of material. To know these amounts of material in mass units, specific sections of bars and specific materials should be defined. In this research, the aim is to find a general theoretical behavior, so the length identifies perfectly those linear elements utilized. A relationship (K) between the area (A) and the length (L) will be given, in order to understand the growth values of these special structures. The surface of each pattern will correspond to the square, circumference or polygon (as appropriate) where the figure is registered. From the division and subtraction of K_{max} and K_{min}, growth factors FC (:) and FC (-) of each structure will be obtained.

Individually applicable patterns and their possible combinations are developed to generate structural developments based on size, as well as transform architectures that follow new geometric developments into deployable auxetic architectures. The auxetic models that will be studied geometrically will be the stellated reentrant auxetic structures. These stellated reentrant auxetic models can be generated from polygons (regular or irregular) by dividing their faces in two and folding those inwards (outward behavior is not auxetic).

The geometric variations on these patterns have no end. It could be verified a lot of behaviors by changing the dimension of each bar of the individual pattern that affects the whole group. However, the structures will be studied for totally regular patterns, with the aim of carrying out a comparative study between all of them where it can be visualized how the shape affects the growth capacity of the structure in an auxetic way.

4. Developed auxetic patterns and their foldability

4.1 Unity pattern

The behavior of different unity patterns of auxetic stellated reentrant auxetic structures is analyzed as a function of their geometric parameters. To generate these structures, the number of vertices that attack the central core in the individual pattern has been varied, achieving different properties. In this way, 3, 4 and 6-point individual stellated reentrant auxetic structures have been considered, in such a way that the behaviors of **Figures 2, 3 and 4** can be appreciated:

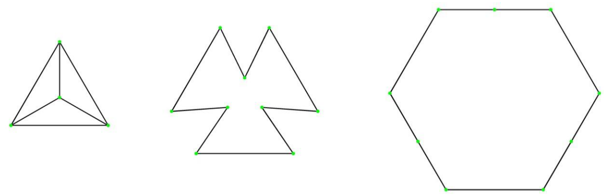

Figure 2. Unity of stellated reentrant structure of three points, own elaboration.

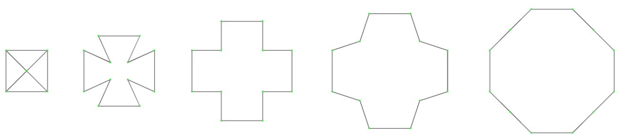

Figure 3. Unity of stellated reentrant structure of four points, own elaboration.

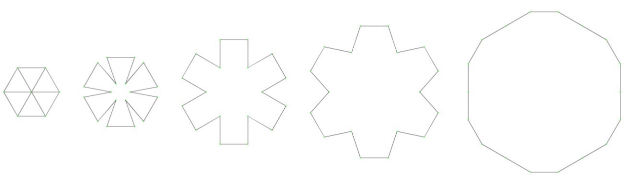

Figure 4. Unity of stellated reentrant structure of six points, own elaboration.

4.2 2D pattern

The behavior of different 2D patterns of auxetic stellated reentrant auxetic structures is analyzed as a function of their geometric parameters. To generate these structures, the repetition of the unity pattern in order to form a planar structure has been established, achieving different properties. In this way, 3, 4 and 6-point individual stellated reentrant auxetic structures have been considered, in such a way that the behaviors of **Figures 5, 6 and 7** can be appreciated:

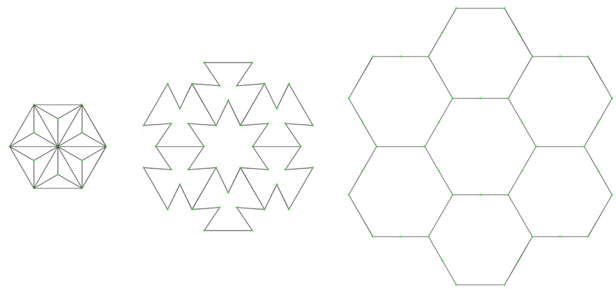

Figure 5. Set of stellated reentrant structure of three points, own elaboration.

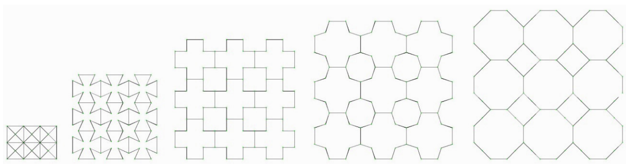

Figure 6. Set of stellated reentrant structure of four points, own elaboration.

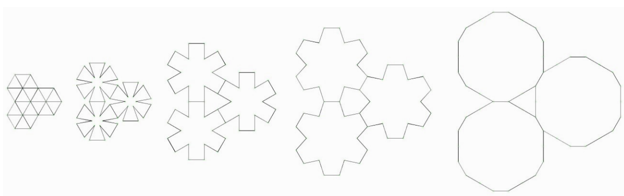

Figure 7. Set of stellated reentrant structure of six points, own elaboration.

5. Discussion of results

5.1 Unity pattern

The growth factor of each unit of the three structures analyzed has been graphically analyzed in **Figure 8**; reaching that growth is proportional to the number of sides as the conclusion. The bigger the number of sides, the more the structure grows when it is unfolded. And it looks like exponential growth.

The growth factors of these three stellated reentrant auxetic structures analyzed have also been analyzed numerically. The numerical data obtained, in relation to the growth of these structures and their mass-volume ratio, have been the following:

The growth factors, numerically, also increase with the number of vertices that attack the central core of the star. They obtain the highest growth factors for the individual 6-pointed stellated reentrant auxetic structures, which corroborates the graphic analysis.

