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ARTICLE

Alpha Power-Kumaraswamy Distribution with An Application on Survival Times of Cancer Patients

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ABSTRACT

The aim of the study is to obtain the alpha power Kumaraswamy (APK) distribution. Some main statistical properties of the APK distribution are investigated including survival, hazard rate and quantile functions, skewness, kurtosis, order statistics. The hazard rate function of the proposed distribution could be useful to model data sets with bathtub hazard rates. We provide a real data application and show that the APK distribution is better than the other compared distributions fort the right-skewed data sets.

1. Introduction

There are many statistical distributions in literature but it is always possible to develop both more flexible and more suitable specific real world scenarios. The Kumaraswamy distribution may be a family of continuous probability distributions characterized on the interval (0,1). Poondi Kumaraswamy [1] initially proposed this distribution for factors that are lower and upper bounded. Kumaraswamy distribution has numerous of the properties of the beta distribution conjointly has a few advantages in terms of tractability. For case, its cumulative distribution encompasses a closed form, the quantile functions are effortlessly obtainable, and one can easily generate random variables from Kumaraswamy's distribution. Kumaraswamy appeared that the well-known probability density funcions (pdf) such as the log-normal, normal, and Beta distributions don't fit well for hydrological data like day by day rain fall and day by day stream flow. Kumaraswamy [2] created a more common density function for double-bounded random processes, which is known as Kumaraswamy's distribution. This distribution is applicable to numerous natural phenomena whose results have lower and upper bounds, such as the heights of individuals , scores gotten on a test, barometrical temperatures, hydrological information, etc. Moreover, this distribution may be suitable in circumstances where researchers utilize probability distributions that have interminable lower or upper bounds (or both) to fit information, when in reality, the bounds are limited [3,4]. In spite of the fact that the Kumaraswamy distribution was presented in 1980, the primary hypothetical examination of it was presented by Mitnik [5,6]. He derived an expression for the minutes, considered the distribution's restricting distributions, presented an explanatory expression for the mean absolute deviation around the median as a function of the parameters of the distribution, established a few bounds for this scattering

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measure and for the variance, and examined the relationship between the Kumaraswamy distribution and the Beta family of distributions. Garg [8] derived generalized order statistics from Kumaraswamy's distribution. .Jones [9] considered the distribution's skewness and kurtosis properties. In addition, he inferred common equations for L-moments and the moments of order statistics of the Kumaraswamy dispersion, determined maximum likelihood estimation (MLE) for the parameters of Kumaraswamy distribution, and compared between the Beta and the Kumaraswamy distributions from a few points of view. The classical Bayesian estimators of the Kumaraswamy distribution for grouped and un-grouped data were obtained by Gholizadeh, Shirazi, and Mosalmanzadeh [10]. Feroze and El-Batal [11] determined the maximum likelihood estimators based on Kumaraswamy dynamic Sort II censored data with random removals. Nadar, Papadopoulos, and Kızılaslan [12] utilized maximum likelihood and Bayesian strategies to get the estimators of the parameters Kumaraswamy distribution based on record data.

The PDF and cumulative distribution function (CDF) of the Kumaraswamy distribution are, respectively given as

$$f(x)=\alpha\beta x^{\alpha-1}(1-x^{\alpha})^{\beta-1}, 0\le x\le 1, \alpha>0, \beta>0 \tag{1}$$

$$F(x)=1-\left(1-x^{\alpha}\right)^{\beta}, 0\le x\le 1, \alpha>0, \beta>0 \tag{2}$$

Plots for the density and cumulative distributive functions of the Kumaraswamy distribution are presented in Figures 1.a and 1.b.

