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REVIEW

A Review on Ranking of Z-numbers

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ABSTRACT

There are numerous studies about Z-numbers since its inception in 2011. Because Z-number concept reflects human ability to make rational decisions, Z-number based multi-criteria decision making problems are one of these studies. When the problem is translated from linguistic information into Z-number domain, the important question occurs that which Z-number should be selected. To answer this question, several ranking methods have been proposed. To compare the performances of these methods, benchmark set of fuzzy Z-numbers has been created in time. There are relatively new methods that their performances are not examined yet on this benchmark problem. In this paper, we worked on these studies which are relative entropy based Z-number ranking method and a method for ranking discrete Z-numbers. The authors tried to examine their performances on the benchmark problem and compared the results with the other ranking algorithms. The results are consistent with the literature, mostly. The advantages and the drawbacks of the methods are presented which can be useful for the researchers who are interested in this area.

1. Introduction

L. Zadeh introduced the Z-number concept to the literature in 2011 [1]. Actually, he was working on the topics combining fuzzy and probabilistic information such as probability measures with fuzzy events [2], fuzzy random variables [3], fuzzy sets and information granularity [4] before put forward the Z-number theory. He claims that Z-numbers can represent the rational decision making ability of the humans under uncertain conditions. Thus, a Z-number contains an uncertainty degree in addition to fuzzy information. A Z-number notation can be shown as

$$Z = (A, B) \text{ or } Z = (X, A, B) \quad (1)$$

X is a set of random variables, A is the restriction part on X and B is the reliability of A. (X, A) is similar for fuzzy researchers, because it is exactly the same with Type-I Fuzzy Logic. And addition of B part makes it Z-number. There is also an extension on Z-number shown as Z^+-number. Whereas Z-number has reliability degree B on A, Z^+-number has probability distribution of reliability degree, B on A.

In recent times, the concept of Z-number is gaining

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much popularity among the researchers. Firstly, generating Z-number was an open issue. For solving this, ordered weighted averaging operators(OWA) based and logistic regression based studies were made [5,6]. Although there were studies about generating Z-number, most of them about Z-number were linguistic [7-9]. We know that the fuzzy systems are good for translating linguistic knowledge into mathematics [10]. After creating Z-number fuzzy if-then rules from the linguistic information, Z-number based calculations play a vital role. For doing these calculations, R. Aliev et al. showed up the formulas for basic algebraic operations such as addition, subtraction etc. [11]. All Z-numbers can be translated into linguistic expressions or vice versa. In this situation, each Z-number contains linguistic or mathematical restrictions. Combining these restrictions with probabilistic restrictions, the probability distribution of the Z-number can be obtained [12]. After having a command of fundamental Z-number terms and calculations, some linguistic based studies have been done. Using Z-number in control problem was one of these studies. In 2018, R. Abiyev et al. controlled an omnidirectional soccer robot with linguistic Z-number rule base[9]. In 2019, M. Shalabi et al. modelled and controlled automotive air-spring suspension system with Z-number based fuzzy system [13]. In 2020, M. Abdelwahab et al. worked on trajectory tracking of a mobile robot with Z-number [14]. As another branch of work, W. Jiang et al. proposed a novel method combining Z-number with Dempster - Shafer evidence theory and they made an application in sensor data fusion problem [15]. As a clustering/classifying problem, Z-number was used with fuzzy c-means and k-means clustering, respectively [16,17]. The effectiveness of the proposed methods was shown on well-known datasets such as iris dataset, wine dataset etc.

As the Z-number based studies are examined in the literature, we make calculations with Z-numbers according to the rule base and we get another Z-number in final. To use final Z-number, B. Kang et al. proposed a method for converting Z-number into classical fuzzy number [18]. Lastly, the researchers have said that converting Z-number into a crisp number may cause information loss. Therefore, using it as Z-number form is more preferable [19]. For doing this, we need Z-number based if-then rules and Z-number based inference engine. At the moment, there is not any study about Z-number inference without converting Z-number into crisp number. Instead of this, ranking Z-numbers are more popular. And there are studies on multi-criteria decision making problems by ranking Z-numbers [20,21]. The results of these applications are mostly consistent with the studies done by mathematical and classical fuzzy operations. But, the issue about this type of works is that there are so many different decision making problems. So, comparing the performances of the proposed ranking methods is impossible. To make this possible, a Z-number fuzzy set has been created. Thus, the researchers can try their proposed method on this set and compare the results with the other methods. As in other Z-number applications, there are two approaches for ranking of Z-numbers. First one is converting Z-number into classical fuzzy, then ranking obtained fuzzy number. The second one is done without converting Z-number into classical fuzzy. In 2014, D. Mohamad et al. converted Z-number into generalized fuzzy number(GFN) because of simpler calculations and they used the standard deviation of GFNs to order them [22]. In 2015, A. Bakar and A. Gegov called their work as multi-layer decision methodology. According to them, conversion process, from Z-number to fuzzy number, is realized in the first layer and in the second layer ranking process is realized. For ranking process, they used centroid point in addition to the spread, called CPS [23]. In 2017, S. Ezadi and T. Allahviranloo proposed a method to rank fuzzy numbers. The method is based on hyperbolic tangent function and convex combination. They turned Z-numbers into generalized normalized fuzzy numbers(GNFS) with B. Kang's formula, and then tried to rank converted fuzzy numbers with their proposed methods [24]. Later, S. Ezadi et al. proposed another method to rank fuzzy numbers by using the similarity between hyperbolic tangent and sigmoid function. By converting the Z-numbers into GNFS, they adapted their method into ranking of Z-numbers [25]. In 2017, Jiang W. et al. proposed a novel method to ranking GFNs. According to this method, a score function is produced based on the centroid of the membership function, spread and Minkowski degree of fuzziness. And the ranking process is realized with produced score value. For ranking Z-numbers, they made some assumptions. According to them, the constraint part of Z-number is more important than the reliability part and it must be the main part of a Z-number. Therefore, the weight of constraint part should be greater than the weight of the reliability. In addition, the information of Z-numbers should be retained without converting into fuzzy or crisp numbers. In the light of these assumptions, they obtained scores for both constraint and reliability part via their proposed method, and they combined these scores with a formula by considering the distance between scores and a reference point [26]. In 2020, R. Chutia proposed a method to rank GFNs according to the concept of value and ambiguity. They obtained values and ambiguities for both constraint and reliability part. After that, they combined the scores as in the method of Jiang W. et al. [27].

