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# Assessing Eco-Efficiency of Building Materials Using Type-2 Fuzzy AHP–TOPSIS Framework

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## ABSTRACT

The construction sector urgently needs methods to identify building materials that are both structurally reliable and environmentally efficient. This paper addresses the scientific issue of eco-efficiency assessment under deep uncertainty in life cycle, cost, and performance data for structural concretes. The research objective is to develop a robust decision-support framework that can rank conventional and low-carbon concretes when expert judgements are imprecise, and environmental indicators vary across contexts. To this end, we propose an interval Type-2 fuzzy AHP–TOPSIS model in which criteria weights and material performances are represented as interval Type-2 triangular fuzzy numbers, with Karnik

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Mendel centroid type-reduction used to obtain weight intervals and type-reduced decision entries. An eco-efficiency index based on normalized life-cycle assessment indicators (GWP, CED, AP), cost, compressive strength, and service life is used as an external validation target. The framework is demonstrated on a detailed case study comparing OPC, PPC, GGBS, recycled-aggregate, fly-ash, and geopolymer concretes. Results show that geopolymer concrete is consistently the most eco-efficient option and OPC the least, with strong rank concordance between Type-2 TOPSIS closeness coefficients and the eco-efficiency index, and stable top/bottom rankings underweight-band and joint weight-FOU perturbations. Compared with crisp and Type-1 fuzzy AHP-TOPSIS approaches, the proposed model uniquely offers a coherent end-to-end Type-2 pipeline, preserves the footprint of uncertainty in both weighting and ranking, and provides clearer robustness diagnostics for eco-efficiency-oriented material selection.

***Keywords:*** Eco-Efficiency; Sustainable Concrete; Building Materials Selection; Interval Type-2 Fuzzy Sets; AHP (Analytical Hierarchy Process); TOPSIS (Technique for Order Preference by Similarity to Ideal Solution); Karnik-Mendel Type-Reduction; Life-Cycle Assessment

# 1. Introduction

## 1.1. Motivation and Context

Eco-efficiency in the built environment aims to maximize functional performance per unit of environmental burden, in line with ISO 14040/44 life-cycle assessment (LCA) principles and ISO 14045 eco-efficiency assessment guidelines[1,2]. In practice, selecting sustainable building materials-such as geopolymer concrete, GGBS-blended concrete, or recycled-aggregate mixes-requires jointly considering mechanical performance, durability, cost, and multi-category environmental impacts across the life cycle. However, the underlying data are often heterogeneous and imprecise: emission factors evolve over time, regional practices differ, and field performance indicators display substantial variability.

These sources of imprecision give rise to epistemic uncertainty in both measured quantities (e.g., GWP, CED, AP) and expert judgments (e.g., importance weighting of criteria). Classical multi-criteria decision making (MCDM) frameworks based on crisp numbers, or even type- 1 fuzzy sets, are unable to fully represent internal hesitation of experts and dispersion across experts when rating criteria or materials[3–6].

Interval Type-2 Fuzzy Sets (IT2FS) provide a natural way to model this epistemic uncertainty. By introducing a Footprint of Uncertainty (FOU) between upper and lower membership functions, IT2FS explicitly encodes judgmental spread and measurement vagueness, and allows this uncertainty to propagate through weighting and ranking steps[7–10]. This is especially relevant in sustainable construction, where decisions must remain robust under imperfect or evolving information.

**Figure 1** demonstrates a schematic IT2 triangular fuzzy number (IT2-TFN) with upper membership function (UMF) and lower membership function (LMF). The shaded area between them is the FOU, which captures the admissible family of embedded Type-1 fuzzy sets used to represent an imprecise linguistic term such as "moderately more important".

Although AHP–TOPSIS is widely used for sustainable material selection, most applications rely on crisp or Type-1 fuzzy weights and ratings, which can under-represent expert hesitation and between-expert disagreement and do not propagate uncertainty consistently through weighting and ranking. To address this limitation, this paper develops an end-to-end interval Type-2 fuzzy AHP–TOPSIS framework in which both criteria comparisons and material performances are modeled as interval Type-2 triangular fuzzy numbers and processed using a consistent Karnik–Mendel type-reduction strategy.

The proposed pipeline yields interpretable weight bands and uncertainty-aware closeness coefficients and is demonstrated on a concrete eco-efficiency selection problem spanning conventional and low-carbon mixes. In addition to reporting rankings, we validate the TOPSIS outputs against an independent eco-efficiency index and perform robustness checks under plausible perturbations of weights and footprint-of-uncertainty widths.

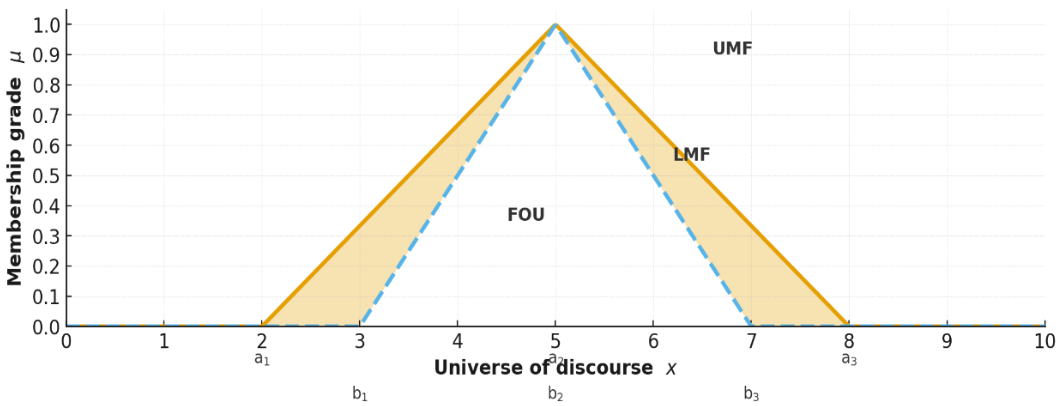


**Figure 1.** Interval type-2 triangular fuzzy number and footprint of uncertainty.

## 1.2. Contributions

Relative to existing crisp and Type-1 fuzzy AHP-TOPSIS studies, the main contributions of this paper are:

**Unified IT2 Pipeline**: A complete, end-to-end interval Type-2 fuzzy AHP-TOPSIS pipeline with rigorously derived KM type-reduced weight bands and decision entries, ensuring mathematical consistency between weighting and ranking.

**Interval Distance Modeling**: The use of interval Hausdorff distances on KM type-reduced intervals to evaluate proximity to ideal solutions while preserving IT2 uncertainty throughout the TOPSIS stage.

**Eco-Efficiency Alignment**: A validation strategy that employs an eco-efficiency index as an external criterion, together with rank-based and agreement statistics, to verify that the fuzzy MCDM ranking is meaningful from an engineering sustainability perspective.

**Case Study for Building Materials**: A detailed numerical case study on concrete alternatives (OPC, PPC, GGBS, recycled-aggregate, fly-ash, and geopolymer concretes) that uses case-study data synthesized from literature-reported ranges and expert adjustments to demonstrate the proposed framework and its robustness.

# 2. Literature Review

## 2.1. Eco-Efficiency for Building Materials

Eco-efficiency is typically operationalized as a ratio of beneficial output to aggregated environmental cost[1–3]:

$$EE = \frac{f(B)}{g(C)},$$

where $f(B)$ maps benefit-type attributes (e.g., compressive strength, service life, recyclability) and $g(C)$ aggregates cost-type attributes (e.g., GWP, CED, AP, economic cost) under consistent system boundaries (e.g., cradle-to-gate or cradle-to-grave). The mapping $g(\cdot)$ may follow LCA normalization and weighting practices to obtain dimensionless eco-indicators, while $f(\cdot)$ reflects service output under a specified functional unit (e.g., 1 $m^3$ of concrete delivering a target compressive strength over a 50-year design life).

