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Experimental Study and Fragility Analysis of Effective-Length Factors in Column Buckling

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ABSTRACT

The design of columns relies heavily on the basis of Leonhard Euler's Theory of Elastic Buckling. However, to increase the accuracy in determining the maximum critical load a column can withstand before buckling, a constant was introduced. This dimensionless coefficient is *K*, also known as the effective-length factor. This constant is often found in building design codes and varies in value depending on the type of column support that is applied. This paper presents experimental and analytical studies on the determination of the effective-length factor in the buckling stability of columns with partially-fixed support conditions. To this end, the accurate *K* value of the columns tested by the Instron Testing Machine (ITM) at California State University, Northridge's (CSUN's) Mechanics Laboratory is determined. The ITM is used in studying the buckling of columns where the supports are neither pinned nor fixed, and the material cross-section rather rests upon the machine while loading is applied axially. Several column specimens were tested and the experimental data were analyzed in order to estimation of the accurate effective-length factor. The calculations from the tested results as well as the conducted probabilistic analysis shed light on how a fragility curve may aid in predicting the effective-length value of future tests.

Keywords: Column buckling; Fragility curve; Critical loading; Effective length factor; Fixity

1. Introduction

Column buckling can be defined as the abrupt lateral displacement of a column under which maximum loading is applied. This occurs mainly in members with a high slenderness ratio, meaning the length of the member is much greater than the width

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of the cross-section. Shorter-length members will not experience buckling but rather crushing under greater loads. Regardless, critical loads for columns of any slenderness ratio can be calculated. Leonhard Euler's Theory for Elastic Buckling produced such an equation which is as follows:

$$P_{cr} = \frac{\pi^2 EI}{(l/r)^2} \tag{1}$$

where E is the modulus of elasticity of the material being used, l is the length of the member, and r is the radius of gyration, which is equivalent to:

$$r = \sqrt{\frac{I}{A}} \tag{2}$$

Here, I is seen as the bending moment of inertia and A is the cross-sectional area of the member. With Equation (1), values obtained for various columns with differing support connections yield unreliable results. By substituting in the effective-length constant "K" of the column being analyzed, the equation then converts to:

$$P_{cr} = \frac{\pi^2 EI}{(Kl)^2} \tag{3}$$

This then provides more accurate critical load results theoretically as well as for physical applications [1]. Specific K values are to be used when calculating critical loads with different column support connections. For example, columns that have fixed connections on both ends will utilize a K value of 0.5, and columns with simply supported pinned connections will use a K value of 1. The goals established before conducting any experimentation were to calculate an accurate K value for the supporting connections on California State University, Northridge's (CSUN) Instron Testing Machine (ITM), since it will be used for this case study specifically, and undergo probabilistic analysis of results. This is achieved by creating a fragility curve that aids in predicting the probability of K values that would be produced in future tests.

2. Literature review

The primary focus of this case study is on the effective length constant for columns, however, there are far more variables that can change the overall effective length of a column. Hibbeler defines the effective length of a column as the distance between points of zero moments [1,2]. Therefore, supports that limit rotation will provide a moment reaction and produce an internal inflection point. This is the case with completely fixed supports. A moment reaction at both the top and bottom of the member decrease the effective length resulting in an effective length factor of K = 0.5. For pinned supports, there are no moment reactions at the end conditions therefore, the effective length results in the full length of the member since the closest values of zero moments are at the connections. An experimental example of this can be cited by Bouras et al. [3]. The experimental design utilizes a notched cylinder that accepts the column, and a welded angle with a Teflon sheet that guides the cylinder in rotation emulating a pinned connection. With this setup, the effective length factor used is K = 1.0 since the resulting zero moments will lie at the connections.

As previously stated, not only is the support condition the only variable that plays a role in the overall effective length. Referring to Tian et al. [4], columns can contain various stiffnesses, cross-sectional areas, distributions in axial loading, and differing ratios between lengths. These variables may only take effect when considering stepped columns composed of materials other than wood elements. However, the theory and determination of critical load behaviors still extend from the foundational Elastic Buckling Theory. Expanding on the notion that many studies in modern engineering stem from Euler's Theory, in Falborski et al.'s study of column base fixity [5], the base support of a column within a frame can be manipulated to obtain a certain degree of flexibility. With this flexibility, the inflection points of the member can be adjusted to lessen the amount of internal moment received at the upper end of the column where the structure is more vulnerable, and result in an increase in the moment at the base where the foundation can resist greater moment reactions. This enables the structure to withstand higher amounts of lateral loadings, such as wind or earthquakes, all while under uniform axial loading.