There are new methods which are relative entropy of Z-numbers [28] and a method for ranking discrete Z-numbers [29]; but, these methods were not tried on the benchmark set of fuzzy Z-numbers before. In this paper, we examine the performances of these methods on ordering Z-numbers. According to the results, we want to present the drawbacks and advantages of these methods.

2. Materials and Methods

2.1 Materials

The materials of this study are Z-numbers. Let Z_1 and Z_2 be two Z-numbers defined as

$$Z_1 = (A_1, B_1) \text{ and } Z_2 = (A_2, B_2) \quad (2)$$

$$A_s = \frac{\mu_{A_s}(u_1)}{u_1} + \frac{\mu_{A_s}(u_2)}{u_2} + \dots + \frac{\mu_{A_s}(u_m)}{u_m}$$
$$B_s = \frac{\mu_{B_s}(v_1)}{v_1} + \frac{\mu_{B_s}(v_2)}{v_2} + \dots + \frac{\mu_{B_s}(v_n)}{v_n} \quad (3)$$

In Equations (2) and (3), A_s and B_s are the membership functions of a Z-number where A defines fuzzy part of a variable which are $u_1, u_2, \dots u_m$ and B defines the reliability of A. Since we have two Z-numbers, s=1, 2. $\mu_{A_s}(u_i)$, $\mu_{B_s}(v_j) \in [0,1]$ are the membership degree of given variables whose indices are i=1,2,…,m and j=1,2,…n.

Instead of Equation (3), we will use more compact expression to show Z-numbers. Most of time, the fuzzy membership functions are triangular, trapezoidal, Gaussian etc. And the benchmark fuzzy sets in this work only consist of triangular and trapezoidal membership functions. For example, a triangular membership function of reliability, B, can be described as given in Equation (4).

$$B = (0.6, 0.8, 1.0; 1.0) \quad (4)$$

The first three component of B describes the critical values and the last component of B, describes the peak membership value of B as seen in the Figure 1.

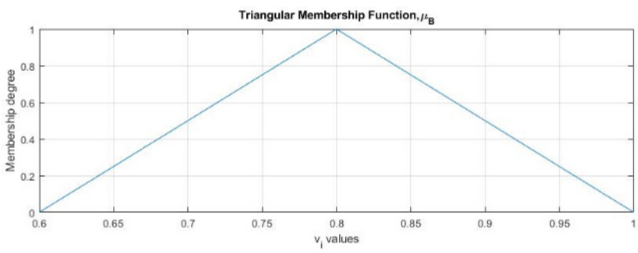

Figure 1. Triangular membership function given in Equation (4).

As in triangular membership function, sample trapezoidal membership functions can be written as in Equation (5).

$$B = (0.4, 0.8, 1.0; 1.0) \quad (5)$$

As in Equation (4), the first four components of Equation (5) give the critical values and the last component of B, corresponds to the peak membership value of B as seen in the Figure 2.

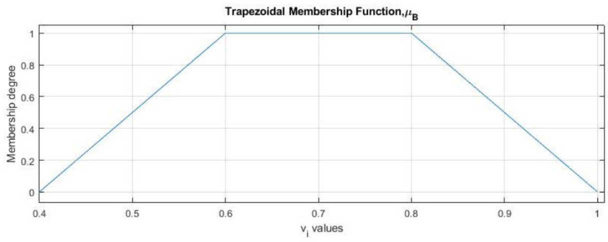

Figure 2. Trapezoidal membership function given in Equation (5).

2.2 Relative Entropy of Z-numbers

L. Yangsue et al. suggested to use relative entropy for ranking Z-numbers. Because the paper is based on the entropy, the underlying probability distributions of the Z-numbers must be found before attempting to find relative entropy. We do not know the underlying probability distributions; but, according to L. Zadeh, there are some restrictions about Z-numbers, called Z-restriction, such as

$$Prob(X \text{ is } A_s) \text{ is } B_s. \quad (6)$$

This information can be formulated as given in Equation (7).

$$B_s = \int \mu_{A_s}(u).\, p_s(u).\, du \quad (7)$$

The second restriction is given in Equation (8) if the centroids of μ_{A_s} and p_s are coincident.

$$\int u.\, p_s(u).\, du = \frac{\int u.\, \mu_{A_s}(u).\, du}{\int \mu_{A_s}(u).\, du} \quad (8)$$

And we know the probability restrictions as in Equation (9).

$$\sum_{i=1}^{m} p_s(u_i) = 1$$
$$0 \le p_s(u_i) \le 1 \quad (9)$$

There can be more than one solution that satisfies these restrictions. So, the solution that gives maximum Shannon entropy is chosen.

$$max{:}\, H(p_s(u_i)) = -\sum_{i=1}^{n} p_s(u_i) \log(p_s(u_i)) \quad (10.a)$$

Or in other words,

$$min{:}\, H(p_s(u_i)) = \sum_{i=1}^{n} p_s(u_i) \log(p_s(u_i)) \quad (10.b)$$

In continuous form, it is hard to solve this equation subject to given equalities and inequalities. To overcome this issue, some assumptions are made such as discretization or having Gaussian probability distributions. But, we do not have any information whether the probability distribution is Gaussian, uniform etc. So, by making the calculations discrete form, we hope that the obtained solution is close to continuous probability distribution with admissible error. Note that there will be n solutions for p_1, because there are n pieces of v value in Equation (3).

After getting the probability distributions p_1 and p_2, the relative entropy must be calculated as step two. For calculating this, the authors used Kullback - Leibler divergence. The divergence is defined in Equation (11.a) and (11.b).

$$D(p_1||p_2) = \sum_{i=1}^{n} p_1(u)\ log_2 \frac{p_1(u)}{p_2(u)} \tag{11.a}$$

$$D(p_2||p_1) = \sum_{i=1}^{n} p_2(u)\ log_2 \frac{p_2(u)}{p_1(u)} \tag{11.b}$$

In meaning, the divergence, $D(p_1||p_2)$, gives the information gain if p_1 is used instead of p_2. As third step, the authors proposed to create another fuzzy subset with obtained relative entropy values as follows.

$$C_1 = \frac{\mu_{B_1}(v_1) + \mu_{B_2}(v_1)}{D_1(p_1||p_2)} + \dots + \frac{\mu_{B_1}(v_n) + \mu_{B_2}(v_n)}{D_n(p_1||p_2)} \tag{12}$$

$$C_2 = \frac{\mu_{B_1}(v_1) + \mu_{B_2}(v_1)}{D_1(p_2||p_1)} + \dots + \frac{\mu_{B_1}(v_n) + \mu_{B_2}(v_n)}{D_n(p_2||p_1)} \tag{13}$$

We mentioned about there are n probability distributions for each p_s. And here, $D_i(p_1||p_2)$ is the relative entropy between i^{th} probability distributions of p_1 and p_2.

