

## REVIEW

# High-Resolution Traffic Flow Prediction Model Based on Deep Learning

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### ABSTRACT

The time resolution of the existing traffic flow prediction model is too big to be applied to adaptive signal timing optimization. Based on the view of the platoon dispersion model, the relationship between vehicle arrival at the downstream intersection and vehicle departure from the upstream intersection was analyzed. Then, a high-resolution traffic flow prediction model based on deep learning was developed. The departure flow rate from the upstream and the arrival flow rate at the downstream intersection was taking as the input and output in the proposed model, respectively. Finally, the parameters of the proposed model were trained by the field data, and the proposed model was implemented to forecast the arrival flow rate of the downstream intersection. Results show that the proposed model can better capture the fluctuant traffic flow and reduced MAE, MRE, and RMSE by 9.53%, 39.92%, and 3.56%, respectively, compared with traditional models and algorithms, such as Robertson's model and artificial neural network. Therefore, the proposed model can be applied for real-time adaptive signal timing optimization.

## 1. Introduction

As one of the most important components of adaptive control systems (e.g., TRANSYT<sup>[1]</sup> and SCOOT<sup>[2]</sup>), the platoon dispersion model is also the basis of traffic flow prediction, simulation, and signal

timing optimization. The first platoon dispersion model based on the hypothesis that the vehicle's velocity follows normal distribution was proposed by Pacey<sup>[3]</sup> in 1965. From the view of traffic flow, Robertson<sup>[1]</sup> proposed a platoon dispersion model supposing that travel times obey a

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shifted geometric distribution. Since the shifted geometric distribution has the merit of simpleness and convenient calculation, Robertson's platoon dispersion model has been incorporated in a large amount of software or systems, including TRANSYT-7F<sup>[1]</sup>, SCOOT<sup>[2]</sup>, SATURN<sup>[4]</sup>, and TRAFLO<sup>[5]</sup>. Most of the later studies<sup>[6,7]</sup> are based on the assumption that the travel speed or travel time follows a certain statistical distribution. These models are accurate or not depend on the assumption. However, traffic flow may operate in unstable states which caused by the signal intersection. Moreover, in real-time transportation systems, access to information constraints will lead to the distribution of restricted traffic data different from that of theoretical traffic data. Under these complex situations, the platoon dispersion models may be unable to capture the dispersion of traffic flow, and may become inapplicable for real-time transportation systems. To some extent, this shortcoming limits the potential application of some real-time traffic signal control systems included with the platoon dispersion models. With the development of big data technology, many methods<sup>[8-15]</sup> were proposed in big data environment. However, the forecasting time resolution of these models is too big, such as 5, 10, 30, and even 60 minutes. Therefore, these models have good predicted effect, but the big time resolution is not enough to be applied in adaptive control systems.

In summary, a proposed traffic flow prediction model must be able to accurately capture the change of traffic flow and must satisfy the optimal time resolution of the signal timing optimization.

In fact, apart from the classic platoon dispersion models, some intelligent methods (e.g., neural network<sup>[16,17]</sup>, support vector machine<sup>[18]</sup>, Kalman filter<sup>[19,20]</sup>, etc.) are also applied to predict traffic flow. However, these methods only consider the time series characters of the traffic flow at downstream intersection, and do not consider the correlation between the arrival and departure rate of the downstream and upstream intersection.

From the perspective of platoon dispersion model, there is a certain relationship between the arrival and departure flow of the downstream and upstream intersection. Considering the deep learning can describe any complex stochastic and non-linear systems, a high-resolution traffic prediction model based on deep learning is proposed. Then, the real-time data of the upstream intersection is used to forecast the arrival flow of the downstream intersection based on the proposed model. In addition, the prediction time resolution of the proposed model can be determined by the actual demand. In this study, we choose 5 seconds as the prediction time resolution, which fully meets the minimum requirements of the adaptive control

algorithms or systems.

This paper is organized as follows. Section 2 reviews the classical Robertson's platoon dispersion model. A high-resolution traffic flow prediction model based on deep learning is proposed in Section 3. In Section 4, the predicted results of the proposed model and traditional models and algorithms are compared. Section 5 closes the paper with conclusions and further research.

