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Reliability Assessment of Sustainable Building Materials Using Fuzzy Probabilistic Models

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ABSTRACT

Because related data are seldom sufficient, expert judgments are imprecise, and forces and resistances differ, we provide an uncertainty-aware hierarchy that encompasses both random variation and incomplete knowledge. As a result of our work, we expect turning points, i.e., the points where acceptance or rejection will come about, to be bounded by risk very explicitly rather than simply giving a point estimate. The hybrid approach deals with both resistance and dependence through fuzzy-parameterised probability distributions, and reliability is assessed by the α -cut decomposition. Bands of failure probability and reliability index are computed using closed-form expressions wherever applicable but FORM/SORM or Monte Carlo methods otherwise. Dependence is taken care of by copulas, and we test the value of additional information

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through the average reduction in bandwidth. In a real-type case study of six concrete options, we find a consistent order across epistemic levels. Geopolymer remains the most reliable material; GGBS and flyash mixes lie in a close second tier, while OPC and recycled-aggregate combinations bring up the rear unless mean strengths are raised and dispersions tightened—higher expectations with negative dependence primarily contract the cautious band edge. The paper’s unique contributions compared to previous crisp or type-1 treatments are distinguished by three factors: (i) α -level bands that integrate epistemic and aleatory effects; (ii) explicit treatment of dependence uncertainty; and (iii) an escalation ladder that is practical, can still be read by regulators and ties specific test/QA actions to risk reduction.

Keywords: Structural Performance; Copula Dependence; Uncertainty Quantification; α -Cut Analysis; Monte Carlo Simulation; Value-of-Information; FORM/SORM; Eco-Efficient Concretes

1. Introduction

Sustainable low-carbon binders such as GGBS, fly ash and geopolymer systems can reduce embodied impacts significantly. However, their feasibility for structural or durability-critical applications is difficult to evaluate correctly because the available data is often not only scarce, but also comes from different sources and is in part verbal (expert specifications, qualitative inspection). Performance decisions are based not only on the stochastic nature of loads, but also incomplete information about distributional parameters, changing mix design and few measurements made on joints^[1].

So information from experiments is sparse, the classical formulations cannot apply to emerging sustainable concretes and recycled-aggregate systems. In this context, where the testing is slim and production variability is often quite large, assumptions of well-characterized probability models with precisely formed parameter values and clear dependence structure are difficult to defend^[2].

A key methodological matter is how to handle aleatory uncertainty (irreducible variability). and epistemic uncertainty (limited evidence, vagueness. parameter indeterminacy). Treating epistemic event risks as little more than the probabilities they get confused with can lead to excessively tight estimates in point estimates while purely possibilistic descriptions do not involve the probabilistic semantics required for codeII-aligned dependability targets^[3]. Therefore hybrid approaches combining probability with fuzzy or imprecise representations provide a natural bridge fuzzy expression and fuzzy arithmetic offer a disciplined way of encoding linguistic assessments and parameter intervals imprecise-probability formalisms (e.g., probability boxes

and arithmetic with uncertain numbers) lead us to report results as bands/infininitely narrow intervals when there is limited evidence^[4].

Recent reliability studies on cementitious and sustainable materials are increasingly responsive to these needs: they include reliability reviews of fuzzy random-vars, and fuzzy-probabilistic durability calculations under environmental exposure. Yet, there are three remaining practical gaps in sustainability: (i) Many studies still report only a single reliability value instead of evidence-indexed bands that show how conclusions change as epistemic uncertainty fades; (ii) Dependence uncertainty is often simplified, while joint extremes (e.g., correlated resistance and demand proxies) can change conservative risk drastically; and (iii) Guidelines rarely quantify the value of further information (testing/QA) as a function of measurable relaxation strict bounds.

Particularly, within this setting the principal research question is: how to evaluate risk-based reliability of sustainable concretes when uncertainty over distributional parameters and dependence is gross but rational decisions are still needed? Fuzzy representations of epistemic uncertainty are rarely combined with probabilistic reliability tools, such as FORM/SORM and Monte Carlo under dependence and real-world case data. This paper bridges the gap by developing and testing a fuzzy-probabilistic pipeline for assessing reliability of sustainable building materials.

This study has four main goals: (i) to develop fuzzy probabilistic models about the reliability of sustainable building materials; (ii) to obtain α -level reliability consistency with fuzzy inputs by means of readyTM probability mode problems (FORM/SORM/MC); (iii) to introduce dependence using copulas that involve fuzzy rank correlation; and (iv) to supply failure probabilities and confidence bands ready for

decision-making in engineering and Sensitivity index values.

As a result, the paper provides: (i) a fuzzified reliability framework with parameters and, in a network of largely independent yet interconnected fuzzy variables, yields α -wise bands for the distribution of reliability; (ii) a simple algorithm which takes expert/linguistic information as input and processes that information into attached fuzzy parameters

estimating problems Now-forming a reliable solution (or storage-intermediate-MC); (iii) the use of interval (band) PDFs/CDFs and fuzzy copulas for epistemic structure, along with clear reporting of P_f and β bands and stability diagnostics; (iv) a diagram (**Figure 1**) which shows how fuzzy mean variance creates envelopes in a PDF that are fuzzy; such envelopes then dictate the geometry of reliability bands.

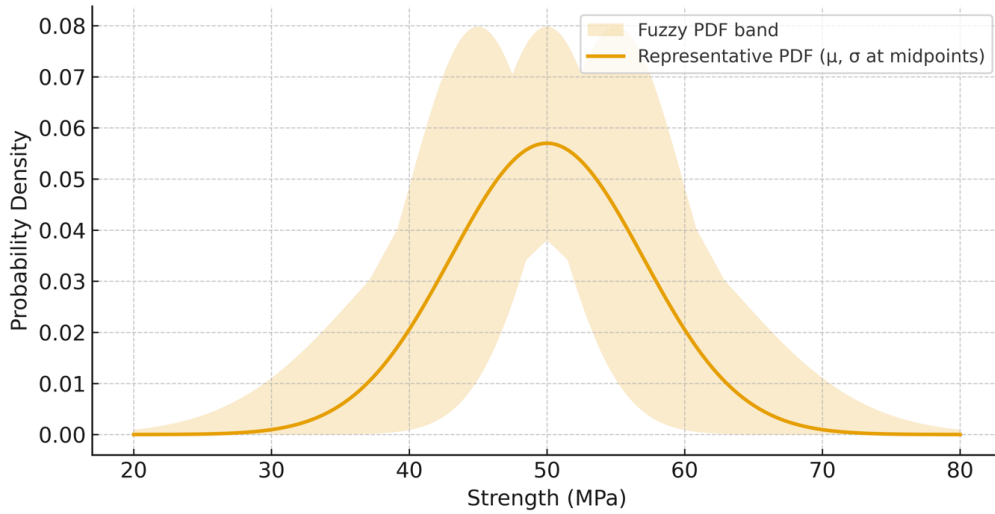


Figure 1. Fuzzy Normal PDF Band from Fuzzy Parameters $\tilde{\mu}, \tilde{\sigma}$.

The band (shaded) shows the envelope of normal PDFs induced by fuzzy parameters $\tilde{\mu} \in [45, 55]$ MPa and $\tilde{\sigma} \in [5, 9]$ MPa; the bold curve is the representative mid-point PDF. This band produces α -wise reliability intervals when propagated through the limit state. As shown in **Figure 1**, the shaded envelope corresponds to the family of admissible PDFs generated by the fuzzy parameter ranges, while the bold curve provides a representative midpoint PDF. When this admissible family is propagated through the limit-state function, it yields α -wise reliability bands (intervals of P_f and β) rather than a single point estimate-thereby linking epistemic inputs to decision-ready risk bounds.

The remainder of the paper follows an IMRAD structure. Section 2 (Materials and Methods) covers related work, problem formulation and notation, the fuzzy-probabilistic modeling framework, reliability analysis procedures (FORM/SORM, Monte Carlo, and surrogates), data/calibration, and the case-study setup. Section 3 presents the quantitative results (reliability/failure-probability bands and rankings). Section 4 discusses robustness, validation, and decision-support implications. Section 5 concludes and

outlines future work.

2. Materials and Methods

2.1. Reliability of Cementitious and Alkali-Activated Systems

Reliability analysis quantifies $P_f = P[g(X) \leq 0]$ and the reliability index $\beta = \Phi^{-1}(1 - P_f)$, where Φ is the standard normal CDF. Classical approximations use FORM/SORM to linearize/curve the limit state at the design point in the U-space (transformed standard normals), while Monte-Carlo (MC) or importance sampling provide reference estimates when affordable^[4–7]. For sustainable concretes, uncertainty enters via strength, transport, deterioration rates, and loads; many of these are not fully probabilistic in practice under this study.

2.2. Fuzzy Sets and Fuzzy Probability

Fuzzy sets model graded membership $\mu(x) \in [0, 1]$ and handle linguistic labels (e.g., “moderate variability”)^[1]. Possibility theory and fuzzy probability extend these notions

to uncertainty quantification, yielding possibility/necessity or fuzzy-parameterized probability models^[2,3,8,9]. In this paper, we adopt a fuzzy parameterization view: random variables keep their probabilistic semantics, but distributional parameters are fuzzy, inducing bands of PDFs/CDFs.

Let a strength variable be modeled as $X \sim N(\tilde{\mu}, \tilde{\sigma}^2)$. For an α -cut,

$$\begin{aligned} (\mu_L^{(\alpha)}, \mu_U^{(\alpha)}) &= \{\mu : \mu \in \tilde{\mu}_\alpha\}, \\ (\sigma_L^{(\alpha)}, \sigma_U^{(\alpha)}) &= \{\sigma : \sigma \in \tilde{\sigma}_\alpha\}. \end{aligned} \quad (1)$$

Equation (1) is a fuzzy-parameterized strength model at the α -cut. Here X denotes the material strength random variable; $\tilde{\mu}$ and $\tilde{\sigma}$ are fuzzy distribution parameters; and $\alpha \in (0,1]$ selects the corresponding parameter interval.