The centroid of these subsets will be the score of each Z-number. According to this score, ranking process will be realized. The Z-number with greater score will have a better rating in ranking.

2.3 A Method for Ranking Discrete Z-numbers

Gong Y. et al convert the Z-number into classical fuzzy set, and then try to rank different Z-numbers. Unlike the other works in the literature, they proposed a different method to convert Z-number into classical fuzzy instead of B. Kang's conversion formula. According to them, the reliability part, B, is the weight of the constraint part A. But, not just the values of B affect the weight, other criteria such as range (cardinality), linguistic order of the fuzzy set should be important to determine the weight. At the first step, measure of uncertainty, $E(B_s)$, is calculated as follows.

$$E(B_s) = \frac{|V_{B_s}| - 1 + \sum_{i=1}^{n} \min\left(B_s(v_i), B_s^C(v_i)\right)}{|V_{B_s}| - 1 + \sum_{i=1}^{n} \max\left(B_s(v_i), B_s^C(v_i)\right)} \tag{14}$$

Here $|V_{B_s}|$ is the number of elements (cardinality) that B_s have. B_s^C is the complementary set of B_s as in Equation (15).

$$R(B_s) = (1 - E(B_s))\frac{Order\ of(B_s)}{|L_P|} \tag{15}$$

According to the authors, there are linguistic fuzzy clusters both for reliability and constraint part. Here, $|L_P|$ is the number of these fuzzy clusters for reliability part and $Order\ of(B_s)$ is the order of (B_s) in L_P. To illustrate, assume a fuzzy set L_P.

$$L_P = \{never, rarely, sometimes, usually, always\} \tag{16}$$

Here, L_P =4 and $Order\ of(rarely)$=1.

We know that $E(B_s)$ is the measure of uncertainty, $(1 - E(B_s))$ gives the measure of certainty, analogously.

At the second step, the constraint part can be weighted with $R(B_s)$, and the new fuzzy set, A_s^*, occurs as a result of this process.

$$A_s^* = R(B_s).A \tag{17}$$

For ranking discrete fuzzy sets, H. Basirzadeh et al. proposed a method in 2012 [30]. Gong Y. et al. followed the same procedure to rank A_s^* in their work. According to this method, the regions where the membership function increases or decreases are checked. And an α-value is defined as follows.

$$\begin{aligned}&\alpha - \text{value} \triangleq \alpha \in [0,1] \text{ such that}\\ &\alpha \le \min\left(A_s^*(u_1), A_s^*(u_2), \dots, A_s^*(u_m)\right)\end{aligned} \tag{18}$$

For both of the regions, increasing and decreasing, a score is calculated. For increasing part, the score is, Q_{inc}^{α}. For decreasing part, the score is, Q_{dec}^{α}. They are given below equations.

$$Q_{inc}^{\alpha} = u_1.\left(\mu_{A_s^*}(u_1) - \alpha\right) + \sum_{i=2}^{m} u_i(\mu_{A_s^*}(u_i) - \mu_{A_s^*}(u_{i-1})) \tag{19}$$

$$Q_{dec}^{\alpha} = u_m.\left(\mu_{A_s^*}(u_m) - \alpha\right) + \sum_{i=1}^{m-1} u_i(\mu_{A_s^*}(u_i) - \mu_{A_s^*}(u_{i+1})) \tag{20}$$

And the total score of Z-number is the sum of Q_{inc}^{α} and Q_{dec}^{α}.

$$Q^{\alpha} = Q_{inc}^{\alpha} + Q_{dec}^{\alpha} \tag{21}$$

As in relative entropy based ranking method, the Z-number with greater score will have a better rating.

The main disadvantage of this method is that it needs $Order\ of(B_s)$ and number of fuzzy clusters, L_P. If we want to rank just two Z-numbers, the multiplier will be 0.5 and 1.0 from Equation (15). Even if the reliability parts of Z-numbers have small differences, the difference between multipliers will be enormous. Moreover, assume two Z-numbers that have B parts, B_1=(0.4,0.6,0.8) and B_2=(0.4,0.5,0.7,0.8). These are so similar and they may be called with same linguistic information "sometimes", although they are different membership functions. In this situation, how we will decide to order of these Z-numbers is another important issue. Thus, we proposed an extension to overcome this drawback. It is known that the range of membership functions is between [0,1]. Even if it is not in this range, it can be scaled by normalization.

As a first step, we will divide [0,1] into 20 parts with 0.05 steps. Thus, the $|L_P|$ will be equal to 20 whatever the Z-number is. To determine $Order\ of(B_s)$, the centroid method will be used, centroid of B_s can be found via Equation (22). To which region the centroid is closer, the number of that region corresponds to the $Order\ of(B_s)$.

$$Centroid\ of\ B_s = \frac{\sum_{i=1}^{n} v_i \cdot \mu_{B_s}(v_i)}{\sum_{i=1}^{n} \mu_{B_s}(v_i)} \quad (22)$$

$Order\ of(B_s)$ is used in Equation (15) to calculate the weight, $R(B_s)$. By arranging the $Order\ of(B_s)$ in the method proposed from Gong Y. et al., we have aimed to obtain more accurate weight term. With this improvement, we are expecting the minor differences that roots from the reliability membership functions can be distinguished and can be taken into account. Actually, the calculations continue to both get a new fuzzy set and produce a score from this new fuzzy set after determining $R(B_s)$. Therefore, the effect of this improvement may not be observed directly from the scores given in Equation (21). However, when Equation (15) is examined, both $Order\ of(B_s)$ and L_P are the multiplier of $(1 - E(B_s))$ which is a measure of certainty. Given the importance of using knowledge in uncertain conditions, any improvements in certainty will have a positive effect on the results.

Optimization of the number of fuzzy membership functions is an important topic in fuzzy applications. Increasing the number of membership functions may cause the system lose the capability of generalization and may require large computation time. At the same time, using few membership functions may cause incomplete modeling and inaccurate results [31]. At this stage, converting linguistic information to the fuzzy Z-number accurately with a sufficient number of membership functions can be another work topic. In this study, L_P was chosen as 20 and the results were given in Results section. Getting a better ranking performance will be possible by optimizing L_P.