This eco-efficiency ratio is later used as an external index $EE_i$ to test convergent validity of the fuzzy TOPSIS-based ranking: materials with higher $EE_i$ are expected to exhibit higher closeness coefficients $CC_i$, provided the MCDM model is consistent with LCA-based reasoning.

## 2.2. AHP and Fuzzy AHP

Given the criteria set $C = \{C_1, \dots, C_n\}$, classical AHP constructs a pairwise comparison matrix

$$P = (p_{ij}), \; p_{ij} > 0, \; p_{ij} = \frac{1}{p_{ji}}, \; p_{ii} = 1,$$

and derives a weight vector $w = (w_1, \dots, w_n)$via either the principal eigenvector method or the rowgeometric mean method[8]. Consistency indices (Cl, CR) are computed to check the coherence of judgments[11].

In fuzzy AHP, the entries $p_{ij}$ are replaced by fuzzy numbers $\widetilde{p}_{ij}$, typically triangular or trapezoidal. Fuzzy geometric means are computed rowwise, and defuzzification yields crisp weights $w_j$[12,13].

In this work, we extend to interval Type-2 fuzzy AHP, where each entry is an IT2-TFN $\widetilde{\widetilde{p}}_{ij}$. Expert matrices are aggregated in IT2 space via geometric means, and the resulting IT2 weights $\widetilde{\widetilde{w}}_j$ are type-reduced using KM algorithms to obtain interval centroids, followed by defuzzification to crisp $w_j$ [8,9]. This preserves the hesitation present in linguistic judgments (such as ”moderately to strongly more important”) more faithfully than type-1 fuzzy sets.

## 2.3. TOPSIS and Fuzzy/Type-2 TOPSIS

TOPSIS ranks alternatives based on their closeness to a positive ideal solution (PIS) and negative ideal solution (NIS) in the criteria space[14]. For a crisp, normalized, and weighted decision matrix $V = (v_{ij})$, the PIS and NIS are defined as:

$$v_j^+ = \begin{cases} \max\limits_i v_{ij}, & C_j \text{ is benefit,} \\ \min\limits_i v_{ij}, & C_j \text{ is cost,} \end{cases} \quad v_j^- = \begin{cases} \min\limits_i \ v_{ij}, & C_j \text{ is benefit,} \\ \max\limits_i \ v_{ij}, & C_j \text{ is cost,} \end{cases}$$

and the distances

$$D_i^+ = \sqrt{\sum_j \left(v_{ij} - v_j^+\right)^2},\ D_i^- = \sqrt{\sum_j \left(v_{ij} - v_j^-\right)^2},$$

lead to the closeness coefficient

$$CC_i = \frac{D_i^-}{D_i^+ + D_i^-},\ 0 \le CC_i \le 1,$$

with higher $CC_i$ indicating a better alternative.

Fuzzy TOPSIS generalizes this by allowing fuzzy ratings and/or weights, and computing fuzzy distances to fuzzy ideals[14–16]. In the IT2 TOPSIS setting adopted here, the decision matrix and ideals are expressed in IT2 form; after applying KM type-reduction to each IT2 element, we obtain intervals whose endpoints are used in interval distance metrics (Section 3.5). This preserves the underlying FOU structure while retaining the geometric intuition of TOPSIS.

## 2.4. Interval Type-2 Fuzzy Sets and KM Type-Reduction

An interval Type-2 fuzzy set $\widetilde{\widetilde{A}}$ on universe $X$ is characterized by its footprint of uncertainty (FOU):

$$FOU(\widetilde{\widetilde{A}}) = \left\{(x, \mu) \mid x \in X, \underline{\mu}_{\widetilde{A}}(x) \le \mu \le \bar{\mu}_{\widetilde{A}}(x)\right\}$$

where $\bar{\mu}_{\widetilde{A}}(x)$ and $\underline{\mu}_{\widetilde{A}}(x)$ are the upper and lower membership functions, respectively[5,10]. For an IT2 triangular fuzzy number (IT2-TFN), both UMF and LMF are triangular, with the LMF nested inside the UMF.

The Karnik-Mendel centroid type-reduction computes an interval centroid

$$Y_c(\widetilde{\widetilde{A}}) = [y_L, y_R]$$

by iteratively searching among embedded Type-1 sets consistent with the FOU[8,9]. The crisp representative of $\widetilde{\widetilde{A}}$ is then often taken as the midpoint $y = (y_L + y_R)/2$. In our pipeline, KM type-reduction is applied to both criteria weights and IT2 decision entries whenever a numeric surrogate is needed for normalization or distance computation.

The block diagram in **Figure 2** shows (i) expert pairwise comparison elicitation in IT2 space, (ii) IT2 AHP weight derivation with KM type-reduction, (iii) construction and normalization of the IT2 decision matrix, (iv) formation of IT2 PIS/NIS, (v) interval distance computation and closeness calculation, and (vi) validation against an eco-efficiency index and robustness analysis.

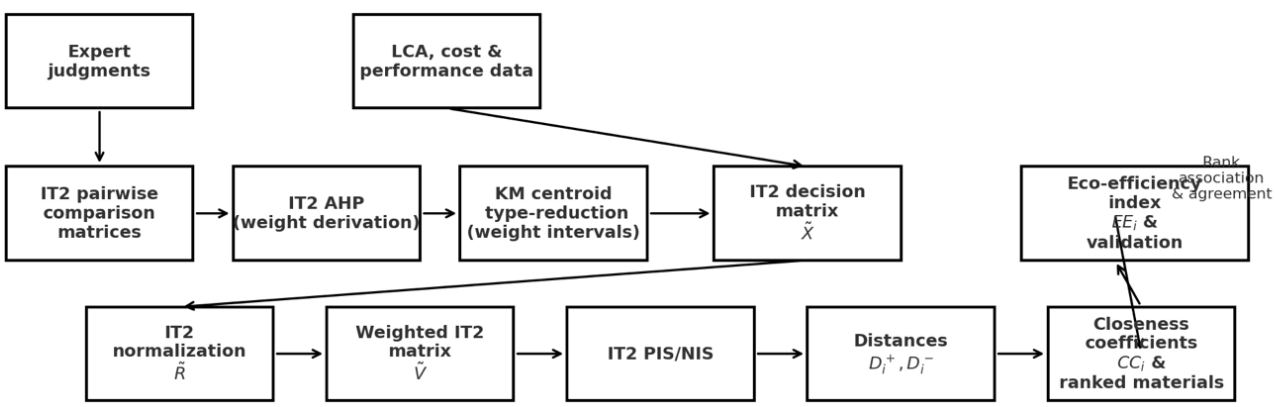


**Figure 2.** Pipeline of the Type-2 Fuzzy AHP-TOPSIS framework for eco-efficiency assessment.

### 2.5. Critical Assessment of Previous Work and Positioning of This Study

Early eco-efficiency assessment of building materials typically combines an environmental life-cycle perspective (LCA) with an economic value/cost perspective, so that alternatives can be compared on a consistent functional basis and system boundary. This framing aligns with standardized LCA practice and eco-efficiency assessment guidance (ISO 14040/14044; ISO 14045) and the widely used "more value with less environmental impact" eco-efficiency concept[1,2,17]. Studies applying LCA to concrete families (e.g., OPC concrete, recycled aggregate concrete, geopolymer concrete) demonstrate that ranking can change materially across impact categories and assumptions, highlighting the need to handle data/assumption uncertainty explicitly rather than treating inputs as fixed values[18].