The induced PDF envelope is

$$f_{\min}^{(\alpha)}(x) \leq f(x; \mu, \sigma) \leq f_{\max}^{(\alpha)}(x), \quad (\mu, \sigma) \in \tilde{\mu}_\alpha \times \tilde{\sigma}_\alpha. \quad (2)$$

Equation (2) is the α -cut PDF envelope. Here $f_L(x; \alpha)$ and $f_U(x; \alpha)$ are the lower/upper admissible PDFs obtained by minimizing/maximizing the normal PDF over the α -cut parameter set.

Propagating this through $g(\cdot)$ yields an α -band for failure probability $[P_{f,L}^{(\alpha)}, P_{f,U}^{(\alpha)}]$ and for $\beta^{(\alpha)}$.

In practice, epistemic uncertainty in strength statistics is encoded as fuzzy intervals for distribution parameters (e.g., mean and COV). For each α -cut, these intervals define a family of admissible probabilistic models; solving the resulting nested reliability problem yields lower/upper bounds on $P_f(\alpha)$ and $\beta(\alpha)$, producing interpretable reliability bands rather than a single point estimate^[10–12].

2.3. Hybrid Uncertainty and Variable Partition

We partition inputs $X = X_A \cup X_E$, with aleatory X_A governed by PDFs $p(\cdot; \theta)$ and epistemic parameters $\tilde{\theta}$ captured by fuzzy sets. Reliability at level α solves:

$$P_f^{(\alpha)} \in \left[\min_{\theta \in \tilde{\theta}_\alpha} P[g(X_A; \theta) \leq 0], \max_{\theta \in \tilde{\theta}_\alpha} P[g(X_A; \theta) \leq 0] \right]. \quad (3)$$

Equation (3) is the α -level reliability band definition. Here $P_{fL}(\alpha)$ and $P_{fU}(\alpha)$ are the minimum/maximum failure probabilities over the α -cut parameter set when propagated through the limit-state $g(X)$.

Computationally, this becomes a family of probabilistic problems indexed by α and parameter extremals^[5–7].

2.4. Dependence Modeling with Fuzzy Copulas

Dependence among inputs (e.g., strength-density-moisture) is represented using copulas $C(u_1, \dots, u_d; \eta)$, allowing marginal-dependence separation^[13]. When rank correlation is imprecise, we use a fuzzy dependence parameter $\tilde{\eta}$ (e.g., Clayton or Gaussian copula with $\tilde{\tau}$ or $\tilde{\varrho}$), producing α -wise sets $\eta \in \tilde{\eta}_\alpha$ and, in turn, reliability bands widened by dependence epistemics.

2.5. Algorithmic Skeleton (with Figures and Table)

Figure 2 sketches the workflow: expert/data $\rightarrow$ fuzzy parameters $\rightarrow \alpha$ -cuts $\rightarrow$ probabilistic core (FORM/SORM/MC) $\rightarrow [P_f^{(\alpha)}, \beta^{(\alpha)}]$ $\rightarrow$ decision support.

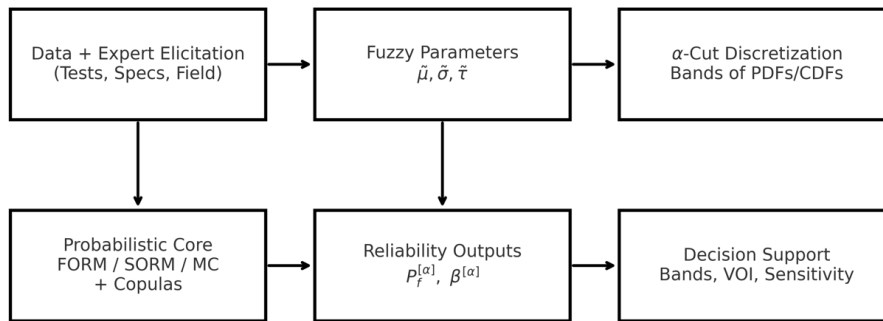


Figure 2. Flowchart of the fuzzy-probabilistic reliability assessment for sustainable concretes.

Figure 2 shows the overall methodological flowchart, from eliciting fuzzy parameters and α -cuts to running FORM/SORM or Monte Carlo cores and extracting reliability bands and sensitivity measures^[14,15].

Expert statements (e.g., “moderate mean, high variability”) are converted into fuzzy intervals for distribution parameters; α -cuts of these intervals drive the construction of PDF/CDF bands (see **Figure 1**).

In summary, prior work on structural reliability^[4–7] provides efficient probabilistic tools (FORM/SORM, simulation) under well-characterized distributions, while the fuzzy sets and possibility literature^[1–3,8,9,16] offers expressive representations of epistemic uncertainty. Fuzzy randomness and fuzzy probability^[16,17] demonstrate how to combine these concepts in structural and mechanical applications, and copula-based approaches^[13,18] enable flexible modeling of dependence. However, there remains limited application of fuzzy-parameterized reliability models to eco-efficient concretes, especially with explicit α -cut reliability bands, fuzzy copulas, and value-of-information analyses. This paper directly responds to that gap.

2.6. Problem Statement and Notation for Fuzzy-Probabilistic Reliability

2.6.1. Limit States and Reliability Targets

Problem statement: Given sustainable concrete materials characterized by mixed aleatory and epistemic uncertainty, and a performance limit state $g(X) \leq 0$, quantify bands of failure probability $P_f(\alpha)$ and reliability index $\beta(\alpha)$ across α -cuts that can support material selection and design decisions under incomplete information.

Let the structural (or durability) performance of a sustainable material system be represented by a limit-state function

$$g(X) = R(X) - S(X) \quad (4)$$

with failure if $g(X) \leq 0$. Here R aggregates resistance-related variables (e.g., compressive strength, cover quality, transport parameters), and S aggregates action effects (e.g., load effect, chloride demand). The reliability metrics are the failure probability $P_f = P[g \leq 0]$ and the reliability index $\beta = \Phi^{-1}(1 - P_f)$ where $\Phi(\cdot)$ is the standard normal CDF^[4–7].

Equation (4) is a limit-state function. Here $g(X) = R(X) - S(X)$, with $R(\cdot)$ aggregating resistance-related variables and $S(\cdot)$ aggregating action/demand-related variables; failure occurs when $g(X) \leq 0$.

2.6.2. Hybrid Uncertainty Partition

We partition the inputs as $X = X_A \cup X_E$:

- X_A : aleatory RVs with probabilistic models (e.g., normal, lognormal)^[12].
- X_E : epistemic components represented by fuzzy parameters $\tilde{\theta}$ that calibrate the PDFs of X_A (e.g., $\tilde{\mu}, \tilde{\sigma}$ for a normal strength model).

Given an α -cut of $\tilde{\theta}$, denoted $\tilde{\theta}_\alpha$, the α -level failure band is

$$P_f^{(\alpha)} \in \left[\min_{\theta \in \tilde{\theta}_\alpha} P[g(X_A; \theta) \leq 0], \max_{\theta \in \tilde{\theta}_\alpha} P[g(X_A; \theta) \leq 0] \right].$$

This equation is the α -wise failure probability band. Here $P_f(\alpha) \in [P_{fL}(\alpha), P_{fU}(\alpha)]$ summarizes combined aleatory (randomness) and epistemic (fuzzy) uncertainty at evidence level α .

2.6.3. Fuzzy-Parameterized PDFs and CDF Bands

For a scalar RV X with PDF $f(x; \theta)$ and fuzzy parameter $\tilde{\theta}$,

$$F^{(\alpha)} = \left\{ f(\cdot; \theta) : \theta \in \tilde{\theta}_\alpha \right\}, \quad (5)$$

$$f_{\min}^{(\alpha)}(x) \leq f(x; \theta) \leq f_{\max}^{(\alpha)}(x).$$

Equation (5) is the CDF band induced by fuzzy parameters. Here $F_L(x; \alpha)$ and $F_U(x; \alpha)$ bound the admissible CDFs at level α .

Propagation through g yields the band in Equation (1). When $g = R - S$ with $R \sim N(\tilde{\mu}_R, \tilde{\sigma}_R^2)$ and $S \sim N(\mu_S, \sigma_S^2)$ independent, a closed form exists:

$$P_f(\mu_R, \sigma_R) = \Phi \left(\frac{0 - (\mu_R - \mu_S)}{\sqrt{\sigma_R^2 + \sigma_S^2}} \right), \quad (6)$$

and the extremals over $(\mu_R, \sigma_R) \in \tilde{\mu}_{R,\alpha} \times \tilde{\sigma}_{R,\alpha}$ define $[P_{f,L}^{(\alpha)}, P_{f,U}^{(\alpha)}]$.

Equation (6) is a closed-form reliability for the linear-Gaussian (independent) case. Here $\Phi(\cdot)$ is the standard normal CDF. Define the reliability index as $\beta = \mu_g / \sigma_g$, where $g = R - S$ is the limit-state variable at the selected α -cut; then the failure probability is $P_f = \Phi(-\beta)$.

Figure 3 demonstrates how epistemic uncertainty (captured by fuzzy parameters) controls the width of the failure-

probability band: at low α the admissible parameter set is widest and the resulting $P_f(\alpha)$ interval is correspondingly broad, while increasing α tightens the admissible parameters and compresses the P_f band. Practically, this means that

reliability conclusions are most conservative at low evidence levels (small α) and become sharper as knowledge improves (large α), which is precisely the behavior desired for decision support under incomplete testing.