3. Results

R. Chutia divided the benchmark set of fuzzy Z-numbers into three examples in his work. We will follow this tradition and we will give the results in this order. In example 1, there are 6 fuzzy sets that have same restriction part, A which is given in Equation (23). This means that the information, e.g. a sensor data, is same for each Z-number, but the reliabilities of information will differ with changing B parts which are given Equations (24), (25), and (26).

$$A = (0.1,0.3,0.5; 1.0) \quad (23)$$

In Set 1,

$$B_1 = (0.1,0.3,0.5; 1.0)$$
$$B_2 = (0.2,0.3,0.4; 1.0) \quad (24)$$

In Set 2,

$$B_1 = (0.1,0.2,0.4,0.5; 1)$$
$$B_2 = (0.1,0.3,0.5; 1.0) \quad (25)$$

In Set 3,

$$B_1 = (0.1,0.3,0.5; 0.8)$$
$$B_2 = (0.1,0.3,0.5; 1.0) \quad (26)$$

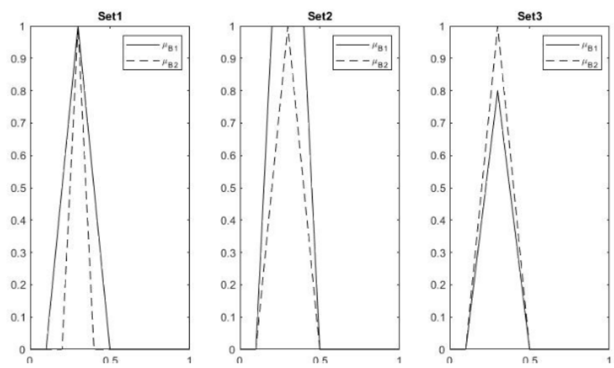

Figure 3. Reliability membership functions of first three sets of example 1

When first three sets of example 1 are examined, the reliability membership functions differ in shape, wideness and peak value. In Set 1, the peak values and shape of the membership functions are same. But, B_1 is wider than B_2. We can infer that B_1 is more fuzzy than B_2. So, we will be able to see the effect, when the fuzziness of the reliability changes. In Set 2, the membership functions differ in shape where the first one is triangular membership function and the latter is trapezoidal. Although their peak values, centroids and wideness are same, we will be able to examine the effect of the membership function's shape. Different from the first two sets, we can predict a result for this set intuitively that the membership function with higher peak value is more preferable than the other. From this point of view, Set 3 can be more distinctive while comparing the ranking methods.

In Set 4,

$$B_1 = (0.1,0.2,0.4,0.5;1)$$
$$B_2 = (0.1,0.3,0.5;0.8) \quad (27)$$

In Set 5,

$$B_1 = (0.1,0.2,0.4,0.5;1)$$
$$B_2 = (0.3,0.3,0.3;1) \quad (28)$$

In Set 6,

$$B_1 = (0.1,0.3,0.5;1)$$
$$B_2 = (0.3,0.3,0.3;1) \quad (29)$$

The above membership functions in Equations (27), (28), (29) which belongs the Set 4, Set 5 and Set 6 can be seen in the Figure 4. All these sets try to measure the effects of membership functions that differ in shape. Set 4 tries to measure the effect of peak values, additionally.

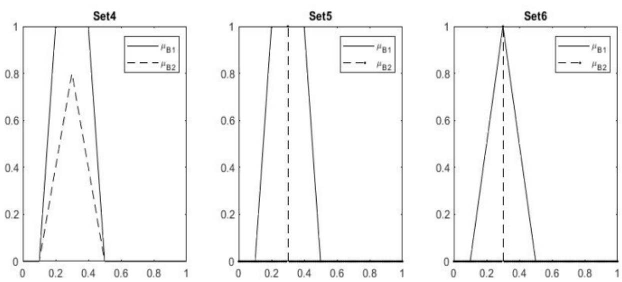

Figure 4. Reliability membership functions of last three sets of example 1

The results from the literature and the methods examined in this study for example 1 are given in Table 1. It is seen from the Table 1 that some of the proposed methods are failed to rank Z-numbers. Although the differences between membership functions in Set 1 are clear, the produced scores are exactly same. It can be said that the performances of the methods producing same scores for different membership functions are poor. These methods were proposed by Mohamad et al. [22], Bakar and Gegov [23], Ezadi and Allahviranloo [24], Ezadi et al. [25].

Jiang et al., R. Chutia, Yongsue et al. and Gong et al. succeed to rank these Z-numbers and their ranking results are same. Another important point about the study of R. Chutia et al., the score is inversely proportional to the rank. From decision making aspect, the score can be interpreted as cost criterion instead of benefit criterion.

Different from Table 1, R. Chutia orders the Z-numbers contrarily to Jiang et al. and Gong et al. in Table 2. Another difference from the previous results that Yangsue et al. do not differ the two Z-numbers. Their score for each Z-number is zero. Because the method of Yangsue et al. is based on the underlying probability distributions, they obtain same probability distributions when the range of reliability and the constraint membership functions are same. Thus, the relative entropy turns zero when the two probability distributions are same.

Table 1. Results for Set 1 of Example 1

Set 1			
Methods	$\boldsymbol{Z_1}$	$\boldsymbol{Z_2}$	**Result**
Bakar and Gegov	0.0508	0.0508	$Z_1 \sim Z_2$
Jiang et al.	0.1953	0.2024	$Z_1 < Z_2$
Mohamad et al.	0.0706	0.0706	$Z_1 \sim Z_2$
Ezadi et al.	0.5224	0.5224	$Z_1 \sim Z_2$
Ezadi and Allahviranloo			
$\alpha = 0.0$	0.3564	0.3564	$Z_1 \sim Z_2$
$\alpha = 0.0$	0.2996	0.2996	$Z_1 \sim Z_2$
$\alpha = 0.0$	0.0897	0.0897	$Z_1 \sim Z_2$
Ezadi et al.			
$\alpha = 0.0$	0.5921	0.5921	$Z_1 \sim Z_2$
$\alpha = 0.5$	0.5719	0.5719	$Z_1 \sim Z_2$
$\alpha = 1.0$	0.5224	0.5224	$Z_1 \sim Z_2$
R. Chutia			
$\alpha = 0.1$	0.0648	0.0557	$Z_1 < Z_2$
$\alpha = 0.5$	0.0333	0.0286	$Z_1 < Z_2$
$\alpha = 0.9$	0.0018	0.0016	$Z_1 < Z_2$
Yangsue et al.	0.0105	0.0110	$Z_1 < Z_2$
Gong et al.	0.2375	0.2672	$Z_1 < Z_2$