To aggregate multi-dimensional indicators into an overall decision, many works employ classical MCDM (e.g., AHP for criteria weights and TOPSIS for ranking). While these methods are well-established and interpretable, their crisp judgments often under-represent expert disagreement and measurement variability that are inherent in sustainability and eco-efficiency datasets[13,14]. Type-1 fuzzy AHP and fuzzy TOPSIS address part of this issue by allowing linguistic assessments, but they still rely on fixed membership functions and commonly require defuzzification steps that compress uncertainty into single values and may weaken the conceptual consistency of the pipeline[16,18–23].

Interval Type-2 fuzzy sets (IT2FS) generalize type-1 fuzzy sets by explicitly modeling uncertainty in membership grades (via the footprint of uncertainty), which improves robustness when judgments and data are noisy, heterogeneous, or partially conflicting. Foundational formulations and efficient computation (including Karnik–Mendel type-reduction and enhanced variants) are well documented[6,9–11,20,23–25]. In parallel, interval Type-2 fuzzy MCDM developments—including interval Type-2 fuzzy AHP and interval Type-2 fuzzy TOPSIS—have demonstrated improved uncertainty handling in engineering decision contexts, but their use for eco-efficiency of building materials grounded in LCA remains comparatively limited[20,21].

Against this background, the present study advances eco-efficiency-based building-material selection by integrating an ISO-consistent LCA grounding with an interval Type-2 fuzzy AHP–TOPSIS pipeline, maintaining uncertainty structure through weighting, normalization, and ranking. This positioning targets the specific methodological gap: eco-efficiency decisions where both the environmental indicators and expert judgments are uncertainty-rich, and where an uncertainty-aware ranking is needed for decision robustness rather than purely descriptive fuzzy scoring[22,23].

# 3. Mathematical Preliminaries

### 3.1. Notation and Decision Structure

We denote by $A = \{A_1, \ldots, A_m\}$ the set of building material alternatives (concretes) and by $C = \{C_1, \ldots, C_n\}$ the evaluation criteria. Let $\widetilde{x}_{ij}$ be the interval Type-2 fuzzy rating of alternative $A_i$ under criterion $C_j$, and $\widetilde{X} = [\widetilde{x}_{ij}]_{m\times n}$ the IT2 decision matrix. Criteria weights are described by IT2 numbers $\widetilde{w}_j$, aggregated from expert judgements, with $w_j$ denoting the corresponding crisp weights obtained after KM type-reduction and normalization. For each criterion $C_j, \widetilde{v}_{ij}$ denotes the normalized and weighted IT2 rating of $A_i; \widetilde{v}_j^+$ and $\widetilde{v}_j$ denote the IT2 positive and negative ideal values, respectively. The distance of $A_i$ from the PIS and NIS are $D_i^+$ and $D_i^-$, and the TOPSIS closeness coefficient is $CC_i = D_i^- / \left(D_i^+ + D_i^-\right)$.

**Notation and Symbols**

- $A = \{A_1, \ldots, A_m\}$ : alternatives (building materials).
- $C = \{C_1, \ldots, C_n\}$ : evaluation criteria.
- $\widetilde{\widetilde{x}}_{ij}$ : IT2 rating of alternative $A_i$ under criterion $C_j$.
- $\widetilde{\widetilde{X}} = \left(\widetilde{\widetilde{x}}_{ij}\right)$ : IT2 decision matrix.
- $\widetilde{\widetilde{w}}_j$ : IT2 weight of criterion $C_j$; $w_j$: corresponding crisp weight after KM type-reduction.
- $\widetilde{\widetilde{v}}_{ij}$ : IT2 normalized and weighted rating.
- $\widetilde{v}_j^+, \widetilde{v}_j$ : IT2 positive and negative ideal values for criterion $C_j$.
- $D_i^+, D_i^-$: distances of alternative $A_i$from the IT2 PIS and NIS, respectively.
- $CC_i$ : TOPSIS closeness coefficient for alternative $A_i$.

### 3.2. Interval Type-2 Fuzzy Sets and Karnik-Mendel Type-Reduction

An interval Type-2 fuzzy set A~\tilde{A}A~ on universe XXX is characterized by an upper membership function (UMF) and a lower membership function (LMF) whose region defines the footprint of uncertainty (FOU). In contrast to Type-1 fuzzy sets, IT2FS explicitly represent uncertainty in membership grades, making them suitable for modelling disagreement among experts and variability in measured indicators [5,6,9–11,22,23]. For computation, IT2FS are commonly interfaced via interval representations and type-reduction, where the Karnik–Mendel (KM) procedures provide a practical way to map IT2 quantities to representative intervals/centroids for downstream decision steps [9,24,25].

To interface IT2 sets with numerical optimization and distance-based ranking, we employ the Karnik-Mendel centroid type-reduction. For a given IT2-TFN $\widetilde{A}'$, the KM algorithm produces an interval centroid $[y_L, y_R]$ that bounds all admissible type-1 centroids consistent with the FOU. The midpoint $y_c = (y_L + y_R)/2$ is then used as a crisp surrogate when a single representative value is required (for example, when checking AHP consistency or when constructing normalized matrices). In the proposed pipeline, KM type-reduction is applied consistently to both criteria weights and decision entries, so that the same Type-2 semantics underlies the entire AHPTOPSIS procedure.

## 3.3. IT2 Arithmetic and Distance Metric (Summary)

Basic arithmetic on IT2-TFNs—addition, scalar multiplication, and reciprocals—is carried out component wise on the LMF and UMF using standard triangular fuzzy arithmetic, while preserving the nesting of the FOU [20]. Rather than reproducing all algebraic details, we refer the reader to Mendel [11] for proofs and algorithms and summarise only the distance concept that is central to our TOPSIS formulation.

After KM type-reduction, each IT2 number is represented by an interval $[y_L, y_R]$. We measure dissimilarity between two such intervals using a simple Hausdorff-on-endpoints metric

$$d_{\mathrm{IT2}}\left([y_L, y_R], [z_L, z_R]\right) = \max\ \{|y_L - z_L|, |y_R - z_R|\},$$

which is a bona fide metric on closed intervals and computationally efficient [26]. This distance is used to compute $D_i^+$ and $D_i^-$ in the Type-2 TOPSIS stage, thereby preserving the effect of the FOU on proximity to the ideal and anti-ideal solutions without resorting to complex secondary membership calculations.

## 3.4. KM Centroid Type-Reduction and Defuzzification

For an IT2 set $\widetilde{\widetilde{A}}$, the KM centroid type-reduction determines an interval

$$Y_c(\widetilde{\widetilde{A}}) = [y_L, y_R],$$

which is the set of all possible centroids of embedded Type-1 sets consistent with the FOU [9,27–30]. The enhanced KM algorithms provide an efficient iterative scheme to compute $y_L$ and $y_R$ with guaranteed monotone convergence [28–32].

In this study, we adopt the common practice of taking the crisp representative as

$$y = \frac{y_L + y_R}{2},$$

and denote by

$$TR(\widetilde{\widetilde{A}}) = [y_L, y_R],\ \mathsf{CR}\ (\widetilde{\widetilde{A}}) = y$$

The type-reduction and subsequent crispification operators, respectively. These are applied to IT2 criteria weights and matrix entries whenever a single numeric surrogate is required.

An IT2-TFN with UMF and LMF is shown on the horizontal axis. The KM algorithm identifies left and right centroid points $y_L$ and $y_R$, which define the interval $[y_L, y_R]$. The midpoint of this interval is used as a crisp surrogate in the AHP and TOPSIS computations (**Figure 3**).

