ARTICLE

The Possibilities of Using the Minimax Method to Diagnose the State of the Atmosphere

Elena S. Andreeva

Department of Life Safety and Environmental Protection, Don State Technical University, Rostov-on-Don, 344111, Russia

ABSTRACT

The article is devoted to the discussion of the possibilities of approbation of one of the probabilistic methods of verification of evaluation works—the minimax method or the method of establishing the minimum risk of making erroneous diagnoses of the instability of the planetary boundary layer of air. Within the framework of this study, the task of probabilistic forecasting of diagnostic parameters and their combinations, leading in their totality to the formation of an unstable state of the planetary boundary layer of the atmosphere, was carried out. It is this state that, as shown by previous studies, a priori contribution to the development of a number of weather phenomena dangerous for society (squalls, hail, heavy rains, etc.). The results of applying the minimax method made it possible to identify a number of parameters, such as the intensity of circulation, the activity of the Earth’s magnetosphere, and the components of the geostrophic wind velocity, the combination of which led to the development of instability. In the future, it is possible to further expand the number of diagnosed parameters to identify more sensitive elements. In this sense, the minimax method, the usefulness of which is shown in this study, can be considered as one of the preparatory steps for the subsequent more detailed method for forecasting individual hazardous weather phenomena.

Keywords: Minimax method; Dangerous weather phenomena; Atmospheric instability; Boundary layer of the atmosphere; Intensity of atmospheric circulation; Earth’s magnetosphere; Geostrophic wind
1. Introduction

As convincingly proved in a number of scientific studies conducted in different years, the unstable state of the atmosphere, more precisely, its planetary boundary layer, is provided by a number of conditions. Among which, of course, both direct telluric or terrestrial origin and cosmic parameters describing aspects of the physical state of the atmospheric boundary layer and part of the so-called free atmosphere above it vertically should be taken into account. Telluric factors are associated with atmospheric processes occurring in the immediate vicinity of the underlying surface and primarily dependent on it. The instability of the atmospheric air layer inside the boundary layer contributes to the emergence and further implementation of a number of dangerous weather phenomena of various scales and damages. In this regard, studies aimed at identifying and diagnosing the state of instability of the atmospheric layer are relevant, since they affect not only the interests of states, but also of humanity as a whole [1-4].

Within the framework of this study, we will consider the problem of probabilistic forecasting of an unstable state of the atmosphere, which a priori will be accompanied by the largest number of hazardous weather phenomena (rainfall, squalls, strong winds, hail, etc.) [1,2,4].

To determine the probability of erroneous decisions when diagnosing the instability of the system - the stability of the state of the atmosphere, we use the minimax method [5-8].

This mathematical estimation method has been used since the 1960s [5] to solve a number of applied problems, if it was necessary to estimate the probability of erroneous and correct solutions in order to minimize incorrect (non-robust) results. Usually, this method was effectively used if the parameters of distributions of random variables of erroneous results are not sufficiently reliable or known. In this regard, based on the difficulties described in a number of scientific publications in accurately identifying the states of the atmosphere, relying on the capabilities of the minimax method, from the list of possible diagnostic parameters of the system under study, we select those that directly characterize the intensity of the circulation of the atmospheric layer, activated by a certain combination of telluric factors: Vertical and horizontal components of geostrophic wind speed (m/s); the actual intensity of atmospheric circulation (hPa/100 km) [9-12]. From the whole variety of cosmic factors affecting the Earth’s atmosphere, we will use the index of magnetic activity (points), which allows us to indirectly reveal the fact of the impact of solar activity (high-energy particles of the solar wind) on the electromagnetic field. It seems that the index of magnetic activity makes it possible to sufficiently fully reveal the fact of disturbance of the Earth’s electromagnetic field due to the release of high-energy particles during increased solar activity.