Table 2. Results for Set 2 of Example 1

Set 2			
Methods	$\boldsymbol{Z_1}$	$\boldsymbol{Z_2}$	**Result**
Bakar and Gegov	0.0508	0.0508	$Z_1 \sim Z_2$
Jiang et al.	0.2049	0.1953	$Z_1 > Z_2$
Mohamad et al.	0.0706	0.0706	$Z_1 \sim Z_2$
Ezadi et al.	0.5224	0.5224	$Z_1 \sim Z_2$
Ezadi and Allahviranloo			
$\alpha = 0.0$	0.3564	0.3564	$Z_1 \sim Z_2$
$\alpha = 0.0$	0.2821	0.2821	$Z_1 \sim Z_2$
$\alpha = 0.0$	0.0897	0.0897	$Z_1 \sim Z_2$

Table 2 continued

Set 2			
Methods	Z_1	Z_2	**Result**
Ezadi et al.			
$\alpha = 0.0$	0.5921	0.5921	$Z_1 \sim Z_2$
$\alpha = 0.5$	0.5719	0.5719	$Z_1 \sim Z_2$
$\alpha = 1.0$	0.5224	0.5224	$Z_1 \sim Z_2$
R. Chutia			
$\alpha = 0.1$	0.0906	0.0648	$Z_1 < Z_2$
$\alpha = 0.5$	0.0571	0.0333	$Z_1 < Z_2$
$\alpha = 0.9$	0.0101	0.0018	$Z_1 < Z_2$
Yangsue et al.	0.0000	0.0000	$Z_1 \sim Z_2$
Gong et al.	0.3117	0.2375	$Z_1 > Z_2$

Jiang et al., R. Chutia and Gong et al. ranked the Z-numbers successfully in Table 3. We say they ranked successfully, because the ranking, $Z_1 < Z_2$, can be done intuitively, too. When the Set 3 is examined, it can be seen that the membership functions of reliability are same in range and shape. They only differ in maximum membership value, so one can expect that the membership function with greater value takes greater order in ranking.

Table 3. Results for Set 3 of Example 1

Set 3			
Methods	Z_1	Z_2	**Result**
Bakar and Gegov	0.0508	0.0508	$Z_1 \sim Z_2$
Jiang et al.	0.1916	0.1953	$Z_1 < Z_2$
Mohamad et al.	0.0706	0.0706	$Z_1 \sim Z_2$
Ezadi et al.	0.5224	0.5224	$Z_1 \sim Z_2$
Ezadi and Allahviranloo			
$\alpha = 0.0$	0.3564	0.3564	$Z_1 \sim Z_2$
$\alpha = 0.0$	0.2821	0.2821	$Z_1 \sim Z_2$
$\alpha = 0.0$	0.0897	0.0897	$Z_1 \sim Z_2$
Ezadi et al.			
$\alpha = 0.0$	0.5921	0.5921	$Z_1 \sim Z_2$
$\alpha = 0.5$	0.5719	0.5719	$Z_1 \sim Z_2$
$\alpha = 1.0$	0.5224	0.5224	$Z_1 \sim Z_2$
R. Chutia			
$\alpha = 0.1$	0.2649	0.2970	$Z_1 < Z_2$
$\alpha = 0.5$	0.1945	0.2250	$Z_1 < Z_2$
$\alpha = 0.9$	0.1261	0.1530	$Z_1 < Z_2$
Yangsue et al.	0.0000	0.0000	$Z_1 \sim Z_2$
Gong et al.	0.2215	0.2375	$Z_1 < Z_2$

In Table 4, $Z_1 > Z_2$ according to Jiang et al., R. Chutia and Gong et al. Set 4 looks like Set 2 from many perspectives. Differently, the reliability membership value of in Set 4 is less than the Z_2 in Set 2. While the decisions of Jiang et al. and Gong et al. are staying same as expected, R. Chutia changes his order preference as $Z_1 > Z_2$ for Set 4 comparing to the decision in Set 2.

Table 4. Results for Set 4 of Example 1

Set 4			
Methods	Z_1	Z_2	**Result**
Bakar and Gegov	0.0508	0.0508	$Z_1 \sim Z_2$
Jiang et al.	0.2049	0.1916	$Z_1 > Z_2$
Mohamad et al.	0.0706	0.0706	$Z_1 \sim Z_2$
Ezadi et al.	0.5224	0.5224	$Z_1 \sim Z_2$
Ezadi and Allahviranloo			
$\alpha = 0.0$	0.3564	0.3564	$Z_1 \sim Z_2$
$\alpha = 0.0$	0.2821	0.2821	$Z_1 \sim Z_2$
$\alpha = 0.0$	0.0897	0.0897	$Z_1 \sim Z_2$
Ezadi et al.			
$\alpha = 0.0$	0.5921	0.5921	$Z_1 \sim Z_2$
$\alpha = 0.5$	0.5719	0.5719	$Z_1 \sim Z_2$
$\alpha = 1.0$	0.5224	0.5224	$Z_1 \sim Z_2$
R. Chutia			
$\alpha = 0.1$	0.2970	0.2649	$Z_1 > Z_2$
$\alpha = 0.5$	0.2250	0.1945	$Z_1 > Z_2$
$\alpha = 0.9$	0.1530	0.1261	$Z_1 > Z_2$
Yangsue et al.	0.0000	0.0000	$Z_1 \sim Z_2$
Gong et al.	0.3117	0.2215	$Z_1 > Z_2$

For Table 5 which gives the results for Set 5 of example 1, Jiang et al., R. Chutia, Yangsue et al., Gong et al. positioned above Z_1. Because the range of reliability part membership functions are different, the method of Yangsue et al. produced a meaningful output for this set.