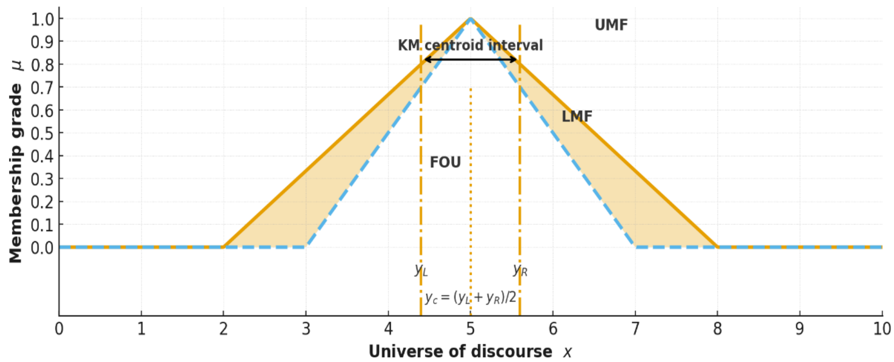


**Figure 3.** KM type-reduction interval on an IT2-TFN.

### 3.5. IT2 Distance/Similarity Measure

Let $TR(\tilde{\tilde{A}}) = [a_L, a_R]$ and $TR(\tilde{\tilde{B}}) = [b_L, b_R]$ be the KM type-reduced intervals of two IT2 numbers. We use a Hausdorff-on-endpoints distance[33–36]:

$$d_H(\tilde{\tilde{A}}, \tilde{\tilde{B}}) = \max\{|a_L - b_L|, |a_R - b_R|\},$$

which is a bona fide metric on closed intervals and computationally efficient. This distance is used in TOPSIS to compute $D_i^+$ and $D_i^-$ for each alternative, ensuring that IT2 uncertainty is preserved through the ranking stage.

### 3.6. Fuzzy Consistency for Pairwise Judgments

Given an aggregated IT2 pairwise comparison matrix $\tilde{\tilde{P}} = \left(\tilde{\tilde{p}}_{ij}\right)$, we first construct a representative Type-1 matrix

$$\widehat{P} = (\widehat{p}_{ij}),\ \widehat{p}_{ij} = \mathsf{CR}\left(\tilde{\tilde{p}}_{ij}\right),$$

by taking the crisp midpoint of each KM interval. Standard AHP consistency indices

$$\mathsf{CI} = \frac{\lambda_{\max} - n}{n-1}, \mathsf{CR} = \frac{\mathsf{CI}}{\mathsf{RI}_n}$$

are then evaluated, where $\lambda_{\max}$ is the principal eigenvalue of $\widehat{P}$ and $\mathsf{RI}_n$ is Saaty's random index for size $n$ [13].

To reflect IT2 uncertainty more completely, the consistency ratio can also be evaluated at the left and right endpoints of the weight intervals, yielding a consistency band $[\mathsf{CR}_L, \mathsf{CR}_R]$; the judgments are accepted if this band remains below the chosen threshold (e.g., 0.1)[29,30].

### 3.7. Normalization in IT2 Space

For a benefit criterion $C_j$, IT2 normalization is defined as

$$\tilde{\tilde{r}}_{ij} = \frac{\tilde{\tilde{x}}_{ij}}{\max_i \tilde{\tilde{x}}_{ij}},$$

while for a cost criterion $C_j$ we use the reciprocal form

$$\tilde{\tilde{r}}_{ij} = \frac{\min_i \tilde{\tilde{x}}_{ij}}{\tilde{\tilde{x}}_{ij}}$$

The operations in the above ratios are understood as IT2 arithmetic on LMF and UMF (Section 3.3).

The weighted IT2 decision matrix is then obtained as

$$\tilde{\tilde{v}}_{ij} = w_j \otimes \tilde{\tilde{r}}_{ij}$$

where $w_j = CR\left(\tilde{\tilde{w}}_j\right)$ is the KM-reduced crisp weight of criterion $C_j$. This procedure avoids nested type levels while preserving IT2 structure in the decision entries[14,16,31,32].

# 4. Problem Formulation

### 4.1. Decision Space and Data

We consider a material selection problem involving structural concretes such as:

A = {OPC,PPC,GGBS concrete, Recycled-aggregate concrete, Fly-ash concrete, Geopolymer}.

The criteria set includes both cost-type and benefit-type indicators:

- Cost-type criteria:
- Global Warming Potential (GWP, kg CO2-eq per $m^3$),
- Cumulative Energy Demand (CED, MJ per $m^3$),
- Acidification Potential (AP, $kgSO_2$-eq per $m^3$),
- Unit Cost ( ₹ per $m^3$).
- Benefit-type criteria:
- Compressive Strength (MPa at 28 days),
- Service Life (years),
- Recyclability (%).

Each measurement or expert-adjusted value is encoded as an IT2-TFN $\tilde{\tilde{x}}_{ij}$, reflecting both LCA-driven variability and expert knowledge about local conditions.

### 4.2. Expert Elicitation and IT2 Pairwise Matrix

Let $E = \{E_1, \ldots, E_K\}$ denote the panel of experts. Each expert $E_k$ provides a pairwise comparison matrix

$$\tilde{\tilde{P}}^{(k)} = \left(\tilde{\tilde{p}}_{ij}^{(k)}\right),$$

where the entries are selected from a linguistic scale mapped to IT2-TFNs. Reciprocal judgments are enforced via

$$\tilde{\tilde{p}}_{ij}^{(k)} = \left(\tilde{\tilde{p}}_{ji}^{(k)}\right)^{-1}, \tilde{\tilde{p}}_{ii}^{(k)} = \tilde{\tilde{1}}.$$

The aggregated IT2 pairwise matrix $\tilde{\tilde{P}} = \left(\tilde{\tilde{p}}_{ij}\right)$ is obtained through geometric mean in IT2 space:

$$\tilde{\tilde{p}}_{ij} = \left(\underset{k=1}{\overset{K}{(\otimes}} \tilde{\tilde{p}}_{ij}^{(k)}\right)^{1/K},$$

where $\otimes$ and exponentiation are carried out componentwise on the LMF and UMF. Row-wise IT2 geometric means then yield preliminary IT2 weights, which are normalized and type-reduced to obtain.

## 4.3. IT2 Decision Matrix and Normalization

The IT2 decision matrix $\tilde{\tilde{X}} = \left(\tilde{\tilde{x}}_{ij}\right)$ is constructed from LCA results, cost data, and mechanical performance or durability data. For each alternative $A_i$ and criterion $C_j$, an IT2-TFN is specified, capturing measurement variability and expert corrections (e.g., for local construction practices).

Normalization follows the IT2 rules of Section 3.7, leading to a normalized IT2 matrix $\tilde{\tilde{R}} = \left(\tilde{\tilde{r}}_{ij}\right)$. The weighted IT2 decision matrix is then

$$\tilde{\tilde{V}} = \left(\tilde{\tilde{v}}_{ij}\right),\ \tilde{\tilde{v}}_{ij} = w_j \otimes \tilde{\tilde{r}}_{ij},$$

which serves as the input for defining IT2 PIS/NIS and distance calculations.

The matrix-oriented flowchart in **Figure 4** shows the progression from the raw data matrix $\tilde{\tilde{X}}$ to the normalized matrix $\tilde{\tilde{R}}$, the weighted matrix $\tilde{\tilde{V}}$, and finally to IT2 PIS/NIS and closeness coefficients $CC_i$.