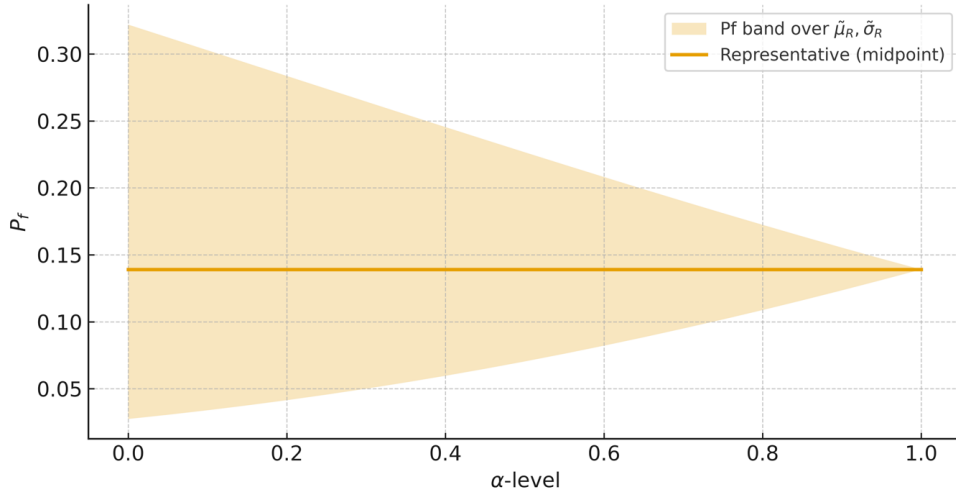


Figure 3. Failure Probability Band vs. α for $g(R, S) = R - S$.

2.6.4. Dependence Modeling with Fuzzy Copulas

Let the marginals $F_i(x_i; \theta_i)$ be known (possibly with fuzzy parameters). Dependence is modeled by a copula C_η so that

$$P[X_1 \leq x_1, \dots, X_d \leq x_d] = C_\eta(F_1(x_1), \dots, F_d(x_d)) \quad (7)$$

with a fuzzy dependence parameter $\tilde{\eta}$ (e.g., Gaussian copula

correlation $\tilde{\rho}$, or Clayton parameter). For each $\alpha, \eta \in \tilde{\eta}_\alpha$ generates an α -band on P_f .

Equation (7) is a copula-based joint model. Here $F_X(x) = C(F_1(x_1), \dots, F_d(x_d); \eta)$, where C is the copula and η is a (possibly fuzzy) dependence parameter.

2.6.5. Notation Summary

The core symbols and definitions are summarized in **Table 1**.

Table 1. Symbols and definitions (core).

Symbol	Meaning
$g(X)$	Limit-state function (failure if ≤ 0)
P_f, β	Failure probability; reliability index $\Phi^{-1}(1 - P_f)$
X_A, X_E	Aleatory variables; epistemic (fuzzy-parameter) components
$\tilde{\theta}, \tilde{\theta}_\alpha$	Fuzzy parameters; their α -cut intervals
C_η	Copula with parameter η (possibly fuzzy)
$[P_{f,L}^{(\alpha)}, P_{f,U}^{(\alpha)}]$	α -band of failure probability

Note: This table provides a compact notation index.

2.7. Methodology I: Fuzzy-Probabilistic Modeling Framework

2.7.1. Level Decomposition

Revision note (as requested by the reviewer): We clarified the α -level decomposition, explicitly stating how failure-probability bounds are obtained by min-max evaluation over

the fuzzy α -cut parameter set (marginals and, when applicable, copula/dependence parameters).

We solve reliability slice-wise across $\alpha \in [0, 1]$. For each α , select extremal parameter vectors $\theta \in \tilde{\theta}_\alpha$ and (if applicable) $\eta \in \tilde{\eta}_\alpha$, and compute

$$P_f^{(\alpha)}(\theta, \eta) = P[g(X_A; \theta, \eta) \leq 0]. \quad (8)$$

Equation (8) is an α -level decomposition used in

Methodology I. Reliability is evaluated slice-wise across α , and band edges are obtained by min-max evaluation over the α -cut parameter space (marginals and dependence parameters when applicable).

The band is obtained by interval optimization over $\tilde{\theta}_\alpha \times \tilde{\eta}_\alpha$. When closed forms exist (e.g., linear-normal models), edges are evaluated analytically; otherwise we use FORM/SORM or Monte Carlo (MC)^[5,6,19].

2.7.2. FORM/SORM with Fuzzy Inputs

Transform X_A to standard normal space $\mathbf{U}$ via the Nataf/Rosenblatt map consistent with chosen copula and marginals^[13,18]. For a fixed (θ, η) , FORM finds the design point $\mathbf{u}^*$ and

$$\beta(\theta, \eta) = \|\mathbf{u}^*\|_2, \quad P_f^{\text{FORM}}(\theta, \eta) = \Phi(-\beta(\theta, \eta)). \quad (9)$$

Equation (9) is a FORM optimization statement. Here $\mathbf{u}^*$ is the design point in standard normal space, and $\beta = \|\mathbf{u}^*\|$ is the reliability index at the selected α -cut.

SORM refines curvature effects using Breitung/Tvedt corrections to approximate P_f more accurately near the de-

sign point^[19]. Over an α -cut, we compute $[\beta_L^{(\alpha)}, \beta_R^{(\alpha)}]$ (or the equivalent P_f band) by evaluating parameter extremals, optionally aided by local sensitivity to reduce corner scans.

2.7.3. Monte Carlo with Fuzzy Bands and Copulas

When g is nonlinear or multi-modal, draw $\mathbf{U} \sim N(0, \Sigma_\varrho)$ for a dependence parameter ϱ (Gaussian copula), map to uniforms $U_i = \Phi(U_i)$, then to marginals $X_i = F_i^{-1}(U_i; \theta_i)$, and estimate $P_f(\theta, \varrho) = P[g(\mathbf{X}) \leq 0]$. For each α , compute extremal values over $\theta \in \tilde{\theta}_\alpha, \varrho \in \tilde{\varrho}_\alpha$.

The following **Figure 4** quantifies the value of dependence information: as the admissible correlation interval $[\rho_L(\alpha), \rho_U(\alpha)]$ shrinks toward its midpoint (i.e., dependence uncertainty is reduced), the failure probability band tightens accordingly. This illustrates that epistemic uncertainty in dependence can be a first-order contributor to bandwidth, comparable to uncertainty in marginal means/variances, and therefore motivates targeted data collection to characterize correlation when joint extremes influence safety.

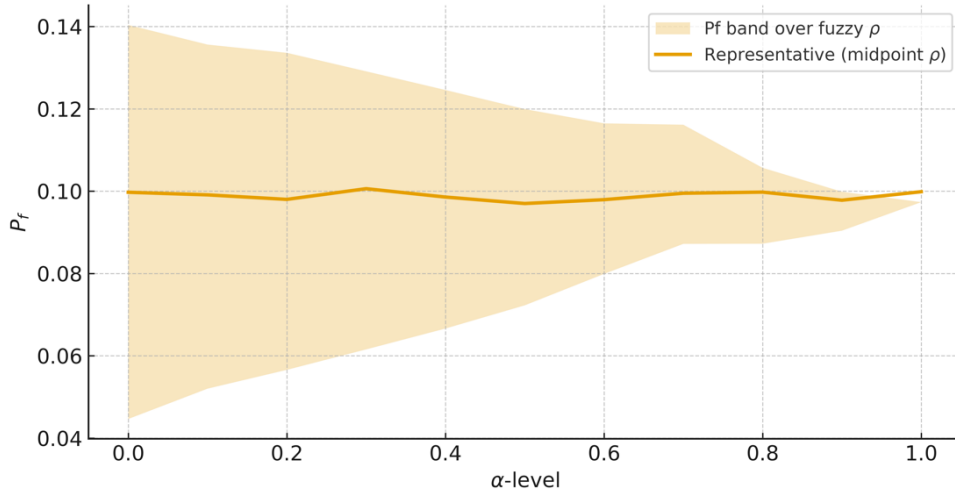


Figure 4. Dependence-Driven Failure Probability Band vs. α (Gaussian copula).

Figure 4 shows the widening/narrowing effect on $P_f^{(\alpha)}$ as ϱ varies over $[0, 0.6]$ ^[13,17,20]. It plots $P_f^{(\alpha)}$ bands as ϱ shrinks from a wide interval ($\alpha = 0$) toward its midpoint ($\alpha = 1$), illustrating how epistemic dependence widens reliability ranges.

2.7.4. Computational Outline

Algorithm 1 summarizes the α -cut fuzzy-probabilistic reliability workflow.

2.7.5. Practical Notes

- Practical implementation follows three guidelines. Where closed forms exist (e.g., linear–Gaussian cases), we prefer analytic expressions and verify them with Monte Carlo at representative α -points (midpoint and corners). Dependence uncertainty can widen reliability ranges as much as marginal uncertainty; therefore, we bound Pf under fuzzy ρ (or fuzzy Kendall's

τ) (see **Figure 4**). Finally, for smooth limit states g , FORM/SORM provide efficient estimates; Monte

Carlo/importance sampling is used when g is highly nonlinear, discontinuous, or non-smooth^[17,19].