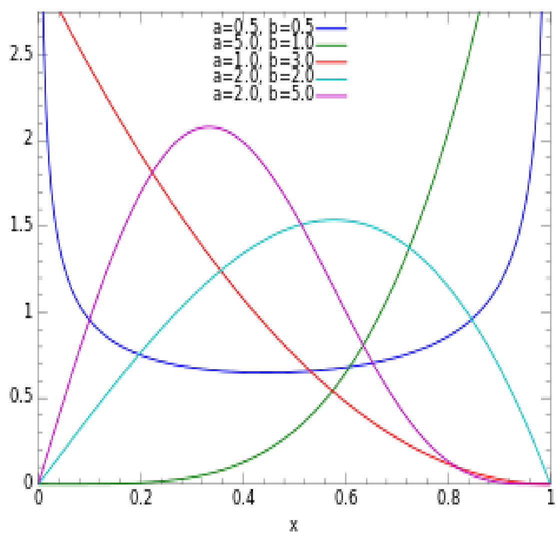

Figure 1. a. Graphs of the density function for the Kumaraswamy distribution

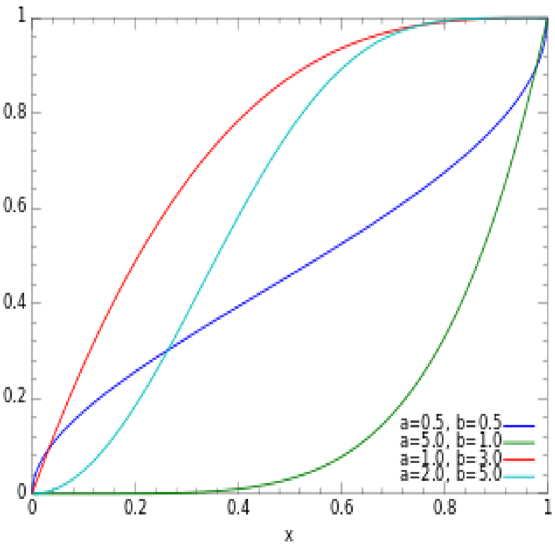

Figure 1. b. Graphs of the cdf for the Kumaraswamy distribution

There have been many methods to obtain more flexible distributions (see Lee et al. [13]). In recent years, Mahdavi and Kundu [12] have proposed Alpha Power Transformation (APT) by adding an extra parameter to a family of distributions. This parameter provides more adaptability to proposed family. Mahdavi and Kundu [12] used APT method to the exponential distribution and obtain the alpha power exponential (APE) distribution. Dey et al. [14] introduced alpha power transformed generalized exponential (αPTGE) distribution. Then Dey et al. [15] obtained alpha power transformed Weibull (APTW) distribution which contains APE distribution for λ =1 and alpha power transformed Rayleigh (APTR) distribution λ =2.

Let f(x) and F(x) be the pdf and the cdf of a continuous random variable X, respectively. The APT of cdf, pdf, survival and hazard rate functions are, respectively, given by for x ∈ R is defined as follows:

$$F_{APT}(x)=\begin{cases}\dfrac{\alpha^{F(x)}-1}{\alpha-1}, & if\ \alpha>0\ and\ \alpha\ne 1\\ F(x), & if\ \alpha=1\end{cases} \tag{3}$$

$$f_{APT}(x)=\begin{cases}\dfrac{log\alpha}{\alpha-1}f(x)\alpha^{F(x)}, & if\ \alpha>0\ and\ \alpha\ne 1\\ f(x), & if\ \alpha=1\end{cases} \tag{4}$$

$$S_{APT}(x)=\begin{cases}\dfrac{\alpha}{\alpha-1}(1-\alpha^{F(x)-1}), & if\ \alpha>0\ and\ \alpha\ne 1\\ 1-F((x), & if\ \alpha=1\end{cases} \tag{5}$$

$$h_{APT}(x)=\begin{cases}\frac{\alpha^{F(x)-1}}{1-\alpha^{F(x)-1}}f(x)\log\alpha, & if\ \alpha>0\ and\ \alpha\neq 1\\ \frac{f(x)}{S(x)}, & if\ \ \alpha=1\end{cases} \quad (6)$$

In this study, we propose Alpha Power Kumaraswamy (APK) distribution motivated by Kumaraswamy distribution and the APT (Alpha Power Transformation) method that mentioned above. In section 2, statistical properties for APK distribution are obtained including skewness, kurtosis, order statistics, survival, hazard rate and quantile functions. In Section 4 real data is used to evaluate performance of proposed distribution. Finally, the study is completed in Section 5.

2. Alpha Power-Kumaraswamy Distribution

Motivated by APT method, we obtain APK distribution. The random variable X has a three-parameter APK distribution if the cdf of X for x>0 as follows:

$$F_{APK}(x)=\frac{c^{1-(1-x^{\alpha})^{\beta}}-1}{c-1}, c\neq 1,\quad \alpha,\beta,c>0,\quad 0<x<1 \quad (7)$$

and the corresponding pdf is obtained as

$$f_{APK}(x)=\frac{\alpha*\beta*logc}{c-1}*x^{\alpha-1}*\left(1-x^{\alpha}\right)^{\beta-1}*c^{1-\left(1-x^{\alpha}\right)^{\beta}};c\neq 1 \quad (8)$$

Figure 2 shows $f(x)$ of APK distribution with different parameter values.