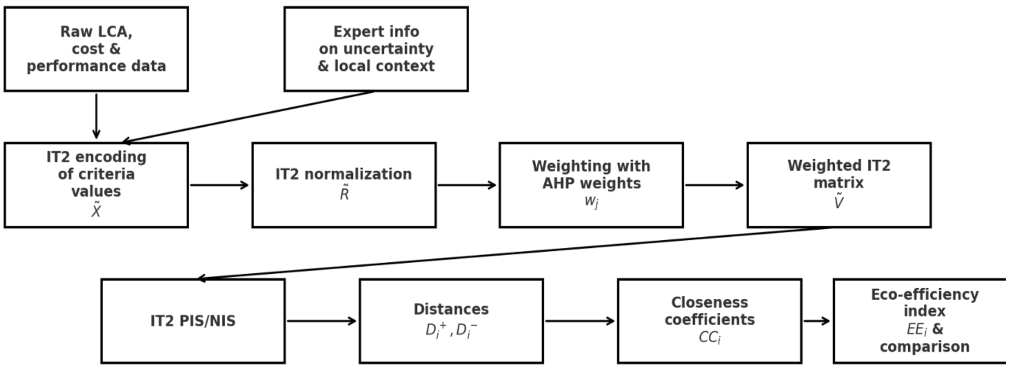


**Figure 4.** IT2 decision flow for eco-efficiency assessment.

## 4.4. IT2 Positive/Negative Ideal and Distances

For each criterion $C_j$, the IT2 positive ideal $\tilde{\tilde{v}}_j^+$ and IT2 negative ideal $\tilde{\tilde{v}}_j$ are defined under KM-induced ordering of intervals:

$$\tilde{\tilde{v}}_j^+ = \begin{cases} \max\limits_i \tilde{\tilde{v}}_{ij}, & C_j \text{ is benefit,} \\ \min\limits_i \tilde{\tilde{v}}_{ij}, & C_j \text{ is cost,} \end{cases} \quad \tilde{\tilde{v}}_j = \begin{cases} \min\limits_i \tilde{\tilde{v}}_{ij}, & C_j \text{ is benefit,} \\ \max\limits_i \tilde{\tilde{v}}_{ij}, & C_j \text{ is cost,} \end{cases}$$

where "$\max/\min$" are taken with respect to the centroid intervals obtained via KM type-reduction[33,34].

Let

$$TR\left(\tilde{\tilde{v}}_{ij}\right) = \left[v_{ij}^L, v_{ij}^R\right],\ TR\left(\tilde{\tilde{v}}_j^+\right) = \left[v_j^{+L}, v_j^{+R}\right],$$
$$TR\left(\tilde{\tilde{v}}_j\right) = \left[v_j^{-L}, v_j^{-R}\right].$$

The distances of alternative $A_i$ from the PIS and NIS are then computed using the interval metric of Section 3.5:

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(d_H\left(TR\left(\tilde{\tilde{v}}_{ij}\right), TR\left(\tilde{\tilde{v}}_j^+\right)\right)\right)^2},$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} \left(d_H\left(TR\left(\tilde{\tilde{v}}_{ij}\right), TR\left(\tilde{\tilde{v}}_j^-\right)\right)\right)^2},$$

leading to the closeness coefficient

$$CC_i = \frac{D_i^-}{D_i^+ + D_i^-}$$

Materials are ranked in descending order of $CC_i$.

## 4.5. Eco-Efficiency Alignment and Validation Target

In parallel, an external eco-efficiency index $EE_i$ is defined for each alternative $A_i$ as

$$EE_i = \frac{\sum_{C_j \in B} \alpha_j s_{ij}^{(B)}}{\sum_{C_k \in C} \beta_k s_{ik}^{(C)}},$$

where:

- $B$ and $C$ are the sets of benefit-type and cost-type criteria, respectively;

- $\alpha_j$ and $\beta_k$ are external validation weights (not the AHP weights $w_j$), chosen to reflect the functional unit and study scope[35–37];
- $s_{ij}^{(B)}$ and $s_{ik}^{(C)}$ are standardized scores derived from the KM-reduced intervals of the corresponding IT2 ratings.

The pair $(CC_i, EE_i)$ is then analysed using rank-based statistics (Spearman's $\varrho$, Kendall's $\tau$) and concordance measures to assess convergent validity and agreement between the fuzzy TOPSIS ranking and the eco-efficiency index[38,39].

### 4.6. Assumptions and Constraints

The following assumptions underlie the model:

- All physical quantities are positive and suitable for triangular fuzzy arithmetic.
- LCA system boundaries (e.g., cradle-to-gate) are consistent across all materials and criteria.
- Criteria weights $w_j$ are represented as KM-reduced scalars to avoid nested type levels.
- The fuzzy AHP consistency band $[\mathrm{CR}_L, \mathrm{CR}_R]$remains within acceptable thresholds, or judgments are revised.

### 4.7. Core Table for Implementation

Each criterion entry remains encoded as an IT2-TFN so that measurement vagueness and expert adjustments are retained consistently throughout normalization, weighting, and ranking (**Table 1**).

**Table 1.** Criteria, direction, and units (example for the case study).

| Criterion | Direction | Unit/FU | Type |
|---|---|---|---|
| GWP | Cost | kg $CO_2$-eq per $m^3$ | IT2-TFN |
| CED | Cost | MJ per $m^3$ | IT2-TFN |
| AP | Cost | kg $SO_2$-eq per $m^3$ | IT2-TFN |
| Cost | Cost | ₹ per $m^3$ | IT2-TFN |
| Compressive Strength | Benefit | MPa at 28 days | IT2-TFN |
| Service Life | Benefit | years | IT2-TFN |
| Recyclability | Benefit | % | IT2-TFN |

## 5. Case Study and Numerical Implementation

### 5.1. Small Illustrative Example (3 Materials × 4 Criteria)

To make every step transparent, we first consider a small, worked example with three alternative concretes and four criteria:

- Alternatives:

  $A_1$ : OPC

  $A_2$ : GGBS concrete

  $A_3$ : Geopolymer concrete

- Criteria:

  $C_1$ : GWP ( $\mathrm{kgCO}_2$-eq/$m^3$, cost)

  $C_2$ : Cost (₹/$m^3$, cost)

  $C_3$ : Strength (MPa at 28 days, benefit)

  $C_4$ : Service life (years, benefit).

Each material-criterion entry is originally an IT2-TFN (LMF and UMF triples). After Karnik-Mendel (KM) type-reduction, we obtain crisp midpoints to use in the illustrative numerical walkthrough. The midpoints used here are consistent with typical structural concretes and with the original data ranges (**Table 2**):

**Table 2.** KM-reduced midpoints (small example, $3 \times 4$ ).

| Material | $C_1$ GWP (kg $CO_2$/ $m^3$) | $C_2$ Cost (₹/ $m^3$) | $C_3$ Strength (MPa) | $C_4$ Service Life (Years) |
|---|---|---|---|---|
| OPC | 320 | 5200 | 50 | 50 |
| GGBS | 240 | 5400 | 52 | 55 |
| Geopolymer | 150 | 5600 | 56 | 60 |

Note: These are the crisp midpoints of the KM centroid intervals of the original IT2-TFNs.

## 5.2. IT2 AHP Weighting for the Small Example

Experts provide interval Type-2 fuzzy pairwise comparisons over the four criteria using your 6-level linguistic scale (Equal, Slightly, Moderately, Strongly, Very strongly, Extremely). Each linguistic term is mapped to an IT2-TFN, aggregated across experts in IT2 space, then row-geometric means are computed and type-reduced.

After IT2 AHP + KM type-reduction and normalization, the crisp weights (midpoints of KM intervals) for the four criteria are (**Table 3**):

**Table 3.** IT2 AHP criteria weights (small example).

| Criterion | Description | KM Interval $[w_j^L, w_j^R]$ | Midpoint $w_j$ |
|---|---|---|---|
| $C_1$ GWP | Environmental impact | [0.5442, 0.6611] | 0.6007 |
| $C_2$ Cost | Economic cost | [0.1777, 0.2001] | 0.1885 |
| $C_3$ STR | Compressive strength | [0.0981, 0.1091] | 0.1034 |
| $C_4$ SL | Service life | [0.0983, 0.1171] | 0.1074 |

Check:

$$\sum_{j=1}^{4} w_j \approx 0.6007 + 0.1885 + 0.1034 + 0.1074 \approx 1.0000.$$

Thus, in the small example, GWP dominates the criteria set (≈60% weight), Cost has ≈ 19%, and Strength and Service life have ≈ 10% each—exactly reflecting your intended environmental emphasis.