2. Materials and methods

The study is based on the daily data of the information array of the NSP NCAR, which includes the values of air temperature. The study is based on daily data from the NSP NKAI information array, which includes the values of air temperature, components of geostrophic wind, cloudiness, precipitation, surface atmospheric pressure, averaged over the period from 1948 to 2005, and presented as a numerical field with sides 60-40° north latitude and 40-60° east longitude. The specified meteorological information was converted into database No. 1 (meteorological fields under various combinations of atmospheric conditions). Database No. 2 included the values of the magnetic activity index (IZMIRAN) in the SI system for individual years, chosen arbitrarily (1984, 1991, 2000, etc.); recurrence (probability) of a number of dangerous weather phenomena, as well as parameters of atmospheric circulation within the lower layer (planetary boundary layer). In particular, the sample for 1948-2005 from meteorological logs TM-1 fixing dangerous weather phenomena made it possible to obtain numerical characteristics of the frequency of occurrence of dangerous weather phenomena, taking into account their intensity and duration. The mentioned recurrences of a number of dangerous weather phenomena were studied over a significant period of time (more than 50 years),
which made it possible to convert them into probabilities. The calculation method was used to obtain the values of atmospheric circulation intensity, expressed as coefficients (Kac circulation indices) \[9\]. Since all the indicated meteorological characteristics had their own dimensions and ranges of values, a special coefficient was introduced for their consistency, taking these circumstances into account. Database No. 1 was a reference (training within the framework of this model); database No. 2 contained factual information about certain dangerous weather phenomena that developed against the background of instability of the lower layer of the atmosphere (planetary boundary layer). The results of the model operation - a probabilistic assessment of the identified cases of atmospheric instability—were presented in database No. 3. Finally, for diagnosing an unstable state of the atmospheric boundary layer, the minimax method was used in this work—a method for mathematically estimating the probability of making an erroneous/correct diagnosis. Mathematical estimation methods are usually based on the belief that the probability distribution of computational errors is known, or at least the mathematical expectation and covariance matrix of the distributions of these errors are known. However, in practice this does not happen very often. In particular, if there are doubts about the correctness of the calculation results or it is necessary to check the adequacy of the obtained solutions, the minimax method of estimating the available results makes it possible to effectively identify erroneous or, conversely, correctly established solutions. Actually, the minimax method with all its capabilities was described in the monograph by Piter H’yuber in 1984 and began to be used either for mathematical evaluation and analysis of the stability of engineering indicators, or for research in the field of economics (risks of losses, damages, etc.). However, in the framework of meteorological studies, the above method was applied for the first time, since the problem of correctly identifying the states of the atmosphere, more precisely, its planetary boundary layer, for developing forecasts of individual hazardous weather phenomena is very relevant and far from a final solution. The computer implementation of this method is based on net technology using the Visual Basic language.

3. Results

Let us take the dichotomy of states inherent in the boundary layer of the atmosphere as a basis: unstable and stable. Obviously, the transition from an unstable state to a stable one can occur with a certain set of parameters and their configuration, in this case called diagnostic ones \[13-20\]. Within the framework of the problem being solved, we will assume that the change of diagnosed states of the atmospheric boundary layer occurs in n-dimensional space when the diagnosed parameters \(x_1, x_2, x_3, \ldots x_n\) are combined. Making a decision about a particular state of the object under study is reduced to identifying in the diagnostic space a certain boundary surface that divides the indicated space into two regions \(S_0\) and \(S_1\). In this case, these areas are a geometric expression to indicate the possibility of the object under study being in one state or another. So, if, for example, the vector \(x\), including \(x_1, x_2, \ldots x_n\) of diagnostic parameters obtained as a result of observations, belongs to the region \(S_0\) (\(x \in S_0\)), the diagnosis is \(D_0\) - “the boundary layer of the atmosphere is unstable”. If the vector belongs to the region \(S_1\) (\(x \in S_1\)), we will assume that the boundary layer is stable, which corresponds to the diagnosis \(D_1\) (Figure 1).