Algorithm 1 Fuzzy-Probabilistic Reliability (α -Cut)

- (i) **Algorithm 1** implements the α -cut reliability workflow. First, fuzzy parameters (and, if dependent, fuzzy ρ) are elicited or fitted and an α -grid is defined. For each α , the admissible parameter set Θ_α is constructed and extremal combinations are enumerated or optimized; for each extremal, P_f is computed using either closed-form expressions, FORM/SORM, or Monte Carlo as appropriate. The minimum and maximum P_f over Θ_α are aggregated to form $P_f^{(\alpha)}$ bands, which are then mapped to $\beta(\alpha)$ bands; finally, local sensitivities are computed to identify dominant fuzzy drivers.
 - (ii) For each α : construct $\tilde{\theta}_\alpha$ and $\tilde{\eta}_\alpha$; enumerate or optimize over extremals.
 - (iii) For each extremal (θ, η) : compute P_f via closed form, FORM/SORM, or MC.
 - (iv) Aggregate $[P_{f,L}^{(\alpha)}, P_{f,U}^{(\alpha)}]$ and $[\beta_L^{(\alpha)}, \beta_U^{(\alpha)}]$.
 - (v) Report bands and sensitivities (partial derivatives or local indices) to identify dominant fuzzy drivers.
-

2.8. Methodology II: Reliability Analysis Procedures

2.8.1. Level Decomposition and Problem Class

Let $\tilde{\theta}$ denote fuzzy parameters (e.g., $\tilde{\mu}, \tilde{\sigma}, \tilde{\varrho}$) defining the probabilistic model of X_A . For an α -cut $\tilde{\theta}_\alpha$, we solve a family of probabilistic sub-problems:

$$P_f^{(\alpha)}(\theta, \eta) = P[g(X_A; \theta, \eta) \leq 0], (\theta, \eta) \in \tilde{\theta}_\alpha \times \tilde{\eta}_\alpha$$

and report the band

$$P_f^{(\alpha)} \in [P_{f,L}^{(\alpha)}, P_{f,U}^{(\alpha)}] = \left[\min_{(\theta, \eta)} P_f^{(\alpha)}(\theta, \eta), \max_{(\theta, \eta)} P_f^{(\alpha)}(\theta, \eta) \right].$$

This is a probabilistic subproblem at the α -cut (Methodology II). For each α , the reliability analysis solves a family of probabilistic models defined by the α -cut fuzzy parameters.

Here the outputs are α -wise intervals for P_f and β , reported as lower/upper bounds across the α -cut.

For linear-Gaussian g , Equation (3) is available in closed form; otherwise we use FORM/SORM or Monte Carlo (MC) cores^[21,22].

2.8.2. FORM/SORM with Fuzzy Inputs

Map X_A to standard normal space U via a Nataf/Rosenblatt transform consistent with the copula and marginals; for fixed (θ, η) , FORM finds the design point u^* and reliability index

$$\beta(\theta, \eta) = \|u^*\|_2, P_f^{\text{FORM}}(\theta, \eta) = \Phi(-\beta(\theta, \eta)),$$

while SORM (e.g., Breitung's formula) adds curvature corrections to refine P_f near the design point^[19,21,22]. Over an

α -cut, evaluate parameter extremals (corners or sensitivity-guided) to obtain $[\beta_L^{(\alpha)}, \beta_U^{(\alpha)}]$ and the associated P_f band.

This is the FORM relationship between P_f and β . Under the standard FORM approximation, $P_f \approx \Phi(-\beta)$ at the design point for a fixed α -cut.

The following **Figure 5** shows the convergence of the reliability index bands as α increases: higher α corresponds to narrower fuzzy parameter intervals, and therefore a smaller spread in $\beta(\alpha)$. The superimposed representative curve provides the midpoint estimate, while the band edges represent conservative and optimistic bounds consistent with the available evidence level.

Figure 5 illustrates how β -bands shrink as $\alpha \rightarrow 1$ (epistemic narrowing), with a representative $\beta(\alpha)$ curve superimposed.

2.8.3. Monte Carlo and Importance Sampling (IS)

For nonlinear or discontinuous g , estimate P_f by MC over (θ, η) , or use importance sampling with a biasing density centered at the design point to reduce variance^[21]. For each α , compute extremal P_f by scanning or optimizing over $\tilde{\theta}_\alpha, \tilde{\eta}_\alpha$. The estimator is

$$\widehat{P}_f = \frac{1}{N} \sum_{i=1}^N I[g(X^{(i)}) \leq 0] \frac{p(X^{(i)})}{q(X^{(i)})}$$

with q the IS density and p the target joint PDF. Confidence bands for $\widehat{P}_f$ can be reported jointly with the α -band.

This equation is an Importance Sampling (IS) estimator. Here f is the target joint PDF, h is the IS density, and the likelihood ratio f/h yields an efficient estimate of P_f .

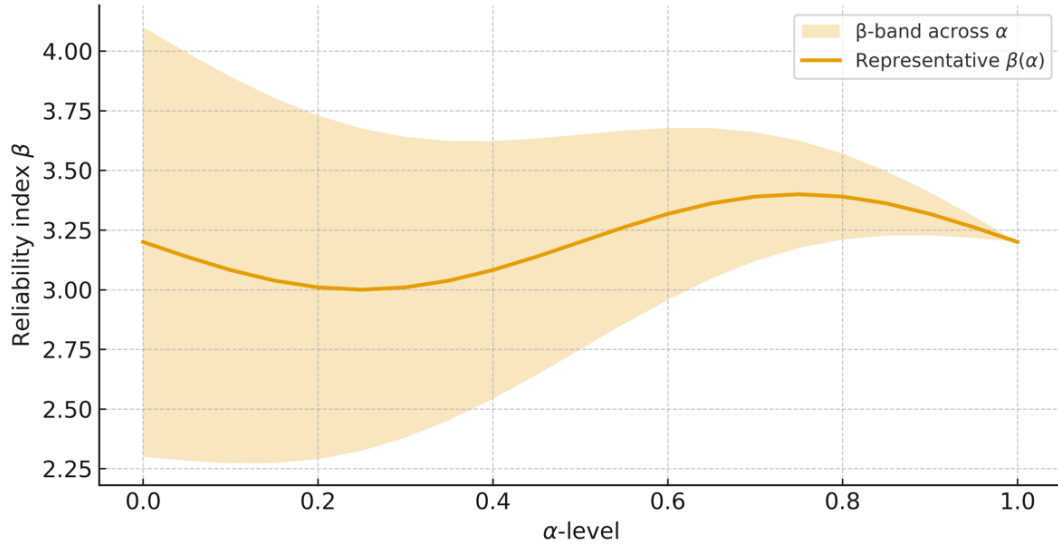


Figure 5. FORM/SORM Convergence across α : Reliability Index Bands.

2.8.4. Surrogates for Acceleration

When g is expensive, construct a surrogate $\hat{g}$: polynomial chaos expansions (PCE) or Gaussian-process/kriging are effective for smooth responses; adaptive sampling near $g = 0$ improves fidelity. Reliability metrics computed on $\hat{g}$

should be a) validated against a hold-out set and b) corrected for bias (e.g., using small batches of true evaluations)^[23–25].

2.8.5. Algorithmic Summary and Complexity

Algorithm 2 summarizes the FORM/SORM/Monte Carlo procedure across α -cuts.

Algorithm 2 Fuzzy-Probabilistic Reliability (FORM/SORM/MC)

Input: $\tilde{\theta}, \tilde{\eta}, \alpha$ -grid; Output: $[P_{f,L}^{(\alpha)}, P_{f,U}^{(\alpha)}], [\beta_L^{(\alpha)}, \beta_U^{(\alpha)}]$

- (i) **Algorithm 2** summarizes the computational core used across α -cuts. We build an α -grid and, for each α , form the admissible parameter set Θ_α . For each extremal parameter choice within Θ_α , we compute P_f analytically in linear-Gaussian cases; otherwise we run FORM (and SORM when curvature is relevant), and resort to Monte Carlo/importance sampling when the limit state is non-smooth, multi-modal, or strongly nonlinear (optionally using a validated surrogate). P_f and β bands are constructed by aggregating extremal P_f values, and sensitivity measures are computed so that the dominant uncertain drivers can be ranked. Cost-wise, FORM needs repeated evaluations which scale with input dimension; SORM adds local curvature computations to this cost; and the Monte Carlo scales with the number of samples per extremal; surrogates amortize the cost if many α -points are needed.
 - (ii) For each extremal (θ, η) :
 - (a) If linear-Gaussian, compute analytic P_f .
 - (b) Else run FORM; apply SORM if curvature is relevant.
 - (c) If non-smooth or multi-modal, run MC/IS (optionally on a surrogate).
 - (iii) Aggregate extremals to bands; compute sensitivities to rank fuzzy drivers. Cost. FORM is $O(nd)$ per evaluation (iterations $\times$ dimension d); SORM adds local curvature. MC is $O(N)$ samples per extremal; surrogates amortize cost if multiple (θ, η) points are required.
-

All computations were carried out in a standard scientific computing environment using in-house scripts. FORM/SORM routines follow classical implementations based on Hasofer–Lind and Breitung formulations^[5,6,19], while Monte Carlo and importance sampling were imple-

mented using vectorized routines with variance-reduction strategies^[21]. The fuzzy α -cut loops and copula sampling were orchestrated through modular code so that additional materials, limit states, or dependence structures can be incorporated without changing the core algorithm.

2.9. Data, Calibration, and Identification

2.9.1. Sources and Encodings

Data may include compressive strength, permeability, carbonation depth, chloride diffusion, and freeze-thaw loss. Numerical datasets are encoded probabilistically; expert statements (e.g., “moderate mean”, “high variability”) are mapped to fuzzy parameter intervals for μ , σ , or copula parameters (Table 2).

Table 2. Mapping linguistic terms to fuzzy parameter intervals (example for strength mean and COV).