In any study involving structural behavior under extreme loading, the allowable stress and strain of material are vital. In Euler's Theory, the modulus of elasticity is the primary component relating stress and strain considering the modulus under axial loading is the quotient of normal stress and strain. Avallone et al. [6] display these relations many times using stress-strain curves. When members are placed under increasing axial loading, the elements' material properties of stress and strain demonstrate a linear-proportional relationship which is called the elastic range. Outside this range lies other circumstances with vibrational forces and accelerations that will not be covered in this experiment but still are immensely important when pertaining to structural designs in earthquake engineering.

In the analysis of earthquake effects on structures, seismic fragility curves have been explored with the goal of determining the vulnerability of structures if induced by such type of lateral loading. In this paper, the basic use of fragility curves will be explored in determining the probability of effective length constants however, studies such as one under Rajeev and Tesfamariam [7] reveal more in-depth uses of this tool. In their study, they analyze multiple building failures due to earthquakes and compile several variables such as a number of weak stories, year of construction, beam-column connections, and the topology of the site. With these factors applied they can create relations between the demand of the earthquake in question and the structural capacity of the building to make predictions on other buildings similar in design and determine its probability of survival or failure with an earthquake of similar magnitude. Guevara-Perez [8] also discusses the importance of beam-column connections in the result of weak stories that are a major factor in the ability of a structure to successfully endure an earthquake.

Many factors play a role in the behavior of columns and through a review of various studies involving advanced concepts in engineering we can see that at the foundation level, column buckling shows there are key differences in behavior when something as simple as a support condition is altered. With this alteration, the behavior of columns will ultimately create effects higher up the chain in structural analysis. By establishing a better understanding of how effective length factors and fragility curves function with a single member, made of wood in this case, the more advanced theories and concepts may begin to see effects as well within larger systems, structures, and materials. They branch off from the basic understanding of column buckling and its reactions to compressive loading.

3. Experimental design

The support connections for the ITM are neither fully fixed nor pinned, therefore, the end connections will be referred to from here on out as partially fixed. For example, as seen in **Figures 1 and 2**, the cross-section of the member is rested flush to the crossheads of the machine, which only partially limits rotational and translational displacements. **Figure 3** shows how the test specimen is positioned in ITM. 15 Douglas Fir specimens roughly 35 inches in length were prepared to undergo compression in the partially fixed machine connections. Calculations such as weight, volume, density, cross-sectional area, slenderness ratio, and moment of inertia were then taken for each sample as shown in **Table 1**. For this experiment, a Modulus of Elasticity (E) of 1.95×10^6 psi is established considering this is the same value dedicated to the wood used in CSUN's laboratory for other experiments. This also maintains the accuracy of results in relation to past and future tests conducted at CSUN. Each specimen was then carefully positioned in the ITM crosshead and testing was conducted through the program TestWorks QTEST. Information about the specimen being tested such as length and cross-sectional area was input into the program and then, at a rate of 0.01 in/min, the crossheads of the ITM compressed the specimens axially until buckling or complete failure occurred. QTEST then provided peak stress (σ_{cr}) and critical load (P_{cr}) values from each sample tested. The details of the ITM as well as the length and cross-section dimensions of a typical test specimen are illustrated in **Figure 4**.

Figure 1. Top crosshead connection.

Figure 2. Bottom crosshead connection.

Figure 3. Specimen placed in ITM.

Table 1. Properties of the test specimens.