Table 5. Results for Set 5 of Example 1

Set 5			
Methods	Z_1	Z_2	**Result**
Bakar and Gegov	0.0508	0.0508	$Z_1 \sim Z_2$
Jiang et al.	0.2049	0.2554	$Z_1 < Z_2$
Mohamad et al.	0.0706	0.0706	$Z_1 \sim Z_2$
Ezadi et al.	0.5224	0.5224	$Z_1 \sim Z_2$
Ezadi and Allahviranloo			
$\alpha = 0.0$	0.3564	0.3564	$Z_1 \sim Z_2$
$\alpha = 0.0$	0.2821	0.2821	$Z_1 \sim Z_2$
$\alpha = 0.0$	0.0897	0.0897	$Z_1 \sim Z_2$
Ezadi et al.			
$\alpha = 0.0$	0.5921	0.5921	$Z_1 \sim Z_2$
$\alpha = 0.5$	0.5719	0.5719	$Z_1 \sim Z_2$
$\alpha = 1.0$	0.5224	0.5224	$Z_1 \sim Z_2$
R. Chutia			
$\alpha = 0.1$	0.0906	0.0521	$Z_1 < Z_2$
$\alpha = 0.5$	0.0571	0.0268	$Z_1 < Z_2$
$\alpha = 0.9$	0.0101	0.0015	$Z_1 < Z_2$
Yangsue et al.	0.0735	0.0813	$Z_1 \sim Z_2$
Gong et al.	0.3117	0.7125	$Z_1 < Z_2$

Set 6 has the same membership function for as Set 5 from other respects except for the shape. So, Jiang et al., R. Chutia, Yangsue et al., Gong et al. give same answer, $Z_1 < Z_2$ in Table 6.

Table 6. Results for Set 6 of Example 1

Set 6			
Methods	Z_1	Z_2	**Result**
Bakar and Gegov	0.0508	0.0508	$Z_1 \sim Z_2$
Jiang et al.	0.1953	0.2554	$Z_1 < Z_2$
Mohamad et al.	0.0706	0.0706	$Z_1 \sim Z_2$
Ezadi et al.	0.5224	0.5224	$Z_1 \sim Z_2$
Ezadi and Allahviranloo			
$\alpha = 0.0$	0.3564	0.3564	$Z_1 \sim Z_2$
$\alpha = 0.0$	0.2821	0.2821	$Z_1 \sim Z_2$
$\alpha = 0.0$	0.0897	0.0897	$Z_1 \sim Z_2$
Ezadi et al.			
$\alpha = 0.0$	0.5921	0.5921	$Z_1 \sim Z_2$
$\alpha = 0.5$	0.5719	0.5719	$Z_1 \sim Z_2$
$\alpha = 1.0$	0.5224	0.5224	$Z_1 \sim Z_2$
R. Chutia			
$\alpha = 0.1$	0.0648	0.0521	$Z_1 < Z_2$
$\alpha = 0.5$	0.0333	0.0286	$Z_1 < Z_2$
$\alpha = 0.9$	0.0018	0.0015	$Z_1 < Z_2$
Yangsue et al.	0.0741	0.0823	$Z_1 < Z_2$
Gong et al.	0.2375	0.7125	$Z_1 < Z_2$

In example 2, there are 3 fuzzy sets that have same constraint part A. This means that the example 2 will try to rank Z-numbers according to the changing reliability parts as in example 1. The equations of the given fuzzy sets can be seen in Equations (30), (31), (32) and (33).

$$A = (0.1, 0.4, 0.6; 1) \quad (30)$$

In Set 1,

$$B_1 = (0.1, 0.4, 0.5; 1)$$
$$B_2 = (0.2, 0.3, 0.6; 1) \quad (31)$$

In Set 2,

$$B_1 = (0.1, 0.4, 0.7; 1)$$
$$B_2 = (0.2, 0.3, 0.5, 0.6; 1) \quad (32)$$

In Set 3,

$$B_1 = (0.2, 0.3, 0.4, 0.5; 1)$$
$$B_2 = (0.2, 0.3, 0.5, 0.6; 1) \quad (33)$$

The sets of example 2 consist of the membership functions that differ in critical values. The critical values term can be defined as the limit and the peak values of the piecewise continuous function. Considering that the right side is more reliable, a result can be predicted for Set 3, intuitively. But, estimating the result for Set 1 and Set 2 is looking hard. The results for example 2 can be seen in the following tables: Tables 7, 8 and 9.

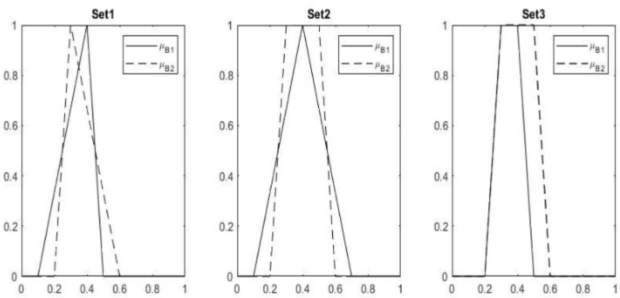

Figure 5. Reliability membership functions of example 2

Table 7. Results for Set 1 of Example 2

Set 1			
Methods	Z_1	Z_2	**Result**
Bakar and Gegov	0.0680	0.0736	$Z_1 < Z_2$
Jiang et al.	0.2303	0.2597	$Z_1 < Z_2$
Mohamad et al.	0.0987	0.1422	$Z_1 < Z_2$
Ezadi et al.	0.5332	0.5399	$Z_1 < Z_2$
Ezadi and Allahviranloo			
$\alpha = 0.0$	0.5062	0.5257	$Z_1 < Z_2$
$\alpha = 0.0$	0.4081	0.4300	$Z_1 < Z_2$
$\alpha = 0.0$	0.1325	0.1586	$Z_1 < Z_2$
Ezadi et al.			
$\alpha = 0.0$	0.6358	0.6420	$Z_1 < Z_2$
$\alpha = 0.5$	0.6066	0.6130	$Z_1 < Z_2$
$\alpha = 1.0$	0.5332	0.5399	$Z_1 < Z_2$
R. Chutia			
$\alpha = 0.1$	0.3854	0.3960	$Z_1 < Z_2$
$\alpha = 0.5$	0.2945	0.3000	$Z_1 < Z_2$
$\alpha = 0.9$	0.0756	0.0760	$Z_1 < Z_2$
Yangsue et al.	0.0602	0.0640	$Z_1 < Z_2$
Gong et al.	0.0888	0.1035	$Z_1 < Z_2$

All of the methods in the literature order $Z_1 < Z_2$.