![Figure 1](image-url)
dividing the space into areas $S_0$ and $S_1$ can be calculated using relation (1):

$$\begin{align*}
P_F &= P(D_0)P(x \in S_0 \mid D_0) = P(D_0) \int_{S_0} w(x \mid D_0) dx \\
P_M &= P(D_1)P(x \in S_1 \mid D_1) = P(D_1) \int_{S_1} w(x \mid D_1) dx
\end{align*}$$

(1)

The error probability in diagnosing the state of the unstable boundary layer of the atmosphere will be taken as $P_F$. The probability of an erroneous statement of a stable state of the atmospheric boundary layer will be called $P_M$. We will consider errors $P_F$ and $P_M$ as errors of the first and second kind, respectively:

$$\begin{align*}
P_F &= P(D_0)P(x \in S_1 \mid D_0) = P(D_0) \int_{S_1} w(x \mid D_0) dx \\
P_M &= P(D_1)P(x \in S_0 \mid D_1) = P(D_1) \int_{S_0} w(x \mid D_1) dx
\end{align*}$$

(2)

Figure 2 shows the probability density distribution of the vector $x$ of the field of diagnostic parameters ($x_1, x_2, \ldots, x_n$) for two possible states of the object under study (unstable and stable). So, the area $S_0$ at $x < x^o$ corresponds to the diagnosis $D_0$; $S_1$ at $x > x^o$ determines the diagnosis $D_1$; finally, $x^o$ is the value of the vector $x$ of diagnostic parameters separating the areas $S_0$ and $S_1$. Segments BC and AB, formed at the intersection of probability density curves for the distribution of the vector of diagnostic parameters, represent the probabilities of errors in diagnosing the state of the atmospheric boundary layer $P_F$ and $P_M$ of the first and second kind, respectively.

![Figure 2. Conditional probability densities of a possible hit of the vector $x$ in the areas $S_0$ and $S_1$.](image)

Hence, in order to calculate the required probability of the diagnosed object transition from a stable state to an unstable state, it is necessary to investigate the area of the geometric figure ABC, divided by the perpendicular $x^o$. To obtain reliable results for calculating the area of a figure, we use the method of dividing the figure into unit segments with their subsequent summation. Taking into account, at the same time, that the number of segments is limited, it is selected empirically and depends on the set of diagnosed parameters $x_1, x_2, \ldots, x_n$.

Since errors in diagnosing an unstable state of the atmospheric boundary layer can lead to omissions in identifying possible hazardous weather phenomena that develop into the lower layer of the atmosphere and pose a real threat to society, economic facilities, etc., which, in connection with the above, cannot be acceptable to society. For this reason, within the framework of this study, the diagnostics of the object under study—the atmospheric boundary layer and its states—will be carried out on the basis of a consequence of the minimum risk method—*the method of the minimum number of erroneous decisions (the minimax method)*.

As is known to us all, this method is used if the costs of losses and gains in the diagnosis are unknown. In our case, the magnitude of damage from possible hazardous weather phenomena is also very close, and their exact size can only be estimated after the elimination of the consequences. The rule for making a diagnosis is found from the minimum proportion of erroneous decisions. The probability of such solutions can be determined by relation (3):

$$P_{\text{min}} = P(D_0) \int_{S_0} w(x \mid D_0) dx + P(D_1) \int_{S_1} w(x \mid D_1) dx$$

(3)

It can be shown that the above probability is minimal if the region $S_0$ of the diagnosis $D_0$ contains $x$ values for which:

$$\lambda(x) > \frac{P(D_1)}{P(D_0)} = \lambda_0$$

(4)

The equation $\lambda(x) = \lambda_0$ defines the boundary surface between the regions $S_0$ and $S_1$ of the diagnostic space. Then the rule for making a diagnosis can be written as inequalities (5):

$$\begin{align*}
\lambda(x) > \lambda_0, \text{ therefore, the object is in state } D_0 \\
\lambda(x) < \lambda_0, \text{ therefore, the object is in state } D_1
\end{align*}$$

(5)

Errors of the first $P_F$ and second $P_M$ kind when us-
ing this rule (5) are calculated by formula (2) taking into account the division of the diagnostic space into areas \( S_0 \) and \( S_1 \).