Wood Specimen	Length (in)	Greatest Cross Section Dimension (Dmax, in)	Least Cross Section Dimension (Dmin, in)	Cross Section Area (in^2)	Weight (oz)	Volume (in^3)	Density (oz/in^3)	Slenderness Ratio (Min)	Slenderness Ratio (Max)	Moment of Inertia (in^4)
1	35.000	0.727	0.693	0.504	6.30	17.633	0.357	50.505	48.143	0.0202
2	35.063	0.729	0.716	0.522	5.30	18.301	0.290	48.970	48.097	0.0223
3	35.000	0.734	0.726	0.533	6.70	18.651	0.359	48.209	47.684	0.0234
4	35.063	0.719	0.712	0.512	5.50	17.949	0.306	49.245	48.766	0.0216
5	35.000	0.721	0.714	0.515	5.30	18.018	0.294	49.020	48.544	0.0219
6	35.000	0.724	0.718	0.520	6.70	18.194	0.368	48.747	48.343	0.0223
7	35.000	0.728	0.717	0.522	6.50	18.269	0.356	48.815	48.077	0.0224
8	35.063	0.727	0.715	0.520	6.30	18.226	0.346	49.038	48.229	0.0221
9	35.063	0.717	0.709	0.508	5.40	17.824	0.303	49.453	48.902	0.0213
10	35.000	0.729	0.718	0.523	6.90	18.320	0.377	48.747	48.011	0.0225
11	35.063	0.729	0.725	0.529	6.30	18.531	0.340	48.362	48.097	0.0232
12	35.000	0.733	0.728	0.534	6.50	18.677	0.348	48.077	47.749	0.0236
13	35.063	0.733	0.716	0.525	5.50	18.402	0.299	48.970	47.834	0.0224
14	35.063	0.719	0.727	0.523	5.40	18.328	0.295	48.229	48.766	0.0230
15	35.000	0.738	0.728	0.537	6.90	18.804	0.367	48.077	47.425	0.0237

4. Evaluation of experimental results

After a certain amount of compression, each column either buckled or completely failed to result in fractures, as shown in respective **Figures 5 and 6**. Some members that were not fractured from testing remained in the elastic range and reverted to their original linear state after loading was removed. However, there was slight deformation left over and a bend in the specimen could be seen. All data obtained from the testing machine was placed into tabular form and comparisons were made between the experimental and theoretical critical load based on support connections, which can be seen in **Table 2**. Average experimental values for peak stress and critical load were 2,278.69 psi and 1,188.77 lbs, respectively. We can assume that variations in both values are due to the inhomogeneity of the wood

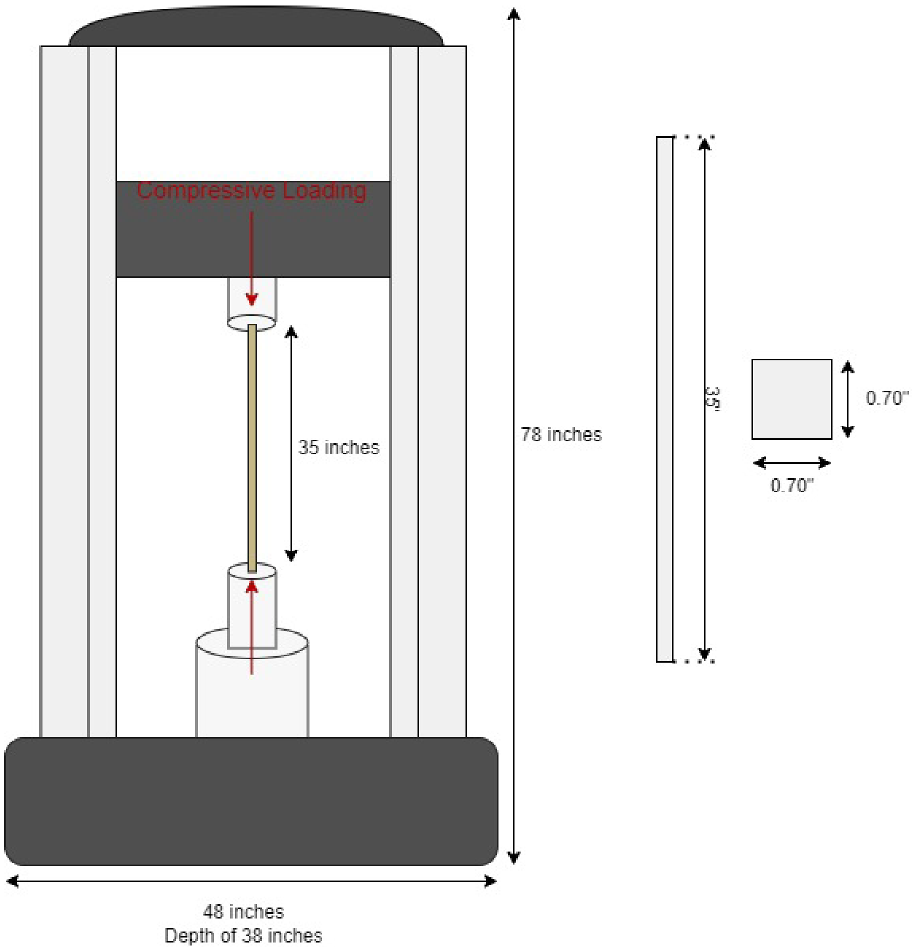

Figure 4. Details of the ITM and geometrical dimensions of a typical test specimen.