## 2. The Platoon Dispersion Model

### 2.1 Notation of Platoon Dispersion

Owing to the existence of the urban signal intersections, the continuous traffic flow is forced to split into separate platoons. Meanwhile, due to the differences of drivers' driving behavior and safety awareness, these factors result in the discrepancy of vehicle's speed. Consequently, the platoon becomes longer as vehicles travel further downstream, and this phenomenon is commonly called "platoon dispersion". This phenomenon is illustrated in Figure 1. When the signal light is red, vehicles have to queue up at the stop line. While the signal light turns green, vehicles through the intersection with saturation flow, but it is not the saturation flow rate when vehicles reach the downstream section. Therefore, the flow rate is decreasing with time as the platoon reaches downstream intersection. In addition, the peak of the platoon will become smoother and smoother when the distance between the upstream and downstream section is getting longer and longer.

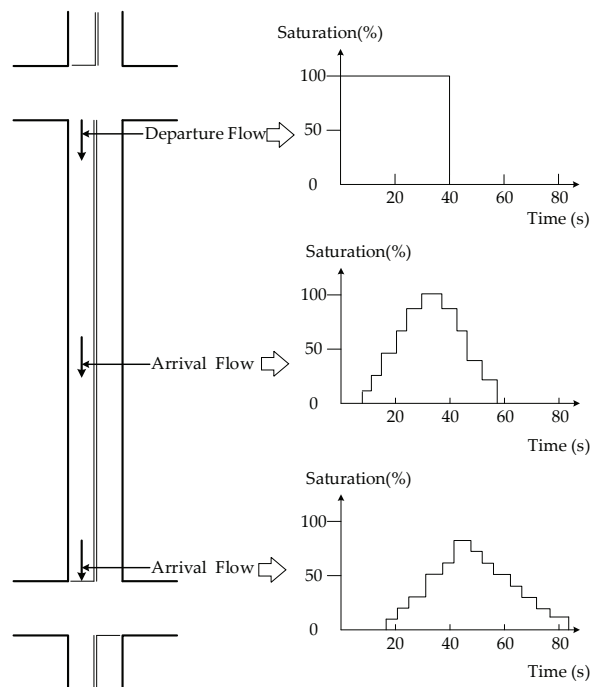


Figure 1. Diagram of platoon dispersion

### 2.2 The Robertson's Platoon Dispersion Model

The platoon dispersion model aims to describe the relationship between the departure flow of the upstream intersection and the arrival flow of the downstream intersection, and realize real-time prediction of arrival flow at downstream sections or intersections. The classical platoon dispersion has been given by Robertson<sup>[1]</sup>, who used observed data to derive an iterative method to capture the behavior of platoon. The traffic flow prediction model was used for optimization of traffic signals to obtain the minimum vehicle delay. For each time interval, the arrival flow rate of the downstream stop-line is calculated by Eq.(1).

$$q_d(t) = \sum_{i=1}^{t-T_{min}} q_u(i) \cdot F \cdot (1 - F)^{t-T_{min}-i}, \quad (1)$$

$$q_d(t) = F \cdot q_u(t - T_{min}) + (1 - F) \cdot q_d(t - 1), \quad (2)$$

where  $q_d(t)$  represents the arrival flow rate at time interval  $t$  of the downstream intersection.  $q_u(i)$  represents the departure flow rate at the time interval  $i$  of the upstream intersection.  $T_{min}$  represents the minimum travel time for the road segment, the value is equal to 0.8 times the mean travelling-time.  $F$  represents a smoothing factor.

In Eqs.(1-2), the departure flow of the upstream intersection can be obtained by loop detectors. Therefore, the arrival flow rate of the downstream intersection can be calculated by Eqs.(1-2) with estimated parameters. The minimum travel time  $T_{min}$  can be calculated according to the historical data, and the smoothing factor can be calculated by the Eq.(3).

$$F = \frac{1}{1 + \alpha\beta T}, \quad (3)$$

where  $\alpha$  is the platoon dispersion coefficient, which has been found to be 0.5 in central business district (CBD),  $\beta$  is the travel time factor, generally taken the value of 0.8. Readers could refer to TRANSYT-7F user's guide<sup>[21]</sup> for more details.  $T$  is the travel time for the road segment.