Table 8. Results for Set 2 of Example 2

Set 2			
Methods	Z_1	Z_2	**Result**
Bakar and Gegov	0.0736	0.0736	$Z_1 \sim Z_2$
Jiang et al.	0.2420	0.2597	$Z_1 < Z_2$
Mohamad et al.	0.1422	0.1422	$Z_1 \sim Z_2$
Ezadi et al.	0.5399	0.5399	$Z_1 \sim Z_2$

Table 8 continued

Set 2			
Methods	Z_1	Z_2	**Result**
Ezadi and Allahviranloo $\alpha = 0.0$ $\alpha = 0.0$ $\alpha = 0.0$			
	0.5257	0.5257	$Z_1 \sim Z_2$
	0.4300	0.4300	$Z_1 \sim Z_2$
	0.1586	0.1586	$Z_1 \sim Z_2$
Ezadi et al. $\alpha = 0.0$ $\alpha = 0.5$ $\alpha = 1.0$			
	0.6420	0.6420	$Z_1 \sim Z_2$
	0.6130	0.6130	$Z_1 \sim Z_2$
	0.5399	0.5399	$Z_1 \sim Z_2$
R. Chutia $\alpha = 0.1$ $\alpha = 0.5$ $\alpha = 0.9$			
	0.0972	0.1094	$Z_1 > Z_2$
	0.0500	0.0658	$Z_1 > Z_2$
	0.0028	0.0104	$Z_1 > Z_2$
Yangsue et al.	0.0121	0.0128	$Z_1 < Z_2$
Gong et al.	0.1294	0.1553	$Z_1 < Z_2$

As in the other sets, the results of Jiang et al., Yangsue et al. and Gong et al. are consistent and $Z_1 < Z_2$. Unlike these results, the results of R. Chutia give $Z_1 > Z_2$ for all α-levels.

Table 9. Results for Set 3 of Example 2

Set 3			
Methods	Z_1	Z_2	**Result**
Bakar and Gegov	0. 0736	0. 0736	$Z_1 \sim Z_2$
Jiang et al.	0.2577	0.2597	$Z_1 < Z_2$
Mohamad et al.	0.1422	0.1422	$Z_1 \sim Z_2$
Ezadi et al.	0.5399	0.5399	$Z_1 \sim Z_2$
Ezadi and Allahviranloo $\alpha = 0.0$ $\alpha = 0.0$ $\alpha = 0.0$			
	0.5257	0.5257	$Z_1 \sim Z_2$
	0.4300	0.4300	$Z_1 \sim Z_2$
	0.1586	0.1586	$Z_1 \sim Z_2$
Ezadi et al. $\alpha = 0.0$ $\alpha = 0.5$ $\alpha = 1.0$			
	0.6420	0.6420	$Z_1 \sim Z_2$
	0.6130	0.6130	$Z_1 \sim Z_2$
	0.5399	0.5399	$Z_1 \sim Z_2$
R. Chutia $\alpha = 0.1$ $\alpha = 0.5$ $\alpha = 0.9$			
	0.0806	0.1094	$Z_1 > Z_2$
	0.0415	0.0658	$Z_1 > Z_2$
	0.0023	0.0104	$Z_1 > Z_2$
Yangsue et al.	0.0164	0.0165	$Z_1 < Z_2$
Gong et al.	0.1294	0.1553	$Z_1 < Z_2$

For Set 3, Jiang et al., Yangsue et al. and Gong et al. ranked the Z-numbers as $Z_1 < Z_2$. R. Chutia ranked as different from the other researchers. We were expecting the result, $Z_1 < Z_2$, intuitively. For the methods that fail to rank this set, we can interpret that as the fuzziness increases, the ranking performances decrease.

In example 3, there are 3 fuzzy sets again. The constraint parts of these fuzzy sets are same with example 1 and example 2. As in the other examples, we will try to measure the effect of the change in reliability on the ranking. The related z-numbers and their membership functions (Figure 6) are given below.

$$A = (0.1, 0.4, 0.6; 1.0) \quad (34)$$

The reliability part in Set 1,

$$B_1 = (0.1, 0.3, 0.5; 1.0)$$
$$B_2 = (0.3, 0.5, 0.7; 1.0) \quad (35)$$

In Set 2,

$$B_1 = (0.1, 0.2, 0.4, 0.5; 1)$$
$$B_2 = (1.0, 1.0, 1.0; 1.0) \quad (36)$$

In Set 3,

$$B_1 = (0.4, 0.5, 1.0; 1.0)$$
$$B_2 = (0.4, 0.7, 1.0; 1.0) \quad (37)$$
$$B_3 = (0.4, 0.9, 1.0; 1.0)$$

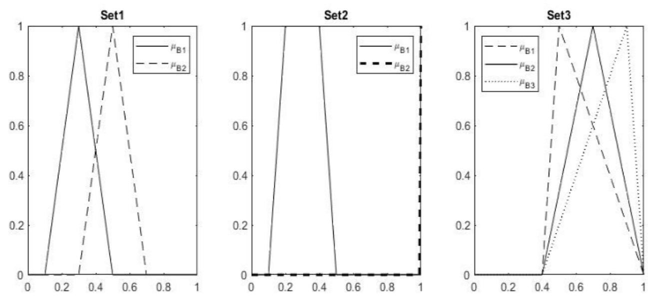

Figure 6. Reliability membership functions of example 3

When the right side is considered more reliable, the sets of example 3 can be interpreted, intuitively. And this is an advantage while comparing the performances of the methods. From the perspective of algorithms, varying fuzziness can be challenging in Set 2. And for Set 3, the place of the peak values is different, although the limit values of the membership functions are same. This situation can also be challenging for the methods. The results of Set 1 from example 3 are given in the Table 10.

Table 10. Results for Set 1 of Example 3

Set 1			
Methods	Z_1	Z_2	**Result**
Bakar and Gegov	0.0650	0.0809	$Z_1 < Z_2$
Jiang et al.	0.2220	0.2774	$Z_1 < Z_2$
Mohamad et al.	0.0715	0.1987	$Z_1 < Z_2$
Ezadi et al.	0.5299	0.5498	$Z_1 < Z_2$
Ezadi and Allahviranloo			
$\alpha = 0.0$	0.4962	0.5541	$Z_1 < Z_2$
$\alpha = 0.0$	0.3969	0.4621	$Z_1 < Z_2$
$\alpha = 0.0$	0.1194	0.1973	$Z_1 < Z_2$
Ezadi et al.			
$\alpha = 0.0$	0.6328	0.6511	$Z_1 < Z_2$
$\alpha = 0.5$	0.6034	0.6225	$Z_1 < Z_2$
$\alpha = 1.0$	0.5299	0.5498	$Z_1 < Z_2$
R. Chutia			
$\alpha = 0.1$	0.3653	0.4308	$Z_1 < Z_2$
$\alpha = 0.5$	0.2768	0.3264	$Z_1 < Z_2$
$\alpha = 0.9$	0.0701	0.0827	$Z_1 < Z_2$
Yangsue et al.	0.2379	0.2523	$Z_1 < Z_2$
Gong et al.	0.0757	0.1514	$Z_1 < Z_2$

All of the methods in the literature give $Z_1 < Z_2$. It is seen from the Set 1 that has a membership function closer to the right. The results are not surprising, when the right side is considered more reliable.