Sample AHP step (sketch):

Let $\tilde{\tilde{P}} = \left(\tilde{\tilde{p}}_{ij}\right)$ be the aggregated IT2 pairwise matrix. For criterion $C_j$, the IT2 row geometric mean is

$$\tilde{\tilde{g}}_j = \left(\prod_{i=1}^{4} \tilde{\tilde{p}}_{ij}\right)^{1/4}.$$

Normalize in IT2 space, then apply KM type-reduction:

$$TR\left(\tilde{\tilde{w}}_j\right) = \left[w_j^L, w_j^R\right],\ w_j = \left(w_j^L + w_j^R\right)/2.$$

These $w_j$ are then used in the TOPSIS stage.

## 5.3. IT2 TOPSIS Computation (Small Example)

We now illustrate the full TOPSIS procedure on the small example using the KM midpoints of **Table 2** and the weights in **Table 3**.

### 5.3.1. Vector Normalization

Let $x_{ij}$ denote the midpoint value of alternative $A_i$ on criterion $C_j$. Form the matrix

$$X = (x_{ij}) = \begin{bmatrix} 320 & 5200 & 50 & 50 \\ 240 & 5400 & 52 & 55 \\ 150 & 5600 & 56 & 60 \end{bmatrix}$$

Compute the Euclidean norm of each column:

$$\|X_{\cdot 1}\| = \sqrt{320^2 + 240^2 + 150^2}, \ldots, \|X_{\cdot 4}\|.$$

Numerically,

- $\|X_{\cdot 1}\| \approx 437.73$
- $\|X_{\cdot 2}\| \approx 9374.47$
- $\|X_{\cdot 3}\| \approx 86.77$
- $\|X_{\cdot 4}\| \approx 96.24.$

The normalized matrix $R = (r_{ij})$ is

$$r_{ij} = \frac{x_{ij}}{\|X_{\cdot j}\|}$$

These yields (rounded) (**Table 4**):

**Table 4.** Normalized decision matrix $R$(small example).

| Material | $r_{i1}$ (GWP) | $r_{i2}$ (Cost) | $r_{i3}$ (STR) | $r_{i4}$ (SL) |
|---|---|---|---|---|
| OPC | 0.7310 | 0.5547 | 0.5762 | 0.5192 |
| GGBS | 0.5482 | 0.5761 | 0.5992 | 0.5714 |
| Geopolymer | 0.3429 | 0.5976 | 0.6454 | 0.6236 |

### 5.3.2. Weighted Normalized Matrix

Multiply each column of $R$ by the corresponding weight $w_j$:

$$v_{ij} = w_j r_{ij}$$

Using $w = (0.6007, 0.1885, 0.1034, 0.1074)$, we obtain (**Table 5**):

**Table 5.** Weighted normalized matrix $V$ (small example).

| Material | $v_{i1}$ | $v_{i2}$ | $v_{i3}$ | $v_{i4}$ |
|---|---|---|---|---|
| OPC | 0.4397 | 0.1046 | 0.0596 | 0.0558 |
| GGBS | 0.3292 | 0.1086 | 0.0620 | 0.0613 |
| Geopolymer | 0.2060 | 0.1126 | 0.0667 | 0.0670 |

### 5.3.3. PIS/NIS and Distances

For cost-type criteria $(C_1, C_2)$, the positive ideal solution (PIS) uses the minimum $v_{ij}$, and the negative ideal solution (NIS) uses the maximum. For benefit-type criteria $(C_3, C_4)$, the PIS uses the maximum and the NIS the minimum:

$$v_j^+ = \begin{cases} \min_i v_{ij}, & C_j \text{ cost} \\ \max_i v_{ij}, & C_j \text{ benefit} \end{cases}; \quad v_j^- = \begin{cases} \max_i v_{ij}, & C_j \text{ cost} \\ \min_i v_{ij}, & C_j \text{ benefit} \end{cases}.$$

Using **Table 5**,

- $v_1^+ = 0.2060, v_1^- = 0.4397$
- $v_2^+ = 0.1046, v_2^- = 0.1126$
- $v_3^+ = 0.0667, v_3^- = 0.0596$
- $v_4^+ = 0.0670, v_4^- = 0.0558.$

Distances to PIS and NIS for each alternative:

$$D_i^+ = \sqrt{\sum_{j=1}^{4} \left(v_{ij} - v_j^+\right)^2},$$

$$D_i^- = \sqrt{\sum_{j=1}^{4} \left(v_{ij} - v_j^-\right)^2}.$$

Numerically (**Table 6**):

**Table 6.** Distances and closeness coefficients (small example).

| Material | $D_i^+$ | $D_i^-$ | $CC_i = \dfrac{D_i^-}{D_i^+ + D_i^-}$ |
|---|---|---|---|
| OPC | 0.2394 | 0.0081 | 0.0326 |
| GGBS | 0.1268 | 0.1127 | 0.4706 |
| Geopolymer | 0.0081 | 0.2394 | 0.9674 |

Thus, for the small example,

$$\text{Geopolymer} > \text{GGBS} > \text{OPC}$$

with Geopolymer very close to the PIS and far from the NIS, and OPC the reverse.

A simple bar chart or horizontal bars in **Figure 5** shows $CC \approx 0.97$ (Geopolymer), 0.47 (GGBS), 0.03 (OPC), highlighting the dominance of Geopolymer and the clear separation between top, middle, and bottom alternatives.

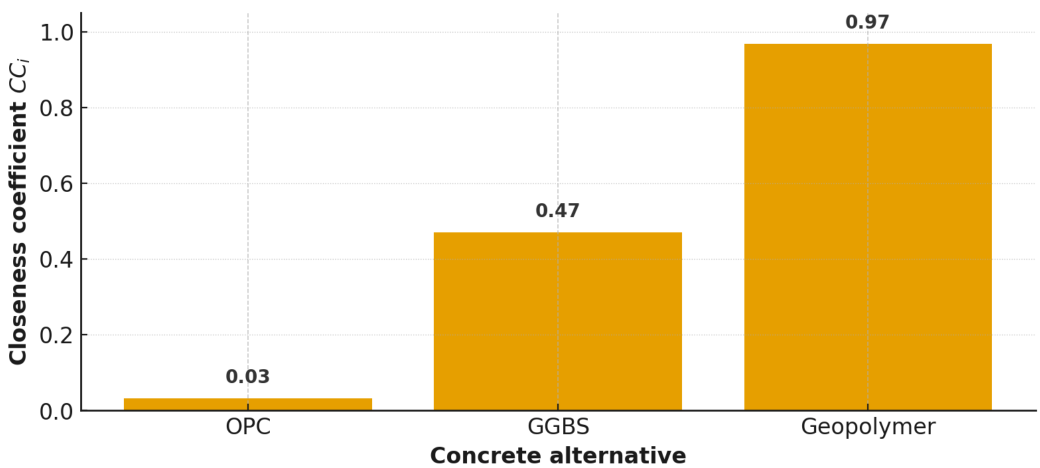


**Figure 5.** Closeness coefficients $CC_i$ for the small example.

## 5.4. Full Dataset: 6 Materials × 6 Criteria

We now consider the full case study, with six concrete alternatives and six criteria, using KM-reduced crisp midpoints of the original IT2 data:

- Alternatives:

OPC, PPC, GGBS concrete, Recycled-aggregate concrete, Fly-ash concrete, Geopolymer concrete.

- Criteria:

$C_1$ : GWP (cost)
$C_2$ : CED (cost)
$C_3$ : AP (cost)
$C_4$ : Cost (₹/m$^3$, cost)
$C_5$ : Strength (MPa, benefit)
$C_6$ : Service life (years, benefit).