Figure 5. Specimen #6 buckling under compression.

Figure 6. Specimen #3 ruptured after compression.

Table 2. Summary of theoretical calculations and test data.

Wood Specimen	Fixity (%)	Theoretical P_{cr} (SS)	Theoretical P_{cr} (FF)	Experimental P_{cr} (PF)	Peak Stress (psi)	Experimental Effective Length Factor (*K*)	Normal Distribution
1	77.115	316.775	1,267.101	839.10	1,665.60	0.614	0.962
2	90.104	349.087	1,396.349	1,156.20	2,245.10	0.549	0.521
3	91.834	367.725	1,470.899	1,257.20	2,359.30	0.541	0.429
4	89.931	338.560	1,354.242	1,117.80	2,183.60	0.550	0.530
5	75.990	343.595	1,374.380	893.70	1,736.00	0.620	0.973
6	93.781	350.856	1,403.424	1,243.90	2,393.00	0.531	0.331
7	101.037	351.322	1,405.290	1,434.90	2,749.00	0.495	0.081
8	92.223	346.673	1,386.692	1,193.80	2,296.60	0.539	0.409
9	97.101	333.369	1,333.476	1,259.40	2,477.50	0.514	0.190
10	98.631	353.279	1,413.116	1,375.20	2,627.30	0.507	0.140
11	79.898	362.417	1,449.669	1,005.00	1,901.60	0.601	0.920
12	84.400	370.267	1,481.068	1,108.30	2,076.90	0.578	0.791
13	96.486	351.003	1,404.011	1,310.30	2,496.60	0.518	0.213
14	92.249	360.412	1,441.649	1,241.70	2,375.50	0.539	0.408
15	96.614	372.793	1,491.171	1,395.10	2,596.70	0.517	0.208
Average	90.493	351.209	1,404.836	1,188.77	2,278.69	0.548	
					Std. Dev.	0.038	

members themselves. Some members had less or greater densities as well as internal defects, seen in most wooden materials. For example, a material such as steel or any other metal alloy is pure and homogeneous throughout the entire length of the member, whereas wood may have internal knots in the wood or initial fractures that may not be seen on the surface. Since we were given the experimental P_{cr}, the experimental *K*-value for each specimen was calculated and an average effective length constant of 0.548 was found. Effective length constants from each member can be seen in **Table 2**. **Figure 7** allows a visual representation of the variability in effective length values. Clear observations were made for the tested columns and a general effective length ratio lies somewhere between 0.5 and 0.6.

Using Equation (3), theoretical critical loads with fixed and pinned connections were calculated and the results show that the experimental critical load lies in-between fixed and pinned connections but resembles fixed circumstances more. This is due to the previously stated fact that the partially fixed crossheads of the ITM do not completely limit rotation, so the columns are able to, just slightly, have an increased effective length. **Figure 8** shows this slight rotational freedom when subjected to the compressive load. To further analyze the degree to which the partially fixed supports constrain rotations and translations, its fixity percentage was calculated.

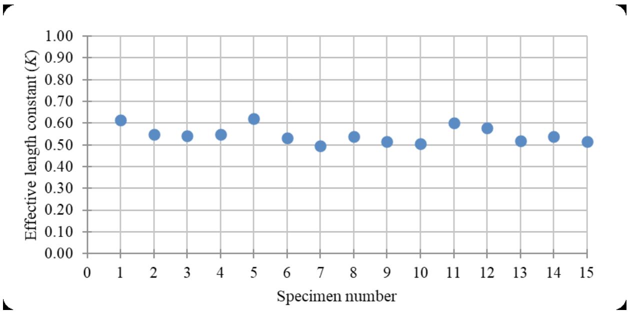

Figure 7. Effective length constants of test specimens.

Figure 8. Upper crosshead support under loading.