In Robertson's platoon dispersion model, the basic assumption is that the travel time follows a shifted geometric distribution. However, the shifted geometric distribution has a long tail and hence the Robertson's model predicts a greater dispersion of the platoon than the actual situation. Later, the actual data fitting proves that the travel time or speed follows various probability distributions, such as normal distribution, lognormal distribution, mixture Gaussian and truncated distribution of these distributions<sup>[6,7,21-27]</sup>, etc. When the traffic condition changes, the travel time or speed distribution will also change greatly.

However, the parameters of the Robertson's model are static which cannot reflect the real-time traffic flow characteristics.

### 3. Deep Learning-based High-resolution Traffic Flow Prediction Model

#### 3.1 Deep Learning

Deep learning describes a high dimensional function via a sequence of semi-affine non-linear transformations<sup>[28]</sup>. The deep learning architecture<sup>[29]</sup> is organized as a graph shown in Figure 2.

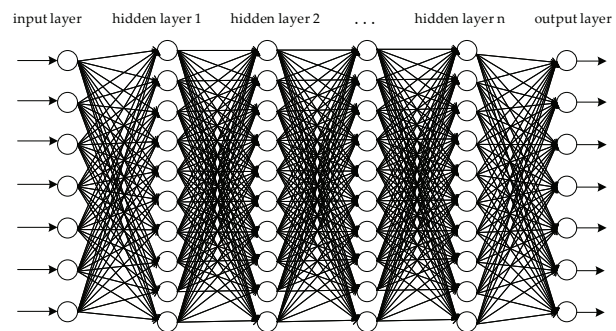


Figure 2. Basic structure of deep learning

A deep learning predictor, denoted by  $\hat{y}(x)$ , takes an input vector  $x = (x_1, \dots, x_p)$  and outputs via different layers of abstraction that employ hierarchical predictors by composing  $L$  non-linear semi-affine transformations. Specifically, a deep learning architecture is as follows. Let  $f_1, \dots, f_n$  be given univariate activation link functions, e.g. sigmoid  $1/(1+e^{-x})$ ,  $cosh x$ ,  $tanh x$ , Heaviside gate functions ( $I(x > 0)$ ), or rectified linear units ( $\max\{x, 0\}$ ) or indicator functions ( $I(x \in R)$ ) for trees. The composite map is defined by

$$\hat{y}(x) = G(x) = (f_{w_n, b_n} \circ \dots \circ f_{w_1, b_1})(x), \quad (4)$$

where  $f_{w, b}$  is a semi-activation rule defined by

$$f_{w_l, b_l}(x) = f\left(\sum_{j=1}^{N_l} w_{lj} x_j + b_l\right) = f(w_l^T x_l + b_l), \quad (5)$$

where  $N_l$  represents the number of units at layer  $l$ . The weights  $w_l \in R^{N_l \times N_{l-1}}$  and offset  $b \in R$  needs to be learned from training data.

Data dimension reduction of a high dimensional map  $G$  is performed via the composition of univariate semi-affine functions. Let  $z^l$  denote the  $l$ -th layer hidden features, with  $x = z^0$ . The final output is the response  $y$ , can be numeric or categorical. The explicit structure of a deep prediction rule is than

$$\begin{aligned}
 z^1 &= f(w_0^T x + b_0) \\
 z^2 &= f(w_1^T z^1 + b_1) \\
 &\vdots \\
 z^n &= f(w_{n-1}^T z^{n-1} + b_{n-1}) \\
 y(x) &= w_n^T z^n + b_n
 \end{aligned} \tag{6}$$

In many cases there is an underlying probabilistic model, denoted by  $p(y|\hat{y}(x))$ . This leads to a training problem given by optimization problem

$$\min_{w,b} \frac{1}{T} \sum_{i=1}^T -\log p(y_i | \hat{y}_{w,b}(x_i)), \tag{7}$$

where  $p(y|\hat{y}(x))$  is the probability density function given by specification  $y_i = G(x_i) + \epsilon_i$ . Efficient algorithms<sup>[31]</sup> exist to solve those problems, even for high dimensional cases.