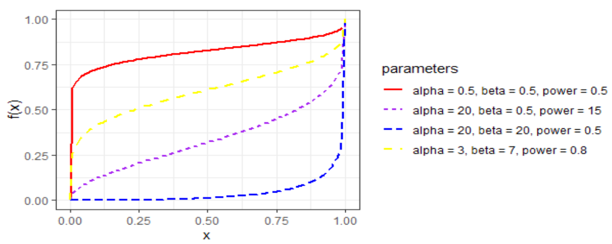

Figure 2. Plots of density function of APK distribution with several values of parameters

As seen in Figure 2, the pdf of APK is flexible and has various shapes for the several values of parameters.

3. Main Properties

3.1 Survival and Hazard Rate Functions

Now, we will provide survival and hazard rate functions for APK distribution. The survival function of the APK distribution for x>0 is given as

$$S_{APK}(x)=1-\frac{c^{1-(1-x^{\alpha})^{\beta}}-1}{c-1}c\neq 1,\quad \alpha,\beta,c>0,\quad 0<x<1 \quad (9)$$

Other important characteristic of the APK distribution is hazard rate function which is given by

$$h_{APK}(x)=\frac{\frac{\alpha*\beta*logc}{c-1}*x^{\alpha-1}*\left(1-x^{\alpha}\right)^{\beta-1}*c^{1-\left(1-x^{\alpha}\right)^{\beta}}}{1-\frac{c^{1-(1-x^{\alpha})^{\beta}}-1}{c-1}};c\neq 1 \quad (10)$$

Plots of survival hazard rate function for the APK distribution are shown in Figure 3 and 4, respectively.

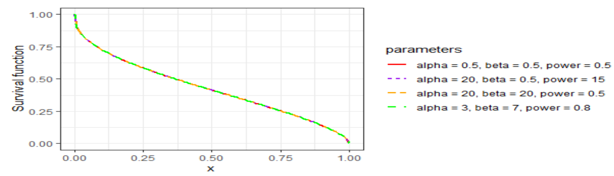

Figure 3. Graphs of density function of the APK distribution with several parameters values

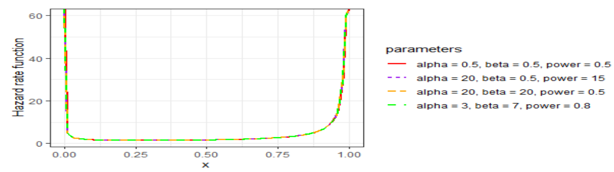

Figure 4. Plots of the hazard rate function of the APK distribution with several parameter values

As seen in Figure 4, the APK distribution is bathtub shaped.

3.2 Quantile Function

Quantile function is important in statistics and this function is described by the inverse of the CDF given by $Q(u) = \inf\{x \in R: u \leq F(x)\} = F^{-1}(x)$. Let $F(x) = u$, then we have

$$Q(u) = \{1 - [1 - \log_c(1 + u * (c - 1))]^{1/\beta}\}^{1/\alpha} \text{ for } 0 < u < 1, \alpha, \beta > 0 \quad (11)$$

where $u \sim$ Uniform(0,1). The p^{th} quantile function of APK distribution is obtained by

$$X = \{1 - [1 - \log_c(1 + u * (c - 1))]^{1/\beta}\}^{1/\alpha}; 0 < u < 1 \quad (12)$$

In particular, the first three quantiles, Q_1, Q_2, Q_3 Q_3 for the APK distribution, are obtained by setting u =0.25 (25^{th} percentile), u =0.50 (50^{th} percentile) and u =0.75 (75^{th} percentile), in Equation 12 respectively. The

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median Q_2 is obtained from Equation (12) by substituting u =0.5. Therefore the median is obtained fort he APK distribution as follows:

$$M = \{1 - \left[1 - \log_c\left(1 + \frac{1}{2} * (c-1)\right)\right]^{\frac{1}{\beta}}\}^{\frac{1}{\alpha}}; 0 < u < 1 \quad (13)$$

Here, 25^{th} percentile and 75^{th} percentile are given by, respectively

$$Q_1 = \{1 - \left[1 - \log_c\left(\frac{c}{4} + \frac{3}{4}\right)\right]^{\frac{1}{\beta}}\}^{\frac{1}{\alpha}}; 0 < u < 1 \quad (14)$$

$$Q_3 = \{1 - \left[1 - \log_c\left(\frac{3c}{4} + \frac{1}{4}\right)\right]^{\frac{1}{\beta}}\}^{\frac{1}{\alpha}}; 0 < u < 1 \quad (15)$$

Figures 5 and 6 presents 25^{th} and 75^{th} percentile functions of the APK distribution for different parameter values.