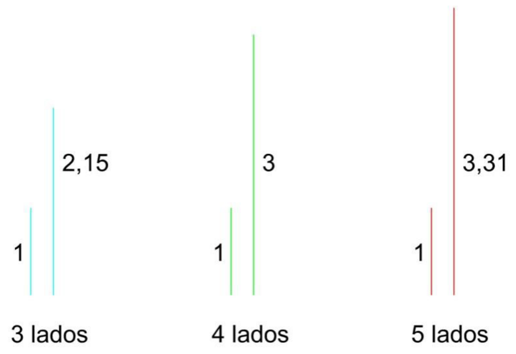

Figure 8. Growth factor of stellated reentrant auxetic structures, own elaboration.

Table 1. Relation between minimum areas (A_{min}) and maximum areas (A_{max}) with the total length of the bars of the structure, own elaboration.

	L	A_{min}	A_{max}	K_{min}	K_{max}	FC (:)	FC (-)
3 points	7.46	0.43	3.01	0.06	0.40	7.00	0.35
4 points	9.66	1.00	7.00	0.10	0.72	7.00	0.62
6 points	18.00	2.60	24.99	0.14	1.39	9.61	1.24

Analyzing these figures, it can also be observed that all their nodes are articulated in the plane, and two bars converge in them, allowing rotations in the xy axis that go from 0º to 90º or from 0º to 120º depending on the bar for the 3 points star, from 0º to 90º or from 0º to 135º depending on the bar for the 4-pointed star and from 0º to 90º or from 0º to 150º depending on the bar for the 6-pointed star. That is, with a bigger number of vertices that attack the central core, the maximum openings between the bars are increased.

5.2 2D pattern

As can be seen in **Figure 9**, as for the unit pattern, the bigger the number of sides in the generated star is, the more this structure grows when it is unfolded; that fact increases the growth factor in the joint structure. Therefore, it can be established that the joint structure grows proportionally with the sides of the polygon that generates it.

The growth factors of these three 2D stellated reentrant auxetic structures analyzed have also been numerically analyzed. The numerical data obtained in relation to the growth of the structures and their mass-volume ratio, have been the following:

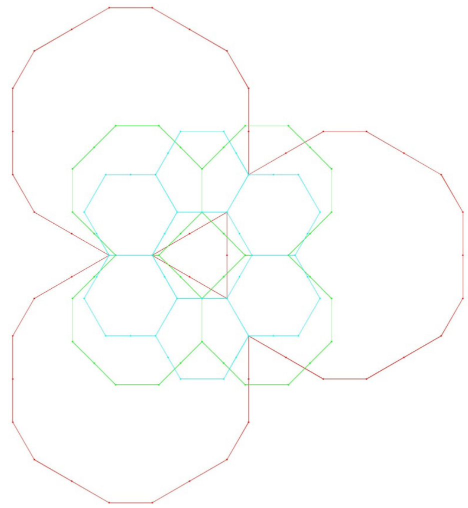

Figure 9. Overlap of the stellated reentrant auxetic structures of three, four and six points, own elaboration.

Table 2. Relation between minimum areas (A_{min}) and maximum areas (A_{max}) with the total length of the bars of the structure, own elaboration.

	L	A_{min}	A_{max}	K_{min}	K_{max}	FC (:)	FC (-)
3 points	32.78	2.6	21.53	0.08	0.66	8.28	0.58
4 points	74.91	9.00	80.89	0.12	1.08	8.99	0.96
6 points	114.00	21.99	233.03	0.19	2.04	10.60	1.85

The growth factors also increase with the number of vertices that attack the central core of the star, numerically; obtaining the highest growth factors the 6-pointed stellated 2D set reentrant auxetic structure, which corroborates the graphic analysis.

Analyzing these combinations of 2D star reentrant structures, it can also be observed that all their nodes are articulated in the plane, and two or three bars converge in them, allowing rotations in the xy axis ranging from 0º to 90º (2 bars) or from 0º to 120º (3 bars, unless it belongs to the outer perimeter of the set) for the 3-pointed star, from 0º to 90º (2 bars) or from 0º to 135º (3 bars, unless it belongs to the outer perimeter of the set), for the 4-pointed star and from 0º to 90º (2 bars) or from 0º to 150º (3 bars, unless it belongs to the outer perimeter of the set) for the 6-pointed star. That is, individual pattern properties are preserved for stellated reentrant auxetic structures.

6. Conclusions

It is concluded that planar stellated reentrant auxetic structures have useful growth factors to apply to surfaces of architecture. The maximum openings between the bars are increased with the number of bars. They never fully collapse auxetically. It can be a problem to transport them. The stabilization concept should be developed at a certain opening moment [20,21]. The next step will be developing the opening and closing points of the mechanisms that would have to be stabilized to convert them into architectural structures.

7. Limitations

This is a theoretical study of the geometry of the two-dimension-stellated reentrant auxetic structures to transformable architecture. But architecture is linked to the use of materials. So, these results are limited by this fact. To use these results, the total length of all bars used in the design of each pattern (L) will be changed to the real quantity of material (mass). This way, the lightness of the deployable structure will be obtained.

This type of pattern is very useful to transformable architecture, and they generate advantages to the transport and posterior deployment of the structure. They are useful to install as quick assembly structures.

Conflict of Interest

There is no conflict of interest.

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