In summary, when the structure of the deep neural network, the training data samples and the error threshold are given, the deep neural network can be optimized through training by the optimization algorithm. Therefore, the actual process of the application of deep neural network is divided into four steps: designing the deep neural network structure, obtaining the data samples, training the network, and using the trained network to predict the value based on the new input. So, how to build a traffic flow prediction model based on deep learning will be discussed in next section.

### 3.2 High-Resolution Traffic Flow Prediction Model

In Figure 1, there is a length of  $\Delta x$  of road segment which between the upstream and downstream intersection. The minimum and maximum travel time of the road segment are calculated according to the speed limit of road segment or historical data.  $T_{min}$  and  $T_{max}$  are the minimum and maximum travel time, respectively.

The departure flow of the upstream intersection can be acquired in real time by detectors which are set at exit lane of the upstream intersection. According to the idea of the platoon dispersion model: the number of vehicles arriving at the downstream section which come from the vehicles of time interval  $([t - T_{max}, t - T_{min}])$  at upstream intersection. So, this relationship can be expressed by the follow formula.

$$q_d(t) = G(q_u(t - T_{max}), q_u(t - T_{max} + 1), \dots, q_u(t - T_{min})), \tag{8}$$

where  $G(\cdot)$  is a mapping relation.

According to the Eq.(8), there is a correlation between the arrival flow rate of the downstream intersection at the time interval  $t$ , and which maybe come from the upstream intersection for each time interval of time period

$[t - T_{max}, t - T_{min}]$ . This relationship cannot be expressed in a general form of function. In the classical platoon dispersion model, there is a basic assumption that vehicle's speed or travel time follows a certain probability distribution. Then, the relationship formula between the downstream and the upstream flow rate is derived. Therefore, these models need to select an appropriate probability distribution to describe the characteristics of traffic flow, and the probability distribution can only characterize the dynamic traffic flow to a certain extent. Considering the deep neural network can be used to describe any linearly separable complex stochastic systems, and does not require any underlying assumptions. The deep neural network is used to describe the mapping relation between the arrival traffic flow of the downstream intersection and the departure traffic flow of the upstream intersection. Then, a high-resolution traffic prediction model based on the deep learning is developed.

Firstly, we need to determine the structure of the deep neural network from the Eq.(8). The number of input neuron is  $T_{max} - T_{min} + 1$ , input variables are  $x = (q_u(t - T_{max}), q_u(t - T_{max} + 1), \dots, q_u(t - T_{min}))$ ; The number of output neuron is 1, output variable is  $y = q_d(t)$ . The number of hidden layers and hidden layer neurons can be selected according to comparative analysis. Among them, the number of input nodes of deep neural network with different time resolution is also different, which is equal to the length of time interval  $[t - T_{max}, t - T_{min}]$  divided by the selected time resolution.

After the deep neural network structure is determined, the network needs to be trained through the historical data. If we want to predict the traffic flow rate at the  $t$ th time interval, the historical data before the  $t$ th time interval can be used to train the network. Considering the network training needs a certain period of time, so we train or optimize the network every once in a while, such as 5 minutes, to ensure that the deep neural network training can be completed in the interval. Finally, the trained network can be used to forecast the traffic flow rate of the downstream intersection based on the departure flow of the upstream intersection which can be acquired in real time by detectors. Moreover, the historical arrival flow rate of the downstream intersection can also be obtained by detectors, which can be used to train the deep neural network.

### 3.3 High-Resolution Traffic Flow Prediction Algorithm

In this study, a high-resolution traffic flow prediction algorithm can be divided into the following 5 steps, which is shown in Figure 3.

Step 1. The minimum and travel time  $T_{min}, T_{max}$ , network

training update time  $\Delta t$  and the current time are determined according to the actual situation;  
 Step 2. The deep neural network structure is determined according to the relevant parameters in Step 1;  
 Step 3. The deep neural network is trained and optimized by using the historical data of the current time  $t$ ;  
 Step 4. The traffic flow of the downstream intersection is predicted based on the trained network and the real-time data obtained by the detectors at the upstream section, then  $t=t+1$ ;  
 Step 5. If the current time meets the network training update time, then skip to Step 3; otherwise, go to Step 4.