In Table 11, the result of Yangsue et al. is different contrary to other methods. It may root from that Set 2 is less fuzzy than the other sets. And this may be compelling to find possible underlying distributions that represent the real distributions. We know that the number of underlying probability distributions will decrease, as the number of v_i decreases.

Table 11. Results for Set 2 of Example 3

Set 2			
Methods	Z_1	Z_2	**Result**
Bakar and Gegov	0.0650	0.1067	$Z_1 < Z_2$
Jiang et al.	0.2309	0.5799	$Z_1 < Z_2$
Mohamad et al.	0.0506	0.5623	$Z_1 < Z_2$
Ezadi et al.	0.5299	0.5987	$Z_1 < Z_2$
Ezadi and Allahviranloo			
$\alpha = 0.0$	0.4962	0.6774	$Z_1 < Z_2$
$\alpha = 0.0$	0.3969	0.6043	$Z_1 < Z_2$
$\alpha = 0.0$	0.1194	0.3799	$Z_1 < Z_2$

Table 11 continued

Set 2			
Methods	Z_1	Z_2	**Result**
Ezadi et al.			
$\alpha = 0.0$	0.6328	0.6951	$Z_1 < Z_2$
$\alpha = 0.5$	0.6034	0.6682	$Z_1 < Z_2$
$\alpha = 1.0$	0.5299	0.5986	$Z_1 < Z_2$
R. Chutia			
$\alpha = 0.1$	0.3653	0.6346	$Z_1 < Z_2$
$\alpha = 0.5$	0.2768	0.4808	$Z_1 < Z_2$
$\alpha = 0.9$	0.0701	0.1218	$Z_1 < Z_2$
Yangsue et al.	2.2010	0.5341	$Z_1 > Z_2$
Gong et al.	0.1192	0.9083	$Z_1 < Z_2$

For Set 3, the method of Yangsue et al. does not produce meaningful output and we have already mentioned about its reasons in previous sets. Except this, the results that all the methods give the same result with $Z_1 < Z_2 < Z_3$ can be seen in Table 12. It was an expected result that the membership functions was closer to the reliable region from Z_1 to Z_3.

Table 12. Results for Set 3 of Example 3

Set 3				
Table 6	Z_1	Z_2	Z_3	**Result**
Bakar and Gegov	0.0892	0.0929	0. 0929	$Z_1 < Z_2 < Z_3$
Jiang et al.	0.3084	0.3295	0.3507	$Z_1 < Z_2 < Z_3$
Mohamad et al.	0.2695	0.3293	0.3774	$Z_1 < Z_2 < Z_3$
Ezadi et al.	0.5629	0.5695	0.5761	$Z_1 < Z_2 < Z_3$
Ezadi and Allahviranloo				
$\alpha = 0.0$	0.5899	0.6071	0.6236	$Z_1 < Z_2 < Z_3$
$\alpha = 0.0$	0.5030	0.5227	0.5418	$Z_1 < Z_2 < Z_3$
$\alpha = 0.0$	0.2481	0.2729	0.2974	$Z_1 < Z_2 < Z_3$
Ezadi et al.				
$\alpha = 0.0$	0.6632	0.6691	0.6750	$Z_1 < Z_2 < Z_3$
$\alpha = 0.5$	0.6349	0.6411	0.6472	$Z_1 < Z_2 < Z_3$
$\alpha = 1.0$	0.5630	0.5695	0.5761	$Z_1 < Z_2 < Z_3$
R. Chutia				
$\alpha = 0.1$	0.4552	0.5082	0.5639	$Z_1 < Z_2 < Z_3$
$\alpha = 0.5$	0.3388	0.3850	0.4339	$Z_1 < Z_2 < Z_3$
$\alpha = 0.9$	0.0834	0.0976	0.1127	$Z_1 < Z_2 < Z_3$
Yangsue et al.	0.0000	0.0000	0.0000	$Z_1 \sim Z_2 \sim Z_3$
Gong et al.	0. 1817	0.2023	0.2168	$Z_1 < Z_2 < Z_3$

4. Conclusions

There has been a lot of study about using Z-numbers in multi-criteria decision making problems since the day they were introduced. Z-numbers are important for this kind of problems, because the idea at the root of their emergence is that better decisions can be made by imitating the human decision making ability. However, after the linguistic information had been converted into Z-number, an important issue occurred about which Z-number was better. To obtain an answer to this question, several ranking methods have been proposed in time. The performances of some of these ranking methods were measured on the benchmark problem, some of them were not. In this paper, we examined the performance of two Z-number ranking methods whose performances are not examined yet. We tried to rank Z-numbers in the benchmark problem and presented the advantages and disadvantages of these methods. The first method was relative entropy of Z-numbers by L. Yangsue et al. Their method was entropy based and it was bounded to probability distributions of Z-numbers which have probabilistic and fuzzy restrictions. The main disadvantage of this method is that the underlying probability distributions are same for the Z-numbers which have same constraint membership function and reliability membership function with the same range. When the probability distributions are same, the relative entropy cannot differ given Z-numbers. The results of this method are consistent with the literature for example 1 and example 2. For the Set 2 from example 3, this method produced an output contrary to the other methods; this may root from the fuzziness of this set. When one examines this set, the membership function looks precise, so this may reduce the possible underlying probabilities and may cause incorrect ordering. As an advantage, the method makes ranking process without converting Z-numbers into fuzzy numbers. Considering converting Z-numbers leads to loss of information, the method can be beneficial for critical applications. The second method was for ranking of discrete Z-numbers. As the name implies, the method is for discrete Z-numbers. Extending the method for continuous Z-numbers may be considered for the future works. To obtain a better ranking performance, an improvement is made by setting the number of reliability functions as constant. The purpose of doing this was to avoid inaccurate ranking while ordering the similar Z-numbers. As the ranking results are examined, the scores are consistent with the other methods in the literature. Therefore, it can be mentioned as an improvement. However, studies may be done on optimizing this constant number for better results in the future. The outputs of this method are same with the results of Jiang et.al. In spite of their good performances, these methods have to convert Z-numbers. So, these methods should be used where a small amount of information loss can be tolerated.

Conflict of Interest

The authors declare no conflict of interest.

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