At present, the minimax method discussed in this study is applied to a surface area with coordinates 60-40° northern latitude and 40-60° eastern longitude. To assess the reliability of the results obtained, individual years from the time interval 1980-2020 were used, within which certain dangerous weather phenomena were recorded within the above geographical coordinates.

The actual result of the study is the established combination of diagnostic parameters, in which the probability of an error of the first kind, \( P_F \), is minimal, which confirms the correctness of the diagnosis and the identified instability of the atmospheric boundary layer. A correct statement of atmospheric instability makes it possible to make an adequate forecast of hazardous phenomena developing in its lower layer, which generally reduces the magnitude of socio-economic losses or damages.

**4. Discussion**

Application of the minimax method for individual years within the time interval 1980-2020 made it possible to identify the most probable diagnostic parameters \( (x_1, x_2, \ldots, x_n) \) that contributed to the development of the instability of the atmospheric boundary layer. These include the following parameters:

- The intensity of atmospheric circulation is not less than 0.3 hPa/100 km (with the predominant role of its meridional component);
- The speed of the vertical component of the geostrophic wind is not less than \(-1.7\) to \(-2.1\) m/s; horizontal component not less than \(-0.3\) to 0.5 m/s;
- The intensity of the Earth’s magnetosphere is not less than 2.3 points.

It is important to note that, in addition to the above values of probable diagnostic parameters and their combinations, as a result of approbation of the minimax method, situations were comprehended when the least probable \( P_F \) errors of the first kind were observed in diagnosing an unstable state. So, in particular, the minimum errors \( P_F \) in ascertaining the instability of the atmosphere were observed in the following situations, when:

- Circulation intensity had small values;
- The weakly disturbed magnetic field of the Earth was recorded;
- Geostrophic wind velocities differed in average values against the background of negative signs of vectors of their vertical and horizontal components.

It seems that the results obtained can be quite explained by the fact that an increase in the magnetic activity of the Earth’s magnetosphere, as well as an increase in the intensity of atmospheric circulation, which contributes to an increase in flow velocities, including the geostrophic wind, in their totality can create so-called noise or interference for the correct making diagnoses of the state of the atmosphere, thereby increasing the likelihood of misdiagnosis.
5. Conclusions

Thus, the mentioned above results of applying the minimum risk or minimax method showed the fundamental usefulness of adapting this method to search for the most probable situational parameters, the combination of which can contribute to the development of atmospheric boundary layer instability in this case. To test this method in the present study, some telluric and cosmic parameters were selected that describe aspects of the physical state of the atmospheric boundary layer and part of the so-called free atmosphere above it vertically. In the future, subsequent selection and expansion of the number of diagnosed parameters are quite possible in order to identify more sensitive elements, the combination of which leads to the development of instability of the atmospheric boundary layer. In this sense, the minimax method, the usefulness of which is shown in this study, can be considered as one of the preparatory steps for the subsequent more detailed method for forecasting individual hazardous weather phenomena.

Author Contributions

The author of this article fully formulated the idea, made calculations, wrote the text, took care of the design of the article.

Conflict of Interest

There is no conflict of interest.

Funding

This research received no external funding.

Acknowledgment

In memory of my Teacher and outstanding scientist and oceanologist Professor Karlin L.N.

References


[18] Шамшура, С.А., Богданова, И.В., 2008. Математическая модель резонансного гасителя (Russian) [Mathematical model of a resonant damper]. Вестник Ростовского государственного университета путей...

[19] Булыгин, Ю.И., Панченко, О.С., Романов, В.А., et al., 2013. Повышение эффективности обеспыливания воздуха рабочих зон металлообрабатывающих и деревообрабатывающих производств (Russian) [Improving the efficiency of dedusting the air of working areas of metalworking and woodworking industries]. Вестник Донского государственного технического университета. 13(7-8), 49-57. DOI: https://doi.org/10.12737/2020