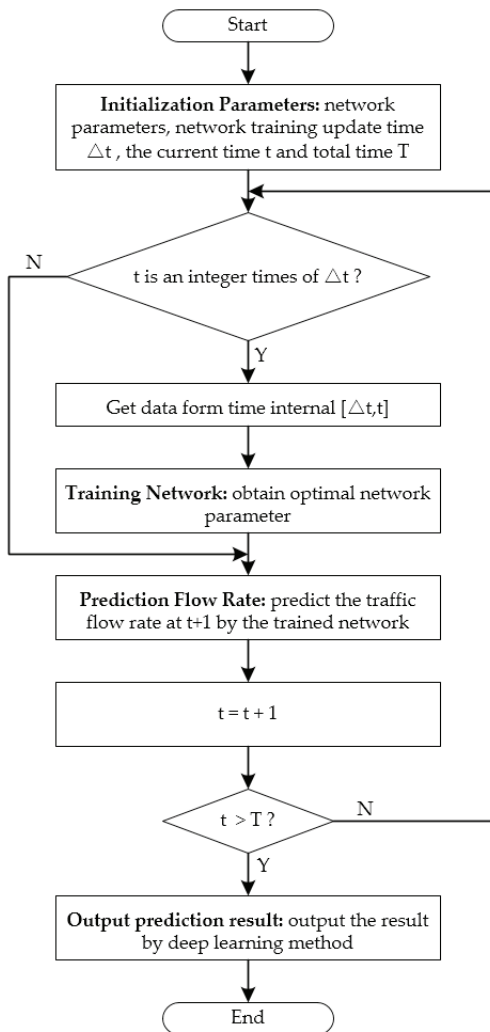


Figure 3. Flowchart of the proposed algorithm

Through the above 5 steps, the traffic flow of the downstream intersection can be predicted in real time.

### 4. Case Study

In this section, the predicted performance of Robertson's model, artificial neural network, and the proposed model

will be discussed based on the survey data.

### 4.1 Data Collection

In order to prove the proposed model have better performance, Wushan Road in Guangzhou is selected for field investigation, as shown in Figure 4. There are 14 bus lines via this segment, and in general the traffic condition is unsaturated. Specifically, the survey time interval is 7:30 am – 11:20 am and the traffic flow is volatile, forming a distribution with a typical morning peak. The travel times can be obtained by comparing vehicle license plates in the upstream and downstream section (the distance between two places is 650 m). After data preprocessing, we get 1,621 pieces of effective data as shown in Table 1. Afterward, the estimated value of all parameters can be calculated by a statistical method. Then, the flow rate of the upstream and downstream section are obtained in Figure 4.

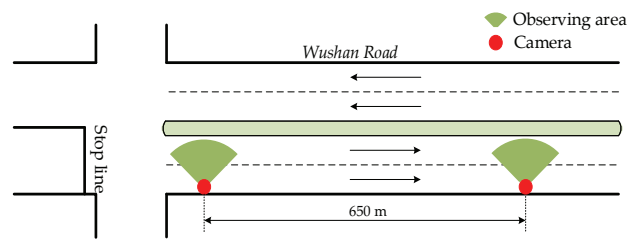
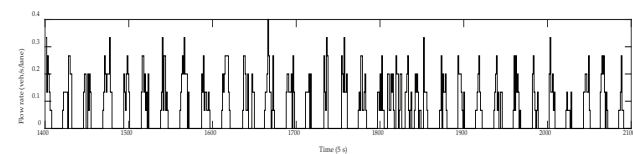
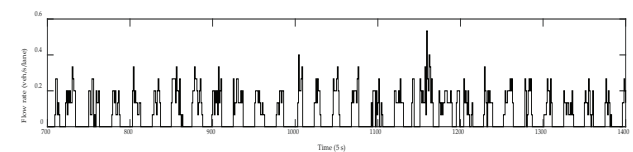
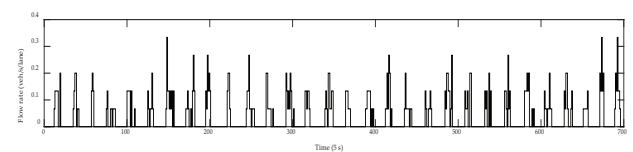