5. Probabilistic analysis

Columns contain a certain level of fixity in the supports. What influences this level of fixity is the degrees of freedom allowed in the support condition. For example, a fixed-end condition restricts translation in the *x*, *y*, and *z* directions as well as rotation therefore it contains fixity in all six degrees of freedom. To further analyze the behavior of columns placed in the ITM, calculations of fixity percentage were obtained. Taking the average *K*-value of 0.548 from the experimental data, interpolations between the fixed and pinned supports were made. The equation to determine this value is as follows:

$$\frac{0.548-0.50}{1-0.50} = \frac{X_n-100}{0-100}$$
$$X_n = \left[-\left(\frac{0.548-0.50}{0.50}\right)\cdot 100\right]+100 \quad (4)$$

Allowing fixed supports to have 100 percent fixity and a *K*-value of 0.50, pin-supported conditions having zero percent fixity and a *K*-value of 1.0, interpolation between the two conditions concluded that the columns tested under partially fixed obtained an average fixity percentage of 90.49 percent. All values for individual members can be seen in **Table 2**. Again, this supports the observation that a partially fixed support such as the one seen in CSUN's ITM lies closer in relation to fixed-end conditions than simply supported, but still results in displaying its own distinct column behavior.

Further probabilistic analysis was done through the generation of a fragility curve. Fragility curves are S-shape plots primarily used in earthquake engineering to assess the damage state of structures or to aid in the prediction of how much damage a structure will encounter when earthquake conditions are applied. This is done through a cumulative probability distribution plotted with earthquake intensity and damage grades. This method of analysis can also be applied to column buckling. To do this, the mean and standard deviation of the effective length values from the entire column set is applied to a cumulative distribution function. These values can also be seen in **Table 2**. Then, the probability distribution was plotted against the original respective *K*-values obtained from experimental testing. The plot is shown in **Figure 9**.

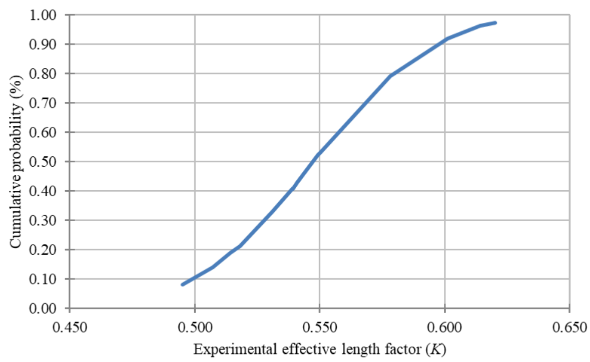

Figure 9. Fragility curve.

Along the ordinate, we can see the range is from zero to one. This is the probability percentage that a given effective length value will show. In other words, by taking any reference point along the curve the area underneath the curve represents the probability that the respective effective length ratio will occur. For example, at the effective length value of 0.55 the cumulative probability outputs a value of about 53 percent. This means that for future testing of these columns, there is a 53 percent probability that the resulting effective lengths between 0.495 and 0.55 will be produced. The same analysis is done from the higher and lower ends of the curve. At the effective length reference point of 0.6 the corresponding probability is around 10 percent that a greater effective length factor is produced and at an effective length factor of 0.5 the probability is less than 10 percent to produce a value less than or equal to 0.5. This coincides with the behavior of the columns and concludes that through further testing, 80 percent of columns will produce an effective length between 0.5 and 0.6 with a larger majority resulting closer to 0.55.

6. Conclusions

In this paper, the determination of the effective-length factors of columns with partially-fixed end supports and subjected to axial loading was investigated through the adoption of experimental and analytical approaches. To this end, a case study on the testing of columns by CSUN's ITM was conducted and discussed. The support conditions of CSUN's ITM allow the columns to produce an alternate behavior due to external compression that is not seen under traditional support connections such as fixed and pinned conditions. A partially-fixed connection with an average effective length of about 0.55, although not far from fixed conditions, still reveals there is slightly different behavior and is significant enough to be classified as its own support condition. Creating a fragility curve of the column behavior also provided a predictability tool to determine how the column will behave under future testing. The effectiveness of the adopted experimental, analytical, and probabilistic approaches in the determination of the accurate effective-length factor in columns with partially-fixed supports was demonstrated in this research endeavor. In particular, the application of the fragility methodology in efficient buckling stability assessment of structures seems to be quite promising.

Conflict of Interest

There is no conflict of interest.

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