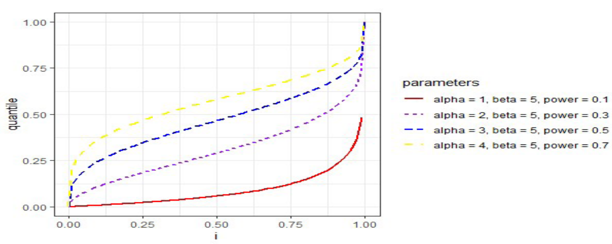

Figure 5. Graphs of the 25^{th} percentile functions of the APK distribution with several values parameters

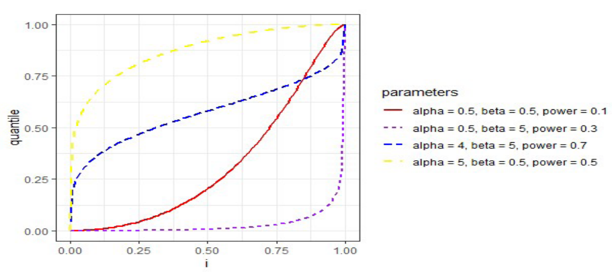

Figure 6. Graphs of the 75^{th} percentile functions of the APK distribution with several values parameters

3.3 Skewness and Kurtosis

The coefficient of skewness is a measure of symmetry and the coefficient of kurtosis is also a measure of whether the data are heavy tailed or thin tailed. The Bowley's skewness is based on quartiles as follows:

$$S = \frac{Q\left(\frac{3}{4}\right) - 2Q\left(\frac{1}{2}\right) + Q\left(\frac{1}{4}\right)}{Q\left(\frac{3}{4}\right) - Q\left(\frac{1}{4}\right)} \quad (16)$$

and the Moors' kurtosis is given below:

$$K = \frac{Q\left(\frac{7}{8}\right) - Q\left(\frac{5}{8}\right) - Q\left(\frac{3}{8}\right) + Q\left(\frac{1}{8}\right)}{Q\left(\frac{6}{8}\right) - Q\left(\frac{2}{8}\right)} \quad (17)$$

where $Q(.)$ represents the quantile function. Note that the Bowley's skewness and the Moor's kurtosis can be obtained by $Q(u)$ which is given in Equation (11).

It is important to state that the distribution is symmetric for S = 0. When S > 0, the distribution is positively (right-skewed). For S < 0, the distribution is left-skewed (negatively-skewed). Similarly, as long as K increases the tail of the distribution brings about heavier. A normal distribution has kurtosis exactly 3. If compared to a normal distribution, when K > 3 (K < 3) its tails are longer (shorter) and central peak is higher (lower).

The skewness, kurtosis, median and Q_1, $Q_3$of the APK distribution are listed in Table 1.

Table 1. The skewness, kurtosis, median and Q_1, Q_3 of the APK distribution with several values of parameters

Parameters			Q_1	Median	Q_3	Skewness	Kurtosis
Power	α	β					
0.1	0.5	0.5	0.04374	0.20418	0.54460	0.35935	1.08994
	0.5	2	0.00324	0.01947	0.08096	0.58232	1.94818
	0.5	7	0.00027	0.00176	0.00832	0.62948	2.31957
	2	0.5	0.45732	0.67220	0.85905	-0.0697	1.07094
	2	2	0.23868	0.37357	0.53342	0.08470	1.19361
	2	7	0.12891	0.20502	0.30204	0.12079	1.24439
	7	0.5	0.79968	0.89272	0.95752	-0.1788	1.19085
	7	2	0.66410	0.75478	0.83564	-0.0572	1.22497
	7	7	0.55693	0.63587	0.71031	-0.0294	1.24337
0.5	0.5	0.5	0.12122	0.43272	0.80346	0.08683	0.80416
	0.5	2	0.01029	0.05530	0.18715	0.49101	1.50404
	0.5	7	0.00090	0.00543	0.02234	0.57732	1.92367
	2	0.5	0.59006	0.81106	0.94676	-0.2391	1.09221
	2	2	0.31854	0.48494	0.65773	0.01883	1.13357
	2	7	0.17351	0.27155	0.38663	0.07999	1.19000
	7	0.5	0.86008	0.94192	0.98449	-0.3156	1.25449
	7	2	0.72118	0.81320	0.88718	-0.1086	1.20846
	7	7	0.60627	0.68903	0.76223	-0.0613	1.22406