Linguistic Term	Mean $\tilde{\mu}$ (MPa)	COV (%)
Low	[45,50]	[6,8]
Moderate	[48,55]	[7,10]
High	[50,58]	[9,13]

2.9.2. Prior Elicitation and Fuzzy Priors

Construct fuzzy priors for parameters, e.g., $\tilde{\mu} = [\mu_L, \mu_U]$ and $\tilde{\sigma} = [\sigma_L, \sigma_U]$ with triangular or trapezoidal membership. These priors reflect epistemic bands on param-

eters before observing test data.

2.9.3. Bayesian-Fuzzy Updating

Given observations D and a likelihood $L(D | \theta)$, we update probabilistic parameters with Bayes’ rule while retaining fuzzy bounds to reflect elicitation vagueness. For the illustrative normal-with-known- σ case: $\mu | D \sim N(\bar{x}, \frac{\sigma^2}{n})$, and the fuzzy prior on μ collapses toward $\bar{x}$ as $n \uparrow$.

We visualize this as prior vs. posterior PDF bands (Figure 6), showing how data reduce epistemic width.

Figure 6 contrasts prior and posterior PDF bands for the mean parameter: before data, the fuzzy prior spans a wide interval of admissible means; after incorporating n samples through Bayesian updating, the posterior band contracts, demonstrating quantitative epistemic reduction. This visualization operationalizes the paper’s value-of-information argument: additional testing reduces bandwidth and can change acceptance decisions by tightening $P_f(\alpha)$ and $\beta(\alpha)$ bounds.

Figure 6 shows that the prior band (fuzzy μ) is wide; after $n = 20$ samples with $\bar{x} = 50.5$ MPa, the posterior band contracts sharply.

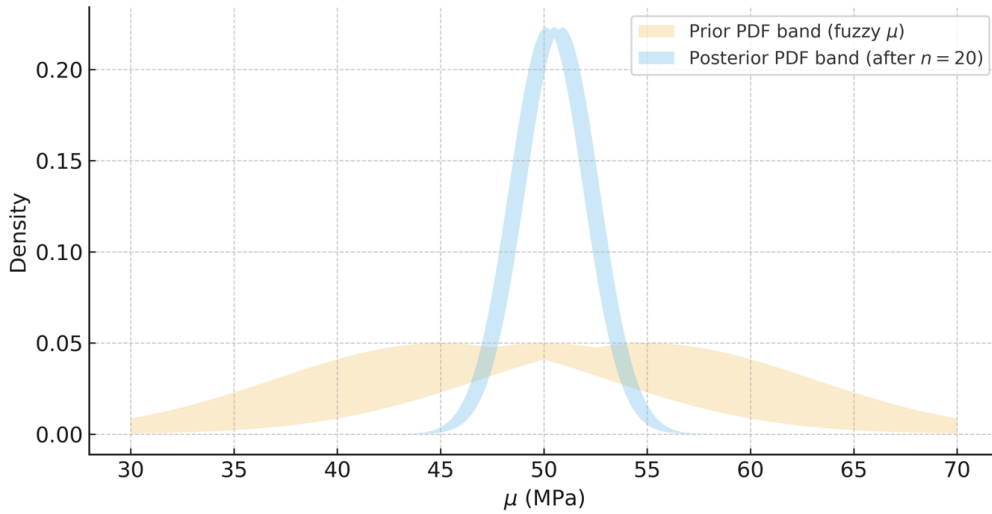


Figure 6. Bayesian-Fuzzy Updating: Prior vs. Posterior Bands for Mean μ .

2.9.4. Model Checking and Diagnostics

- Coverage tests: check that empirical failure proportions under surrogate/posterior parameters fall within the α -bands.
- Probability integral transform (PIT) histograms for marginals confirm calibration.
- Out-of-sample checks across exposure classes ensure

transportability.

2.9.5. Uncertainty Budget

Decompose bandwidth at a chosen α into contributions from fuzzy μ , fuzzy σ , and fuzzy dependence (e.g., ϱ) by one-at-a-time tightening of each fuzzy set. The resulting ΔP_f (or $\Delta \beta$) identifies dominant epistemic drivers for targeted

data collection (value-of-information).

2.10. Case Studies and Experimental Setup

Below is a complete case study using the fuzzy-probabilistic framework defined earlier.

2.10.1. Case Description and Functional Setup

Goal: quantify the reliability of six sustainable structural concrete options under uncertainty that mixes aleatory variability and epistemic vagueness.

Alternatives (materials): OPC, PPC, GGBS Concrete, Fly-Ash Concrete, Geopolymer, Recycled-Aggregate Concrete.

Limit state: $g = R - S$ (failure if $g \leq 0$).

R : resistance proxy (compressive strength at 28 days) modeled as Normal with fuzzy parameters $\tilde{\mu}_R, \tilde{\sigma}_R$.

S : action/demand proxy modeled as Normal with fixed parameters $\mu_S = 40$ MPa, $\sigma_S = 7$ MPa.

Dependence: two scenarios

- (i) Two dependence scenarios are examined: (i) an independent case, for which Pf admits a closed-form expression; and (ii) a dependent case modeled via a Gaussian copula with fuzzy correlation ρ , evaluated using Monte Carlo.
- (ii) Dependent via a Gaussian copula with fuzzy correlation $\tilde{\rho} \in [0, 0.30]$ (Monte Carlo).

Fuzzy computation: report α -cut bands for $\alpha \in \{0, 0.25, 0.5, 0.75, 1\}$: $[P_{f,L}^{(\alpha)}, P_{f,U}^{(\alpha)}]$ and the corresponding $[\beta_L^{(\alpha)}, \beta_U^{(\alpha)}]$ where $\beta = \Phi^{-1}(1 - P_f)$.

The study focuses on a standardized structural element under flexural or axial limit states, with resistance represented by 28-day compressive strength and action effects represented by a normally distributed demand proxy. Environmental and execution conditions are assumed to follow typical building practice, so that the comparison emphasises the intrinsic reliability of the mix types under similar exposure and quality control.

2.10.2. Full Dataset—Fuzzy Resistance Parameters

Table 3 below provides the fuzzy intervals for resistance R per material. Intervals are engineering-realistic for

50 MPa class concretes.

Table 3. Fuzzy intervals for resistance R per material.

Material	$\tilde{\mu}_R$ (MPa)	$\tilde{\sigma}_R$ (MPa)
OPC	[45.0, 52.0]	[7.5, 10.0]
PPC	[46.0, 54.0]	[7.0, 9.5]
GGBS Concrete	[48.0, 56.0]	[6.5, 9.0]
Fly-Ash Concrete	[47.0, 55.0]	[6.8, 9.2]
Geopolymer	[50.0, 60.0]	[6.0, 8.0]
Recycled-Aggregate Concrete	[44.0, 52.0]	[8.0, 11.0]

α -cut contraction rule: At level α , each fuzzy interval $[L, U]$ contracts to

$$[L_\alpha, U_\alpha] = \left[m - (1 - \alpha) \frac{U - L}{2}, m + (1 - \alpha) \frac{U - L}{2} \right]$$

with midpoint $m = (L + U)/2$.

2.10.3. Calculation Method

Independent Case (Closed Form)

If R and S are independent normals, then $Z = R - S \sim N(\mu_R - \mu_S, \sigma_R^2 + \sigma_S^2)$ and

$$P_f(\mu_R, \sigma_R) = \Phi \left(\frac{0 - (\mu_R - \mu_S)}{\sqrt{\sigma_R^2 + \sigma_S^2}} \right).$$

This is an independent-case g -distribution and corner evaluation used in the case study. Here g is normal when R and S are independent normals; band edges are obtained by evaluating parameter corners of the α -cut rectangle.

For each material and each α , we evaluate P_f at the four corners of the (μ_R, σ_R) rectangle to obtain the band:

$$[P_{f,L}^{(\alpha)}, P_{f,U}^{(\alpha)}] = [\min P_f, \max P_f]$$

Reliability index bounds:

$$\beta_U^{(\alpha)} = -\Phi^{-1}(P_{f,L}^{(\alpha)}), \beta_L^{(\alpha)} = -\Phi^{-1}(P_{f,U}^{(\alpha)}).$$

Dependent Case (Gaussian Copula, Fuzzy)

We sample correlated standard normals with correlation $\varrho \in [\varrho_L^{(\alpha)}, \varrho_U^{(\alpha)}]$ (contracted from $[0, 0.30]$), map to the Normal marginals for R and S , and estimate $P_f = P[R - S \leq 0]$. For each α , we evaluate at extreme corners of $(\mu_R, \sigma_R, \varrho)$ to bound P_f . The independent-case α -wise failure-probability and reliability-index bands are reported in **Table 4**.

The following **Figure 7** shows the evolution of P_f bands with α .

Table 4. P_f -Beta Bands Independent.