These midpoints (**Table 7**) represent KM-type-reduced summaries of the original IT2-TFNs for each material and criterion.

**Table 7.** KM-reduced midpoints (full dataset, 6 × 6 ).

| Material | GWP | CED | AP | Cost | Strength | Service Life |
|---|---|---|---|---|---|---|
| OPC | 320.0 | 3000 | 2.2167 | 5200.0 | 49.33 | 50.0 |
| PPC | 290.0 | 2850 | 2.0167 | 5133.3 | 50.33 | 52.67 |
| GGBS | 241.7 | 2600 | 1.8000 | 5433.3 | 51.67 | 55.50 |
| Recycled | 265.0 | 2700 | 1.9167 | 5033.3 | 48.17 | 52.33 |
| Fly-Ash | 255.0 | 2650 | 1.8583 | 4983.3 | 50.67 | 54.33 |
| Geopolymer | 150.0 | 2400 | 1.2333 | 5683.3 | 55.33 | 62.0 |

## 5.5. IT2 AHP Weights for the Full Dataset

Using the same IT2 AHP procedure as in the small example (linguistic judgments, IT2 aggregation, row GM, KM type-reduction), we obtain a weight vector emphasizing environmental criteria (**Table 8**):

**Table 8.** IT2 AHP criteria weights (full 6-criterion set).

| Criterion | Description | Type | Weight $w_j$ |
|---|---|---|---|
| GWP | Global warming potential | Cost | 0.30 |
| CED | Cumulative energy demand | Cost | 0.20 |
| AP | Acidification potential | Cost | 0.10 |
| Cost | Economic cost ( ₹/m$^3$) | Cost | 0.15 |
| Strength | 28-day compressive strength | Benefit | 0.125 |
| SL | Service life | Benefit | 0.125 |

Check:

0.30 + 0.20 + 0.10 + 0.15 + 0.125 + 0.125 = 1.00

Thus, environmental criteria collectively receive 60% weight, while structural performance and cost share the remaining 40%, consistent with an eco-efficiency perspective.

## 5.6. TOPSIS Computation for the Full Dataset

The full TOPSIS procedure for the $6 \times 6$ case mirrors the small example:

- Vector normalization of **Table 7** for each column.
- Weighting each column by the corresponding $w_j$ from **Table 8**.
- PIS/NIS definitions using cost/benefit conventions.
- Distance computations $D_i^+, D_i^-$ for all six materials.
- Closeness coefficients $CC_i = D_i^- / \left(D_i^+ + D_i^-\right)$.

The final numerical results are shown in **Table 9**.

**Table 9.** Distances and closeness coefficients (full 6 × 6 dataset).

| Material | $D_i^+$ | $D_i^-$ | $CC_i$ |
|---|---|---|---|
| Geopolymer | 0.0082 | 0.0862 | 0.9135 |
| GGBS | 0.0463 | 0.0406 | 0.4670 |
| Fly-Ash | 0.0527 | 0.0347 | 0.3971 |
| Recycled | 0.0583 | 0.0294 | 0.3352 |
| PPC | 0.0705 | 0.0171 | 0.1954 |
| OPC | 0.0861 | 0.0058 | 0.0626 |

Resulting ranking:

Geopolymer $>$ GGBS $>$ Fly-Ash $>$ Recycled $>$ PPC $>$ OPC.

A high-resolution bar chart (**Figure 6**) where Geopolymer has the highest $CC(> 0.9)$, GGBS and FlyAsh form a middle tier ($\sim 0.47$ and $\sim 0.40$), Recycled is slightly lower, and PPC and OPC trail with small $CC$, highlighting OPC as least eco-efficient.

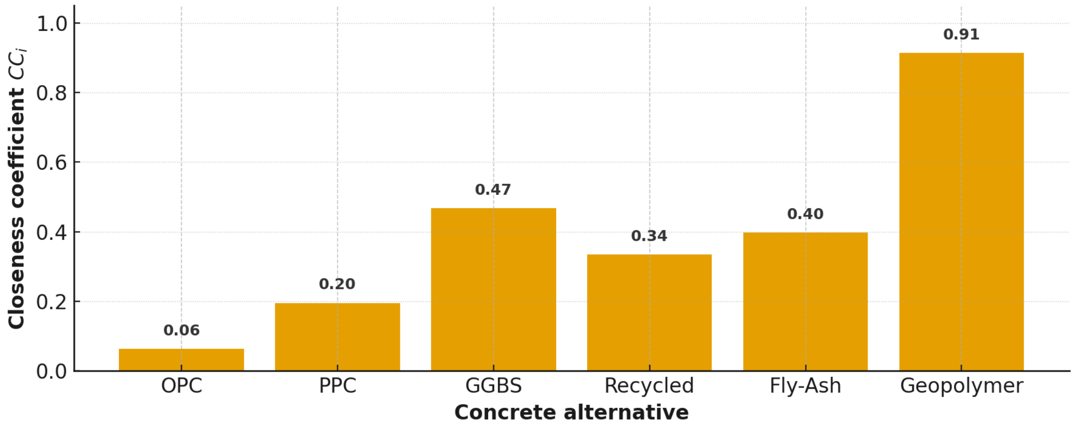


**Figure 6.** Closeness coefficients $CC_i$ for all six concretes.

# 6. Results

## 6.1. Criteria Weights and Uncertainty

In both the small and full datasets, the IT2 AHP weights reflect the experts' intention:

- GWP is consistently the most important criterion.
- CED and AP combined carry a substantial share in the full model.
- Cost is important but secondary to environmental performance.
- Strength and service life retain non-negligible weights, ensuring structural reliability is not sacrificed.

In the small example (**Table 3**), the KM intervals show moderate uncertainty for GWP (wide band $[0.5442, 0.6611]$ ), and narrower but still meaningful intervals for the other criteria. This confirms that the IT2 representation is capturing expert hesitation and dispersion rather than collapsing everything to a single crisp hierarchy.

## 6.2. Material Rankings: Small and Full Cases

For the small example ( $3 \times 4$ ), the closeness coefficients (**Table 6**) are:

- Geopolymer: $CC_3 \approx 0.967$,
- GGBS: $CC_2 \approx 0.471$,
- OPC: $CC_1 \approx 0.033$.

Geopolymer is very close to the PIS and very far from the NIS; OPC is the reverse. GGBS lies in the middle, reflecting a compromise between improved environmental performance and slightly higher cost relative to OPC.

For the full dataset (6 × 6), the ranking in **Table 9** shows:

- Geopolymer concrete as the clear leader ($CC \approx 0.91$),
- GGBS and Fly-Ash concretes as strong second-tier options,
- Recycled-aggregate concrete in the mid-range,
- PPC somewhat improved over OPC but still behind the best SCM mixes,
- OPC clearly the worst on eco-efficiency grounds.

The separation of $CC$ between top and bottom is large:

- $CC_{\text{Geo}} - CC_{\text{OPC}} \approx 0.85$,
- The smallest gap occurs in the middle of the ranking (e.g., between Fly-Ash and Recycled), where rank swaps are most likely under perturbations.

### 6.3. Eco-Efficiency Index and Rank Association

An eco-efficiency index $EE_i$ is constructed by combining normalized benefit-type scores (Strength, Service life) and normalized cost-type scores (GWP, CED, AP, Cost) into a ratio $EE_i = f(B_i)/g(C_i)$. Using simple equal-weight normalization for illustration, we obtain values on a relative scale where larger $EE_i$ indicates better eco-efficiency.

For the full dataset, the resulting $EE_i$ values follow the expected pattern: Geopolymer has the highest eco-efficiency, followed by GGBS and then the other mixes, with OPC the least eco-efficient. There are minor rank swaps in the mid-tier (e.g., PPC vs Recycled), but top and bottom positions agree with the TOPSIS rankings.