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0.9	0.5	0.5	0.17880	0.54267	0.86932	-0.0539	0.79080
	0.5	2	0.01648	0.08045	0.24014	0.42797	1.34859
	0.5	7	0.00148	0.00826	0.03063	0.53475	1.75134
	2	0.5	1.75134	0.85829	0.96559	-0.3194	1.16177
	2	2	0.35831	0.53258	0.70003	-0.0199	1.12777
	2	7	0.19621	0.30150	0.41836	0.05206	1.17794
	7	0.5	0.88429	0.95727	0.99004	-0.3802	1.33279
	7	2	0.74584	0.83526	0.90312	-0.1370	1.21939
	7	7	0.04374	0.20418	0.54460	0.35935	1.08994

Table 1 indicates that the kurtosis and skewness increase with increasing of the β,whereas these values decrease when power and α increase. So, APK distribution is platykurtic for all values of the parameters.

3.4 Order Statistics

Let $X_1,X_2,\ldots,X_n$ be a random sample from any APK distribution. Let Xi:n indicate the i^{th} order statistics. Now, we derive the pdf of the i^{th} order statistics Xi:n ($1\leq i\leq n$) for *APK* distribution given by

$$f_{i:n}=\frac{n!}{(i-1)!(n-i)!}f\left(x_i\right)[F(x_i)]^{i-1}\ [1-F(x_i)]^{n-i} \tag{18}$$

where $F(x)$ and $f(x)$ are given in Equation (7) and (8) respectively. Therefore pdf for the i^{th} order statistics becomes

$$f_{i:n}\left(x\right)=\frac{n!}{(i-1)!(n-i)!}*\alpha\beta * \frac{c^{1-\left(1-x^{\alpha}\right)^{\beta}}*x^{\alpha-1}*\left(1-x^{\alpha}\right)^{\beta-1}\log(c)}{c-1}\left(\frac{c^{1-\left(1-x^{\alpha}\right)^{\beta}}-1}{c-1}\right)^{i-1}\left(1-\frac{c^{1-\left(1-x^{\alpha}\right)^{\beta}}-1}{c-1}\right)^{n-i} \tag{19}$$

From this equation, for i=1, the density function of the minimum order statistics of the APK distribution is given by

$$f_{1:n}=\frac{n!}{(n-1)!}f\left(x_1\right)[1-F(x_1]^{n-1}=nf\left(x_1\right)[1-F(x_1]^{n-1} \tag{20}$$

Then, we have

$$f_{1:n}=n\alpha\beta*\frac{c^{1-\left(1-x^{\alpha}\right)^{\beta}}*x^{\alpha-1}*\left(1-x^{\alpha}\right)^{\beta-1}\log(c)\left(1-\frac{c^{1-\left(1-x^{\alpha}\right)^{\beta}}-1}{c-1}\right)^{n-1}}{c-1} \tag{21}$$

and likewise, the pdf of the maximum order statistics (i=n,) of the APK distribution as follows:

$$f_{n:n}=\frac{n!}{(n-1)!}f\left(x_n\right)[F(x_n]^{n-1}=nf\left(x_n\right)[F(x_n]^{n-1} \tag{22}$$

and we have

$$f_{n:n}=n\alpha\beta*\frac{c^{1-\left(1-x^{\alpha}\right)^{\beta}}*x^{\alpha-1}*\left(1-x^{\alpha}\right)^{\beta-1}\log(c)\left(\frac{c^{1-\left(1-x^{\alpha}\right)^{\beta}}-1}{c-1}\right)^{n-1}}{c-1} \tag{23}$$

4. Estimation

If the X_i,i=1,2,…,n are independent APK random variable with unknown parameter c, then the probability density function of each X_i is given by

$$f_{APK}\left(x_i,c\right)=\frac{\alpha*\beta*logc}{c-1}*x_i^{\alpha-1}*\left(1-x_i^{\alpha}\right)^{\beta-1}*c^{1-\left(1-x_i^{\alpha}\right)^{\beta}};c\neq1 \tag{24}$$