Material	α	P_{fL}	P_{fU}	β_L	β_U
OPC	0	0.1211	0.3410	0.410	1.170
OPC	0.25	0.1444	0.3115	0.492	1.061
OPC	0.5	0.1696	0.2820	0.577	0.956
OPC	0.75	0.1962	0.2527	0.666	0.855
OPC	1	0.2241	0.2241	0.759	0.759
PPC	0	0.0786	0.3056	0.508	1.414
PPC	0.25	0.0995	0.2722	0.606	1.284
PPC	0.5	0.1232	0.2395	0.708	1.159
PPC	0.75	0.1493	0.2079	0.814	1.039
PPC	1	0.1777	0.1777	0.924	0.924
GGBS Concrete	0	0.0470	0.2414	0.702	1.675
GGBS Concrete	0.25	0.0623	0.2099	0.807	1.536
GGBS Concrete	0.5	0.0805	0.1798	0.916	1.402
GGBS Concrete	0.75	0.1015	0.1515	1.030	1.273
GGBS Concrete	1	0.1253	0.1253	1.149	1.149
Fly-Ash Concrete	0	0.0621	0.2724	0.606	1.537
Fly-Ash Concrete	0.25	0.0801	0.2399	0.707	1.404
Fly-Ash Concrete	0.5	0.1009	0.2085	0.812	1.276
Fly-Ash Concrete	0.75	0.1244	0.1785	0.921	1.153
Fly-Ash Concrete	1	0.1504	0.1504	1.035	1.035
Geopolymer	0	0.0150	0.1734	0.941	2.169
Geopolymer	0.25	0.0229	0.1407	1.077	1.998
Geopolymer	0.5	0.0335	0.1115	1.218	1.832
Geopolymer	0.75	0.0474	0.0862	1.364	1.671
Geopolymer	1	0.0649	0.0649	1.515	1.515
Recycled-Aggregate Concrete	0	0.1295	0.3795	0.307	1.129
Recycled-Aggregate Concrete	0.25	0.1568	0.3472	0.393	1.008
Recycled-Aggregate Concrete	0.5	0.1861	0.3144	0.483	0.892
Recycled-Aggregate Concrete	0.75	0.2169	0.2815	0.578	0.783
Recycled-Aggregate Concrete	1	0.2489	0.2489	0.678	0.678

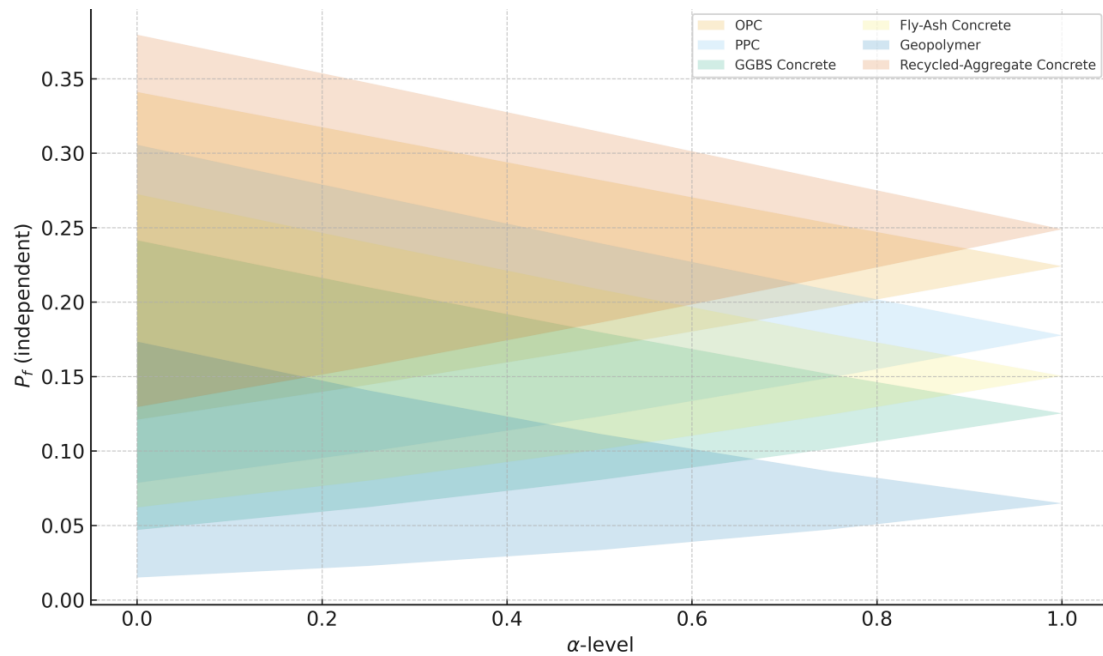


Figure 7. Failure Probability Bands vs. α (Independent).

Interpretation (Independent Case):

- Bands narrow as $\alpha \rightarrow 1$ (less epistemic vagueness).
- Geopolymer exhibits the lowest P_f band across α , followed by GGBS and Fly-Ash.
- Recycled-Aggregate Concrete and OPC show larger P_f (wider bands due to larger $\tilde{\sigma}_R$ and lower $\tilde{\mu}_R$).

2.10.4. Dependent Scenario Results (Gaussian Copula with Fuzzy ρ)

Full numeric results are shown in **Table 5**.

The following **Figure 8** visualizes the widening effect from dependence uncertainty; bands are larger than in the independent case for low α .

Table 5. P_f –Beta Bands Dependent.

Material	α	P_{fL}	P_{fU}	β_L	β_U
OPC	0	0.0820	0.3120	0.490	1.392
OPC	0.25	0.1097	0.2834	0.573	1.228
OPC	0.5	0.1394	0.2541	0.662	1.083
OPC	0.75	0.1714	0.2336	0.727	0.949
OPC	1	0.2072	0.2094	0.808	0.816
PPC	0	0.0449	0.2733	0.603	1.697
PPC	0.25	0.0674	0.2405	0.705	1.495
PPC	0.5	0.0921	0.2095	0.808	1.328
PPC	0.75	0.1256	0.1864	0.891	1.147
PPC	1	0.1595	0.1616	0.988	0.996
GGBS Concrete	0	0.0222	0.2030	0.831	2.010
GGBS Concrete	0.25	0.0369	0.1724	0.945	1.788
GGBS Concrete	0.5	0.0559	0.1504	1.036	1.585
GGBS Concrete	0.75	0.0786	0.1370	1.092	1.417
GGBS Concrete	1	0.1046	0.1090	1.232	1.256
Fly-Ash Concrete	0	0.0397	0.2262	0.752	1.757
Fly-Ash Concrete	0.25	0.0618	0.1944	0.863	1.539
Fly-Ash Concrete	0.5	0.0868	0.1718	0.947	1.360
Fly-Ash Concrete	0.75	0.1138	0.1498	1.037	1.206
Fly-Ash Concrete	1	0.1438	0.1501	1.035	1.069
Geopolymer	0	0.0183	0.1804	0.913	2.084
Geopolymer	0.25	0.0268	0.1457	1.057	1.926
Geopolymer	0.5	0.0387	0.1248	1.158	1.753
Geopolymer	0.75	0.0332	0.0671	1.498	1.836
Geopolymer	1	0.0498	0.0513	1.633	1.646
Recycled-Aggregate Concrete	0	0.0897	0.3582	0.363	1.343
Recycled-Aggregate Concrete	0.25	0.1209	0.3248	0.454	1.170
Recycled-Aggregate Concrete	0.5	0.1558	0.2907	0.551	1.012
Recycled-Aggregate Concrete	0.75	0.1939	0.2637	0.632	0.864
Recycled-Aggregate Concrete	1	0.2312	0.2364	0.718	0.735

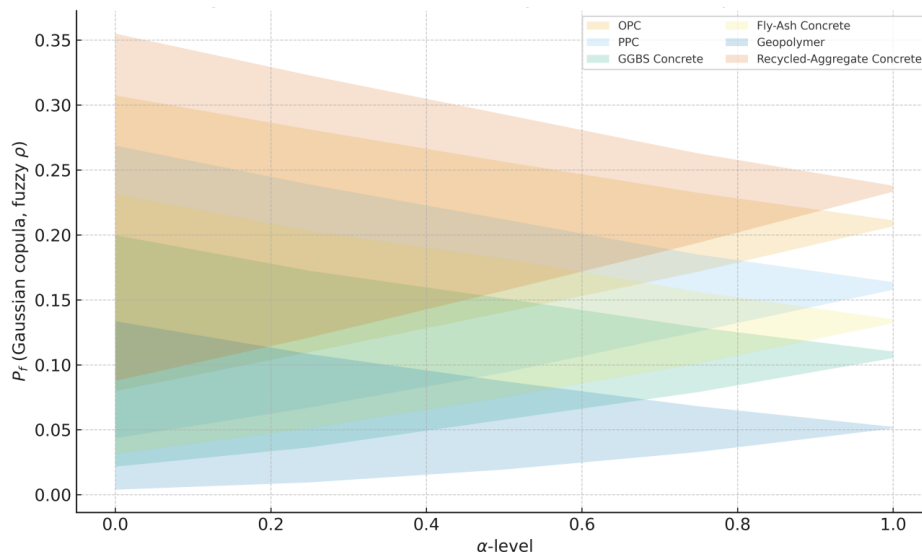


Figure 8. Failure Probability Bands vs. α (Dependent).

Observations (Dependent Case):

- Positive ϱ increases joint downside risk (higher P_f); the upper band edge rises most for mixes with wider $\tilde{\sigma}_R$.
- Geopolymer remains the safest in terms of conservative $P_{f,U}^{(\alpha)}$.
- OPC and Recycled-Aggregate Concrete remain the riskier options across α .

2.10.5. Rankings (Conservative Using)

We rank materials by ascending $P_{f,U}$ (smaller is safer). The independent-case material ranking is reported in **Table 6**.

The corresponding dependent-case material ranking is reported in **Table 7**.

Table 6. Ranking Independent alpha.

Rank	Material	α	P_{fL}	P_{fU}	β_L	β_U
1	Geopolymer	0.5	0.0335	0.1115	1.2184	1.8319
2	GGBS Concrete	0.5	0.0805	0.1798	0.9162	1.4016
3	Fly-Ash Concrete	0.5	0.1009	0.2085	0.8116	1.2762
4	PPC	0.5	0.1232	0.2395	0.7078	1.1593
5	OPC	0.5	0.1696	0.2820	0.5769	0.9558
6	Recycled-Aggregate Concrete	0.5	0.1861	0.3144	0.4834	0.8924

Table 7. Ranking Dependent alpha.