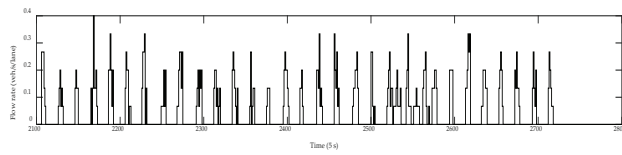
Figure 4. Diagram of the survey road segment.

Table 1. The statistical parameters

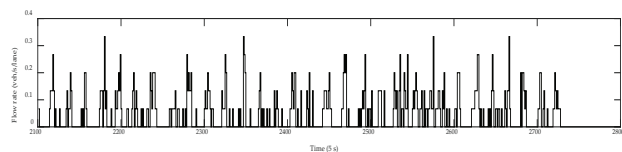
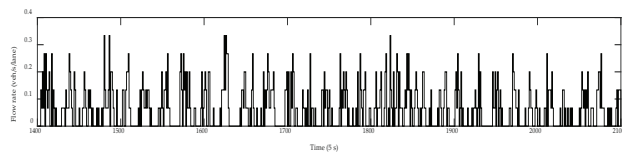
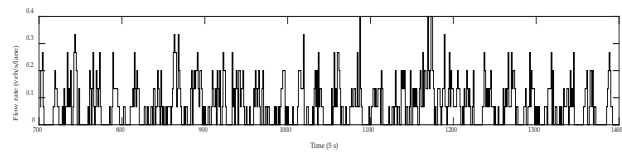
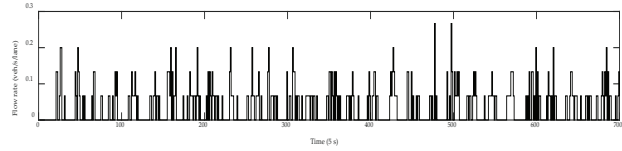
Statistical Parameter	Value
The number of vehicles (vehicle)	1621
The minimum travel time (second)	30
The maximum travel time (second)	130
The average travel time (second)	41.6







(a) The departure flow rate of the upstream section.



(b) The arrival flow rate of the downstream section.

**Figure 5.** The flow rate of departs and arrivals during time intervals of 5 s

As demonstrated in Figure 5, the fluctuation of traffic flow at the upstream and downstream section is large, and the regularity is not very significant. The forecast effect of these three models will be discussed based on the survey data as following.

### 4.2 Model Evaluation

Firstly, with 5 seconds as the time interval, the data are aggregated, and we obtain 2,720 data samples. Then, these data can be divided into two parts, the first part serves as the parameter calibration (the first 2,000 data samples), the second part serves as the model prediction effect validation (the last 720 data samples). Therefore, the parameters of Robertson's model can be calculated by the method in the literature<sup>[32]</sup> or the TRANSYT-7F manual<sup>[21]</sup>. The parameters of the artificial neural network and deep learning are calibrated by training based on the first part of

data samples<sup>[10,12]</sup>. The key parameter values of these three models are shown in Table 2.

**Table 2.** Key parameter values for different models

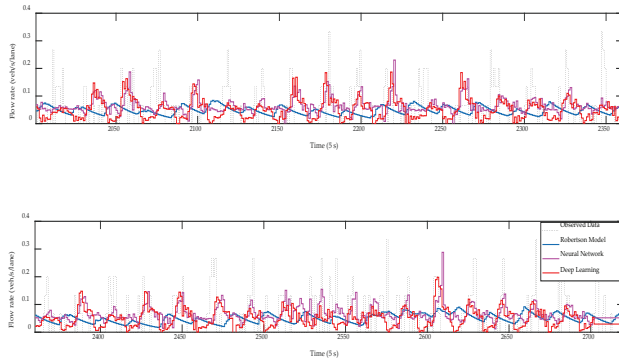
Model	Parameter	Value
Robertson's model	$T$	41.6 s
	$\alpha$	0.5
	$\beta$	0.8
Artificial Neural Network	Input neurons	10
	Hidden neurons	5
	Hidden layer number	1
	Output neurons	1
	Transfer function	S-function
	Training algorithm	Levenberg-Marquardt algorithm
	Training epoch	500
	Learning rate	0.05
Deep Learning	$T_{min}$	30 s
	$T_{max}$	130 s
	Input neurons	20
	Hidden neurons	[10,10,10,10,10]
	Hidden layer number	5
	Output neurons	1
	Transfer function	S-function
	Training algorithm	Levenberg-Marquardt algorithm
	Training epoch	1000
Learning rate	0.05	