The likelihood function $L(c)$ is defined as

$$L\left(c\right)=\prod_{i=1}^{n}f_{APK}\left(x_i,c\right) \tag{25}$$

Presently, in arrange to execute the strategy of maximum likelihood, we ought to find the c that maximizes the probability $L(c)$. We have to be put on our calculus hats presently, since in arrange to maximize the function, we are getting to ought to separate the probability function with regard to c. In doing so, we'll utilize a "trick" that frequently makes the differentiation a bit simpler. Note that the natural logarithm is an increasing work of x That is, if $x_1\leq x_2$. then $f(x_1)\leq f(x_2)$. Meaning that the value of c that maximizes the normal logarithm of the probability function $lnL(c)$ is additionally the value of c that maximizes the probabilityfunction $L(c)$.So, the "trick" is to take the derivative of $lnL(c)$ (with respect to c) rather than taking the derivative of $L(c)$ Then we have

$$\begin{aligned}lnL\left(c\right)&=\sum_{i=1}^{n}lnf_{APK}\left(x_i,c\right)\\&=\sum_{i=1}^{n}ln\frac{\alpha*\beta*logc}{c-1}*x_i^{\alpha-1}*\left(1-x_i^{\alpha}\right)^{\beta-1}*c^{1-\left(1-x_i^{\alpha}\right)^{\beta}};c\neq1\end{aligned} \tag{26}$$

Now, taking derivative of the log likelihood,and setting to 0

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$$\frac{\partial lnL(c)}{\partial c}=\frac{n}{lnc}\left(1-\frac{1}{c}\right)-n+\frac{1}{c}*\left\{n-\sum_{i=1}^{n}\left(1-x_i^{\alpha}\right)^{\beta}\right\}\equiv 0 \quad (27)$$

We can get maximum values of the parameters. By using most computer algebra systems we can examine this equations to dtermine which is MLE of the c

5. Application

In this section, we perform an application of the APK model to prove empirically its potentiality. We used data set that were pre-modeled by different distribution. Then, we provide a comparison of fits of other competitive models. So as to compare the fits of the APK model with other competing distributions, we consider Akaike Information Criteria (AIC), Corrected Akaike Information Criteria (CAIC), Bayesian Information Criteria (BIC), Hannan-Quinn (HQIC), and log-likelihood (LL).

The data set is modelled with different distributions in some previous studies. These distributions are the weighted Lindley (WL), Lindley (L) distributions from Shanker et al. [15]; three-parameter weighted Lindley (TPWL) distribution from Shanker et al. [16]; IE, generalized inverted exponential (GIE), inverse Rayleigh (IR) distributions from Sharma et al. [17] and Singh et al. [18] and inverse Lindley (IL) distribution from Sharma et al. [17].

The data set shown in Table 2 includes 44 survival times of patients get Head and Neck cancer disease. This data used in this paper was given by Efron [19]. The data set and its descriptive statistics are presented in Table 2 and Table 3, respectively.

Table 2. Survival times of patients get Head and Neck cancer disease

12.20 23.56 23.74 25.87 31.98 37 41.35 47.38 55.46 58.36 63.47 68.46 78.26 74.47 81.43 84 92 94 110 112 119 127 130 133 140 146 155 159 173 179 194 195 209 249 281 319 339 432 469 519 633 725 817 1776

Table 3. Descriptive statistics of survival times data

Mean	St.Deviation	Variance	Skewness	Kurtosis
223,477	305,428	93286,413	3,504	15,387

It can be noticed from Table 3 that the data set is also right-skewed and leptokurtic with the coefficients of skewness and kurtosis.We normalized this dataset into (0,1) range in order to compress it. The goodness-of-fit statistics are presented in Table 4.

Table 4. The goodness-of –fit statistics fort he survival times data

	AIC	BIC	AICc	LL
APK	-82.25304	-65.54790	-81.65304	44.12652
Normal	-30.96772	-27.39934	-30.67504	17.48386
Beta	-63.08005	-59.51167	-62.78736	33.54002
Kumaraswamy	-65.65957	-62.09119	-65.36688	34.82978

The results in Table 4 shows that when the compare with the other distribution, the APK distribution has the lowest values of AIC, BIC, CAIC and the highest value of LL. From the result, we conclude that the APK distribution is a very flexible distribution to model right-skewed data sets. The parameter estimations are obtained as α=1.206, β=1.672, c=0.001 by maximum likelihood method.

6. Conclusion

In this study, the Alpha Power Kumaraswamy (APK) is obtained. Some important statistical properties of the APK distribution are obtained including survival, hazard rate and quantile functions, skewness, kurtosis. As seen from the plots of hazard function the proposed distribution could be useful to model data sets with bathtub hazard rates. Then, we provide a real data application and show that the APK distribution is better than the other compared distributions for right-skewed data sets.

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