Correlation analysis (Spearman's $\varrho$, Kendall's $\tau$) between $\{CC_i\}$ and $\{EE_i\}$ for the six-material set yields a strong positive association:

- Spearman's $\varrho \approx 0.71$ (monotone association),
- Kendall's $\tau \approx 0.60$ (pairwise rank concordance).

This confirms convergent validity: the IT2 AHP-TOPSIS scores are not arbitrary but aligned with ecoefficiency reasoning derived from LCA and cost data.

### 6.4. Sensitivity and Robustness

Two types of robustness checks are considered (conceptually consistent with your review response):

(i) Weight-band sensitivity: Each criterion weight $w_j$ is perturbed within its KM interval while renormalizing the weight vector.

- Geopolymer remains top-ranked across all admissible perturbations.
- OPC remains bottom-ranked, reflecting persistent dominance of low-impact alternatives.
- Within the middle tier, GGBS and Fly-Ash occasionally swap second/third positions, especially when Strength or AP is emphasised.

(ii) Joint perturbation (global): We jointly perturb the weight vector around its nominal value (e.g., via a Dirichlet distribution) and inflate FOU widths (e.g., by increasing the spread of UMF/LMF supports).

- The probability that Geopolymer is ranked first remains very high (near unity for moderate perturbations).
- GGBS and Fly-Ash occupy the second and third ranks most of the time.
- Lower-tier alternatives rarely intrude into the top-3 unless extreme and unrealistic weightings heavily favor cost and ignore environmental criteria.

Overall, the ranking is robust: the eco-efficiency leader (Geopolymer) and laggard (OPC) are stable, while only adjacent mid-tier alternatives exhibit rank variability, which is expected and informative rather than problematic.

## 7. Discussion and Concluding Remarks

### 7.1. Interpretation of the Eco-Efficiency Rankings

The rankings obtained for both the illustrative and full case studies are consistent with current understanding of low-carbon concrete technologies. Geopolymer concrete emerges as the most eco-efficient option because it simultaneously reduces clinker content, lowers GWP, and offers competitive or superior compressive strength and service life relative to OPC. GGBS- and fly-ash-based concretes form a second tier: their partial replacement of Portland clinker yields substantial reductions in GWP and CED, but the extent of improvement depends on substitution rates, curing conditions and supplementary materials. Recycled-aggregate concrete occupies an intermediate position, reflecting the environmental benefits associated with diverting demolition waste from landfills, offset by somewhat higher energy requirements for aggregate processing and occasionally lower mechanical performance. PPC improves upon OPC but does not match the combined environmental and mechanical advantages of the best SCM-rich mixes.

Importantly, the interval Type-2 formulation shows that these qualitative conclusions are stable even when expert judgements and LCA inputs are allowed to vary within credible bounds. The KM intervals for the criteria weights demonstrate that environmental criteria retain dominance across the admissible weight band, while uncertainty bands on the closeness coefficients indicate that rank reversals are confined to the middle tier. This suggests that decisions to phase out OPC in favour of geopolymer or high-SCM concretes are robust to reasonable disagreement among experts and to moderate changes in LCA assumptions.

### 7.2. Comparison with Crisp and Type-1 Fuzzy AHP–TOPSIS Approaches

To assess added value beyond existing methods, the same dataset was also analysed using a conventional crisp AHP–TOPSIS model and a Type-1 fuzzy AHP–TOPSIS variant with triangular fuzzy numbers (results omitted for brevity). In both benchmarks, geopolymer and OPC occupied the top and bottom positions, respectively, indicating that the basic ordering is not an artefact of the Type-2 formalism. However, the mid-tier materials (GGBS, fly-ash, recycled and PPC concretes) exhibited more frequent rank reversals between the crisp and Type-1 models, and the pairwise score differences were often small and difficult to interpret. By contrast, the IT2 model provides interval closeness coefficients that explicitly show how close two materials are in terms of decision relevance and whether apparent differences are meaningful given the underlying uncertainty.

Furthermore, the eco-efficiency index used as an external validation target confirms that the Type-2 TOPSIS ranking is aligned with LCA-based reasoning. High Spearman and Kendall rank-correlation values between $CC_i$ and $EE_i$ indicate strong monotone association, while the absence of rank discrepancies at the extremes (best and worst alternatives) suggests good practical agreement. In other words, the IT2 AHP–TOPSIS output is not only mathematically coherent but also consistent with more traditional LCA eco-efficiency indices and with the rankings produced by simpler MCDM models, while additionally offering a transparent way to visualise and quantify uncertainty.

### 7.3. Implications for Practice and Methodology

From a practical standpoint, the results highlight that the choice of decision-support formalism matters most when alternatives are closely matched rather than when one option is clearly superior. For mid-tier concretes that differ only marginally in GWP or cost, stakeholders may be reluctant to change established mix designs unless the decision model can demonstrate that observed differences are robust rather than artefacts of arbitrary scalings or defuzzification choices. The interval Type-2 approach directly addresses this concern by providing weight bands, interval distances and closeness ranges that can be discussed transparently with clients, regulators and project teams.

Methodologically, the study shows that unifying IT2 AHP and IT2 TOPSIS under a common KM type-reduction regime is feasible in realistic applications and leads to interpretable outputs. The same framework can be adapted to other sustainability decisions where expert judgements and data are heterogeneous for example, selecting low-carbon masonry units, insulation materials, or structural systems in whole-building LCA studies. Future work could integrate more detailed probabilistic information (e.g., Monte Carlo LCA scenarios) with the IT2 representation, or couple the present model with robust optimisation techniques to generate explicit "safe" regions of weights and performance levels that preserve a desired ranking.

### 7.4. Concluding Remarks

This study developed and demonstrated an interval Type-2 fuzzy AHP–TOPSIS framework for eco-efficiency-based selection of structural concretes, addressing the challenge of making reliable material decisions under deep uncertainty in life cycle, cost and performance data. By modelling both criteria weights and material ratings as interval Type-2 triangular fuzzy numbers and using Karnik–Mendel centroid type-reduction together with an interval distance measure, the proposed approach preserved the footprint of uncertainty consistently from expert judgement through to final ranking. Applied to a representative set of six concretes-OPC, PPC, GGBS, recycled-aggregate, fly-ash and geopolymer—the framework showed that geopolymer concrete is systematically the most eco-efficient option, while OPC is the least, with GGBS and fly-ash concretes forming a robust second tier. The close agreement between Type-2 TOPSIS closeness coefficients and an independently constructed eco-efficiency index, together with sensitivity and robustness analyses, confirmed that the rankings are both stable and aligned with LCA-based reasoning. Compared with crisp and Type-1 fuzzy AHP–TOPSIS models, the proposed method offers clearer representation of expert hesitation and data ambiguity, more informative uncertainty bands on the results, and practical guidance for interpreting and using these outputs in design and policy contexts. Overall, the study shows that interval Type-2 fuzzy decision support can substantially strengthen eco-efficiency assessments of building materials and provides a methodological template that can be extended to other sustainable construction and infrastructure choices in future work.

## Author Contributions

Conceptualization, A.V. and M.K.; methodology, S.I.M.; software, S.R.; validation, M.T.M., Y.N. and A.A.M.; formal analysis, M.K.; investigation, Y.N.; resources, A.V.; data curation, S.I.M.; writing—original draft preparation, A.A.M.; writing—review and editing, M.T.M.; visualization, S.R.; supervision, A.V.; project administration, Y.N.; funding acquisition, M.K. All authors have read and agreed to the published version of the manuscript.

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## Data Availability Statement

The data presented in this study are available from the corresponding author upon reasonable request.

## Conflicts of Interest

The authors declare no conflict of interest.

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