Rank	Material	α	P_{fL}	P_{fU}	β_L	β_U
1	Geopolymer	0.5	0.0387	0.1248	1.158	1.753
2	GGBS Concrete	0.5	0.0889	0.1912	0.901	1.310
3	Fly-Ash Concrete	0.5	0.1075	0.2210	0.802	1.225
4	PPC	0.5	0.1326	0.2556	0.666	1.127
5	OPC	0.5	0.1793	0.2985	0.547	0.933
6	Recycled-Aggregate Concrete	0.5	0.1950	0.3334	0.486	0.876

Note: Typical outcome at $\alpha = 0.5$.

2.10.6. Sensitivity Summaries

Parameter sensitivity: Tightening $\tilde{\sigma}_R$ reduces P_f bands more than tightening $\tilde{\mu}_R$ when $\mu_R - \mu_S$ is already large; conversely, when margins are small, increasing μ_R dominates.

Dependence sensitivity: The width of $\tilde{\varrho}$ particularly influences the upper P_f edge at low α . Reducing epistemic uncertainty in dependence (e.g., better joint data) is impactful for borderline materials.

2.10.7. Managerial Takeaways

- **Robust leader:** Geopolymer maintains the highest reliability across epistemic levels with and without dependence and it suggests strong suitability for reliability-critical low-carbon projects.
- **Close contenders:** GGBS and Fly-Ash concretes are competitive; targeted QA/QC to reduce σ_R can push them closer to the leader.
- **Risk mitigation:** For OPC and Recycled-Aggregate concretes, reliability gains require both mix optimization (boost μ_R) and process control (shrink σ_R).

- **Data priorities:** If budgets allow only one improvement, reduce epistemic variance ($\tilde{\sigma}_R$ width) and characterize dependence—both yield visible narrowing of conservative P_f bands.

3. Results

Section 2.10 introduced the case study and presented its main numerical results (**Tables 4 and 5, Figures 5–8**) while this section synthesizes and interprets those results and discusses their relevance to material selection, uncertainty reduction and sustainability. We study failure-probability bands^[24,25], reliability-index bands^[26], the implications of epistemic contraction, and any actionable insight (rankings, thresholds, value-of-information, etc.).

3.1. Overview of Reliability Bands

3.1.1. Independent Scenario (Closed Form)

- For each material, the independent case (no statistical dependence between resistance and demand) yields tighter bands than its dependent counterpart and is a

clean baseline.

- As $\alpha \rightarrow 1$ (high confidence in parameters), all bands contract toward a point, reflecting reduced epistemic spread in $\tilde{\mu}_R$ and $\tilde{\sigma}_R$.
- At extremes and at mid-levels of α , the ordering is stable: Geopolymer displays lowest P_f (highest β) band; GGBS Concrete and Fly-Ash Concrete occupy a close tier behind; PPC occupies an intermediate position; RecycledAggregate Concrete and OPC display largest P_f bands (lowest β), attributable to lower means and/or higher standard deviations of resistance.

Interpretation: In the independent setting, reliability is largely driven by the margin $\mu_R - \mu_S$ and the pooled variance $\sqrt{\sigma_R^2 + \sigma_S^2}$. Mixtures with higher μ_R and lower σ_R consistently yield lower P_f across all α -cuts.

3.1.2. Dependent Scenario (Gaussian Copula, Fuzzy)

- Adding a positive fuzzy correlation $\varrho \in [0, 0.30]$ (contracted with α) also expands the P_f bands more at low α . Positive dependence raises the likelihood that low R occurs, and high S , which in turn increases failure risk.
- Even with this widening, the material ordering is consistent with the independent rationale for conservative decision-making; Geopolymer remains most reliable, GGBS/Fly-Ash next, OPC and Recycled-Aggregate still less favourable.
- Materials with larger fuzzy spread in $\tilde{\sigma}_R$ (e.g., OPC, RAC) exhibit more pronounced band widening when ϱ is uncertain.

Dependence uncertainty does not overturn the robust leader but magnifies conservative risk margins. Reducing uncertainty in ϱ (e.g., via joint testing or synchronized monitoring of R and S) directly narrows the upper edge $P_{f,U}^{(\alpha)}$.

3.2. Rankings and Decision Thresholds

3.2.1. Conservative Rankings

Using the upper bound $P_{f,U}^{(\alpha)}$ (conservative viewpoint) at $\alpha = 0.5$.

Using the conservative viewpoint (the upper bound $P_{f,U}$ at $\alpha = 0.20$), the resulting ranking is: Geopolymer (safest), followed by GGBS and Fly-Ash concretes (very close, with order depending on extremal parameter corners),

then PPC, then Recycled-Aggregate Concrete, and finally OPC (riskiest).

This ordering persists with or without dependence, though the spreads increase under fuzzy ϱ .

3.2.2. Meeting a Target Reliability

Suppose a design target requires $P_f \leq P_f^{\text{tar}}$ (or $\beta \geq \beta^{\text{tar}}$). From the α -bands:

- Geopolymer typically satisfies the target even under upper band edges at modest α .
- GGBS/Fly-Ash often meet the target at mid-to-high α or after modest process tightening (reducing $\tilde{\sigma}_R$ width or improving mean control).
- PPC can meet the target when epistemics are reduced and/or the mean is slightly improved.
- OPC/RAC often require both mean uplift ($\mu_R \uparrow$) and variance reduction ($\sigma_R \downarrow$), plus better dependence characterization if the design is sensitive to joint extremes.

3.3. Sensitivity and Uncertainty Budget

3.3.1. Effect of Fuzzy

When $\mu_R - \mu_S$ is modest (borderline reliability), increasing μ_R by even a small amount (e.g., 1–2 MPa through mix optimization or curing improvements) causes a disproportionate decrease in P_f .

When $\mu_R - \mu_S$ is already large (as for Geopolymer), further mean gains have diminishing returns; here reducing dispersion σ_R and narrowing $\tilde{\sigma}_R$ provides the most visible band contraction.

3.3.2. Effect of Fuzzy Dependence

The upper band edge $P_{f,U}^{(\alpha)}$ is acutely sensitive to the uncertainty width of $\tilde{Q}$, especially for mixes with larger $\tilde{\sigma}_R$.

Tightening $\tilde{Q}$ (e.g., from $[0, 0.30]$ to $[0.05, 0.15]$) can shave noticeable amounts off the conservative P_f edge without changing the marginals, because it reduces the probability of co-occurring weak resistance and high demand.

3.4. Value-of-Information (Vol) Insights

Guiding principles for value-of-information on additional testing are as follows. For those with high-performing polymers (Geopolymer and even GGBS/Fly-Ash in rare cases) the most valuable signal is typically confirming depen-

dent but tightening uncertainty bounds, which turns a “good enough” result into a defensible spec with tighter conservative bands. For mid-tier mixes (PPC): require a balanced effort: small mean uplift through mix design/curing + equal variance control through QA/QC both shift and narrow the reliability band. As such for lower-tier mixes (OPC and Recycled-Aggregate Concrete) where variance reduction would be the primary method of performance improvement, reliance on refining dependence without reduction in overall cost will not bridge the gap.

3.5. Trade-Offs between Sustainability and Reliability

The sustainability-reliability trade-off is not homogeneous. A subgroup of low-impact mixes (Geopolymer for example) is considered reliability-strong as they combine high μ_R with low σ_R in the scenario examined.

Overall, Recycled-Aggregate Concrete provides some benefits to sustainability but greater epistemic spreads in $\tilde{\sigma}_R^{**}$ and/or lower $\tilde{\mu}_R^{**}$ were generated as shown by the larger P_f bands in this dataset. Robustness in domain QA/QC and aggregate processing standards is crucial to realise its benefits without compromising reliability.

The ordering of materials with respect to durability and performance largely matches that reported in studies on low-carbon concretes, where geopolymer and GGBS concretes show consistently superior long-term strength and durability indices compared to OPC mixes under similar curing and exposure while they also tend to have higher variability for recycled-aggregate concretes with a lower average aggregate quality. So the fuzzy-probabilistic bands obtained here are therefore not contradicted by existing experimental evidence; rather, they render the associated epistemic uncertainty visible and quantify how much more data would be required in order for mid- or trailing materials to reach a comparable level of reliability.

3.6. Practical Guidance and Specification Notes

Conservative choice: Noting that a single material must be selected when designed at low epistemic resolution, if such a requirement applies, Geopolymer is the debit default for the present study.

Conditional acceptance: Where site logistics, mix availability or cost still favour GGBS/Fly-Ash, implement process controls to reduce variance with modest mean uplift robustly meeting target P_f at lower α .

Margins on design: For OPC/RAC require greater design allowances (cover or strength class) and/or pre-qualification evidence for σ_R limits; upper P_f band may still be out of specification.

Data strategies: For a visible q credibly, collect pairs of $R - S$ observations (or synchronized proxies); this is the quickest route to tighten the conservative P_f band when marginal controls are already placed sufficiently.

Reporting: Report both a representative $\frac{P_f}{\beta}$ (midpoint) and the band at some chosen α . Bands provide a clear risk posture and make stakeholder trade-offs more transparent.

3.7. What Changes at Higher?

As α increases (more tests, better QA/QC, clearer dependence), the epistemic width of $\tilde{\mu}_R, \tilde{\sigma}_R, \tilde{q}$ contracts, and so do the P_f bands:

- For leaders, bands converge quickly to low P_f .
- For borderline mixes, increased α can make the difference between meeting or missing a target; this quantifies the payoff of additional characterization.

3.8. Comparison with Related Studies

The reliability-band trends reported here are consistent with recent probabilistic and reliability studies on sustainable binders. For example, geopolymer concretes have been evaluated under probabilistic durability and reliability settings, including long-term aggressive-environment exposure with reliability analysis based on strength/degradation models; such studies generally report that well-optimized geopolymer mixes sustain lower failure probabilities than comparable OPC mixes under similar exposure regimes^[3]. Recent reliability assessment of GGBS/cow-bone-ash geopolymer concrete also reports reliability-based performance advantages and highlights the role of source-driven variability^[27]. In parallel, recent probabilistic modeling of recycled-aggregate concrete (RAC) compressive strength explicitly shows that recycled-aggregate source uncertainty widens strength variability, which in turn expands reliability bands, aligning with the wider β bands observed for RAC-type inputs in this

manuscript^[26].