Because the time interval is 5 seconds, and the difference between the maximum and the minimum travel time is 100 seconds, so the number of input neurons is 20. The number of hidden layers is 3 and hidden neurons are 10 calculated by the empirical formula<sup>[32]</sup> and the number of output neurons is 1. After the parameters of these three models are determined, we can predict the traffic flow rate of the downstream section by using Robertson's model, artificial neural network, and deep learning, respectively. The predicted results of these three models are shown in Figure 6. The performance of these three models can be assessed quantitatively by examining the prediction error statistics. Standard prediction measures include MAE, MRE, and RSME<sup>[33,34]</sup>. These measures for the predicted results shown in Figure 6 are given in Table 3.

**Table 3.** The Evaluation Index Value of Models

Measure	Robertson's Model	Artificial Neural Network	Deep Learning	Improvement
MAE	0.0576	0.0519	0.0494	14.24% / 4.82% / 9.53%
MRE	17.69%	23.51%	12.13%	31.43% / 48.41% / 39.92%
RMSE	0.0728	0.0732	0.0704	3.30% / 3.83% / 3.56%

Note: the value of improvement: improvement compare Robertson's model / improvement compare artificial neural network / average improvement compare these two models.



**Figure 6.** The actual and predicted arrival flow rate based on two models during time intervals of 5 seconds.

As shown in Figure 6, compared with the other two models, the deep learning prediction results can better capture the fluctuant characteristics of traffic flow. Because to a certain degree, the deep learning can get the relationship between the arrival and departure flow rate of the downstream section and the upstream intersection by training. On the contrary, Robertson's model is based on the strict assumption that travel times follow a shifted geometric distribution, and cannot accurately characterize the flow relationship between the upstream and downstream intersection. In addition, the artificial neural network only considers the time series characters of the traffic flow, and does not consider the correlation between the arrival and departure rate of the downstream and upstream intersection. The analysis shows that the performance of Robertson's model and artificial neural network do not work well when the traffic flow fluctuation is frequent. However, the deep learning can adapt to the fluctuation of traffic flow through continuous learning, so it has better prediction effect.

Moreover, the error analysis results in Table 3 show that the prediction errors of the deep learning are less than the Robertson's model and artificial neural network. The MAE, MRE, and RMSE of the deep learning is average reduced by 9.53%, 39.93%, and 3.56%, respectively, compared with Robertson's model and artificial neural network. Therefore, deep learning can be used for real-time traffic flow prediction, and the prediction time resolution

can be accurate to 5 seconds. The results can be applied to the optimization of adaptive signal timing.

## 5. Conclusion and Future Work

### 5.1 Conclusion

In this paper, a high-resolution traffic flow prediction model is proposed based on the perspective of the platoon dispersion model. The proposed model uses deep learning to describe the relationship between the arrival flow of the downstream intersection and the departure flow of the upstream intersection, and to realize the prediction of traffic flow at the downstream intersection. The results of the field data validation show that the proposed model is better than the Robertson's model and artificial neural network, and the time resolution is 5 seconds, which meets the basic needs of adaptive signal timing optimization algorithm. So, the proposed model can be used for adaptive signal timing optimization.

### 5.2 Future Work

Future work will be considered to study more other traffic information (e.g., vehicle's speed and acceleration) to improve the proposed model prediction capability. And the calculation method of intersection stopping times and queuing length based on the proposed traffic flow prediction model should be studied in future research. In addition, the stability and robustness of the proposed model will be discussed by more data set in the future.

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