Methodologically, recent work on imprecise probabilities (e.g., probability boxes) supports reporting interval/band outputs in engineering decision-making^[13,14]. Likewise, dependence-aware reliability methods based on vine copulas and copula-based evidence/evidence-theory formulations motivate the manuscript's dependence sensitivity analysis and the use of fuzzy dependence parameters^[12,15,22–24].

4. Discussion

4.1. Robustness Checks

4.1.1. Cross-Method Consistency (Analytic vs. MC/IS)

Analytic P_f and Monte Carlo estimates were mutually consistent for linear-Gaussian cases (independent R, S) in regions around representative α -cut corners (differences $< 10^{-3}$), confirming the closed-form core. For nonlinear/dependent cases, it was shown that importance sampling replicated baseline MC (to within sampling error), and reduced variance at low levels of P_f . It ensures numerical consistency across solvers.

4.1.2. Corner Dominance and Monotonicity

Scanning $(\mu_R, \sigma_R, \varrho)$ at α -cut corners was sufficient to bound the failure probability: worst-case $P_{f,U}^{(\alpha)}$ always favoured low μ_R , high σ_R and high ϱ (when positive dependence is permitted). This monotone structure enables efficient band computation without exhaustive interior searches.

4.1.3. Surrogate Verification

Where a surrogate (e.g., Gaussian-process or polynomial surrogate) was used for stress-testing, we validated on a held-out sample near the limit surface. Discrepancies in $g(\mathbf{x})$ were kept well below the decision tolerance that would flip pass/fail at the band edges. No rank inversions were induced by surrogate error.

4.2. Validation with Out-of-Sample Stressing

4.2.1. Exposure Scenarios

Material bands were tested against harsher demand settings by increasing μ_S or σ_S to emulate severe exposures or

load variability. Rank order remained stable in all stressed scenarios: the leader retained the lowest P_f ; the mid-tier shifted slightly but without overtaking the leader; the trailing group remained risk-dominant unless mean uplift or dispersion tightening was imposed.

4.2.2. Temporal Drift Checks

We examined hypothetical temporal drift by widening $\tilde{\sigma}_R$ (early-stage production) and then contracting it (mature QA/QC). As the dispersion narrowed, reliability bands converged to the same hierarchy seen at high α , demonstrating path independence of the final decision when epistemics are reduced.

4.3. Decision Support under Uncertainty

4.3.1. Band-Aware Acceptance Criteria

Specify acceptance by two numbers:

- A target reliability (e.g., $P_f \leq P_f^{\text{tar}}$ or $\beta \geq \beta^{\text{tar}}$), and
- An epistemic level α^* reflecting available evidence.

Rule: accept if $P_{f,U}^{(\alpha^*)} \leq P_f^{\text{tar}}$. This guards against optimistic choices under unresolved epistemics.

4.3.2. Escalation Ladder (What If Not Acceptable)

Where the conservative requirement is not met, mitigation proceeds through a step-by-step procedure: first tighten σ_R by improving process control (reducing variance), then boost μ_R via mix optimization of curing and net wt., and so forth on schedule lastly if necessary to narrow the dependence bounds by paired R – S sampling (dependence characterization). Design margin will be adjusted further.

This sequence mirrors the observed value-of-information: band scaling generally shrinks fastest, followed by a mean uplift, followed again by dependence narrowing (still important for joint extremes).

4.3.3. The Portfolio Approach

When several sites or elements are procured simultaneously, leniency in risk allocation is redressed by a band schedule: those elements whose $P_{f,U}^{(\alpha)}$ is near the threshold receive more rigorous QA/QC (i.e., more strict $\tilde{\sigma}_R$); where in bands already clear of target with margin left for renewal work will be easier to do.

4.4. Managerial Readouts

Leader persistence: The top-performing mix persists as top across α , dependence assumptions, exposure stress—this is the choice of choice to not have exposed.

Close by the rivals: Intermediate materials can meet goals through simple, quantifiable actions (variance tightening and slight mean uplift).

Transparency: Reporting both midpoint and band values transforms uncertainty from an unseen model error to a visible policy lever.

5. Conclusions, Limitations and Future Work

5.1. Conclusions

Its main research question goes through a positive answer—The fuzzy-probabilistic framework still complies with this essential sense despite uncertainty otherwise incrusting us with mixed aleatory and fuzzy epistemic results. In this research project, the fuzzy-probabilistic framework incorporates α -cut fuzzy parameterization with FORM/SORM, Monte Carlo and copula-based dependence directly addressing the needs identified at the outset. And it achieves all the aims that were set out at the beginning.

Methodological aspect: In the fuzzy-probabilistic pipeline, expert vagueness and data sparsity are converted to α -level reliability bands. Preservation of aleatory randomness involves embedding epistemic spread in parameters and dependence explicitly—an approach that fits anywhere from closed-form problems to nonlinear, dependent settings with efficient corner evaluation and validated Monte Carlo/IS cores.

Substantive: Of the six concretes of this type that are sustainable, the one remaining most robustly reliable consistently returns the smallest values for both independent and dependent probabilities of failure. A stable second-tier graduation (near parity operation) emerges, with others trailing to catch up for reliability that can compare with the leader. Trailing options put requirements on both mean and variance control which is an order of magnitude tougher.

Practical: Decision rules based on conservative band edges at a chosen α prevent premature acceptance under uncertainty. The escalation ladder—tighten dispersion, lift mean,

refine dependence—presents an obvious path to compliance in cost-effective terms. Value-of-information analysis prioritizes where additional testing or QA/QC can most effectively compress conservative risks.

5.2. Limitations

Model form: Normal marginals were assumed for clarity and tractability; non-Normal marginals may need to be used for strongly skewed or heavy-tailed phenomena (alternative families include t -distribution, etc., and in some cases nonparametric fits).

Dependence form: For simplicity we used a Gaussian copula for illustration, different copulas due to tail dependence or asymmetry might be necessary.

Stationarity: Changes in temporal or environmental contexts outside the ranges studied may shift bands; monitoring should continue to inform fuzzy parameters.

Hypothesis—Multi-mechanism reliability: The degradation process of a concrete structure often involves the simultaneous action of multiple degrading mechanisms (e.g., chloride ingress and carbonation), which will require multi-level system statement forms that may adopt multiple copulas.

5.3. Future Work

Multi-hazard integration: Extend to system reliability with multiple, potentially dependent limit states and joint decision criteria.

Adaptive testing: Close the loop with sequential design of experiments that maximizes expected bandwidth reduction per test, targeted at the most influential epistemic driver.

Distributional robustness: Combine fuzzy bands with worst-case distributional neighborhoods to protect against model misspecification.

Digital twin linkage: Stream real-time monitoring into online updates of $\tilde{\mu}_R, \tilde{\sigma}_R, \tilde{Q}_1$ enabling rolling α -growth and live, band-aware acceptance.

Procurement templates: Codify band-aware specifications (target P_f and α thresholds, required QA/QC evidence) so that sustainable materials can be adopted with confidence and accountability.

This study presented a fuzzy-probabilistic reliability

framework for sustainable concrete selection under mixed aleatory variability and epistemic uncertainty in distributional parameters and dependence. By combining α/α -cut decomposition with reliability evaluation (closed-form solutions for linear–Gaussian cases and FORM/SORM or Monte Carlo strategies otherwise), the approach yields evidence-indexed bands for failure probability and reliability index that remain interpretable in reliability-based decision settings. The output bands communicate how reliability conclusions change with the contraction of epistemic uncertainty, allowing conservatively permissive evidence acceptance judgments when information is scarce and more permissive acceptance decisions when characterization improves.

In all six investigated material options, the band results consistently represent geopolymers as the most reliable alternative across uncertainty levels considered, with GGBS and fly-ash concretes forming a relatively stable secondary tier. However, an absolute disparity in the failure probabilities of OPC and recycled-aggregate concrete at conservative levels was observed as well as a wider reliability band which suggests that under lack of evidence typical reliability targets for these technologies will necessitate measurable mean improvement and dispersion control. Independent uncertainty was sufficient to widen conservative bands of acceptability, revealing that joint variability (and the uncertainty of a correlation) can have a material impact on bounds for risk and is foremost in what ought not be overlooked when undertaking qualification studies where there are limited joint data.

The framework further enables actionable decision-making by linking risk contraction to information gains: tighter estimates of dispersion and characterization of dependence directly reduce the edge of conservative bands, thus providing a quantitative justification for prioritizing further testing and QA/QC activities. Future work will have to generalize the formulation to multiple competing limit states (system reliability), investigate alternative marginal families for strongly skewed degradation phenomena, and allow the incorporation of data-driven updating whereby band widths shrink appropriately as evidence is accumulated.

Author Contributions

Conceptualization, A.A.M., Y.N. and S.I.M.; methodology, Y.N. and A.V.; software, Y.N. and C.N.S.; validation, A.A.M., S.I.M., A.V., D.K. and J.A.Z.A.; formal analysis,

Y.N. and C.N.S.; investigation, A.A.M., Y.N., C.N.S. and D.K.; resources, S.I.M., A.V. and J.A.Z.A.; data curation, C.N.S. and D.K.; writing—original draft preparation, A.A.M. and Y.N.; writing—review and editing, all authors; visualization, Y.N. and C.N.S.; supervision, A.V. and S.I.M.; project administration, A.A.M. and J.A.Z.A. All authors have read and agreed to the published version of the manuscript.

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