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Copula Method and Neural Networks for X-Band Polarimetric Radar Rainfall Retrieval in West Africa

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ABSTRACT

In the context of climate change, countries in West Africa are faced with recurrent flooding with catastrophic consequences, that makes it imperative to have access to rainfall information on fine spatial and temporal scales for better monitoring and prediction of these phenomena, as could be provided by weather radars. Based on an extensive archive of data from the X-band polarimetric radar and rain gauges observations gathered during the intensive AMMA campaigns in 2006–2007 and the Megha-Tropiques satellite measurement validation programme in 2010 in West Africa, we (i) simulated jointly realistic data for polarimetric radar variables and rain intensity using copula, and (ii) assessed rain rate estimation methods based on neural network (NN) inversion techniques and non-linearly calibrated parametric algorithms. The assessment of rainfall rate retrieval by these estimators is carried out using the part of the observations database not employed for calibration steps. The multiparametric algorithms $R(Z_H,K_{DP})$ and $R(Z_{DR},K_{DP})$ perform better than $R(Z_H,Z_{DR})$ and $R(Z_H,Z_{DR},K_{DP})$, especially since they are calibrated using copulas with upper tail dependencies, with KGE ranging in 0.68–0.75 and 0.79–0.82, respectively versus ranges of 0.40–0.64 and 0.20–0.51, for the two latter estimators. The neural network-based estimators $R_{NN}(Z_{DR},K_{DP})$ and $R_{NN}(Z_H,K_{DP})$, show KGE score characteristics comparable to those obtained from the best parametric relations, specifically optimized for the synthetic copula-based dataset. However, the neural network-based estimators were shown to be more robust when applied to a specific rainfall event. More specifically, neural network-based estimators trained on synthetic data are sensitive to the copulas' ability to capture the dependence

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between the variables of interest over the entire distribution of joint values. This leads to a near-cancellation of sensitivity to variability in the raindrop size distribution, as shown the coefficients of correlation near 1, especially for $R_{NN}(Z_{DR},K_{DP})$, and for less extent $R_{NN}(Z_H,K_{DP})$.

Keywords: Quantitative Precipitation Estimation; Copulas; Polarimetric Radar Data; Multiparametric Algorithms; Artificial Neural Network; Non-Linear Fitting

1. Introduction

In West Africa, climate change is leading to an increase in the occurrence of extreme events [1, 2] that causes recurrent flooding in the cities associated with catastrophically high casualty rates and material damage each year [3]. As a result, government investments and the households of the local population are wiped out, as road and drainage infrastructure and housing are destroyed, rendering many cities more vulnerable to those phenomena. The impact of floods and extreme rainfall events could be reduced with better monitoring and prediction of these phenomena [3] through access to rainfall information on a fine spatial and temporal scale.

In this respect, real weather radar measurements are an important contribution to complement the in-situ rain gauge sparse networks which are inadequate for meteorological and hydrometeorological applications at the catchment scale, such as warnings for intense rainfall and operational forecasting. Because quantitative precipitation estimates (QPE) from polarimetric capabilities have become a mandatory standard for weather radars [4, 5], several West African countries, including Senegal and Côte d'Ivoire, have taken the initiative to equip themselves with polarimetric radar for rainfall monitoring and hydrometeorological applications. As a prelude to the installation and exploitation of these radar measurements, we propose to use the large archive of X-band polarimetric radar data gathered from specific measurement campaigns by programs such as the African Multidisciplinary Monsoon Analysis (AMMA) [6, 7] and Megha-Tropiques [8, 9], to derive new data-driven radar-based QPE algorithms providing reliable precipitation estimates. Compared with C- and S-band radars, X-band radar has been preferred because of its compactness, smaller size (small antenna) and portability over these several experimental fields, and its low power consumption. However, X-band radar, with its higher transmission frequencies, experiences significant signal attenuation due to rain. To solve this problem and make this type of radar attractive, various methods based on polarimetric radar observables that are insensitive to attenuation effects, such as differential phase shift (ϕ_{DP}) or specific differential phase shift (K_{DP}), have been used. Attenuation correction at X-band is done using two approaches involving self-consistent procedures (i.e., ZPHI methods) [10–12] and the more direct methods based on a nearly linear fit between attenuation and the differential phase shift (PIA-ϕ_{DP}) [13–15] that make it possible to efficiently determine the specific horizontal A_H and differential A_{DP} attenuations affecting the horizontal reflectivity Z_H and differential reflectivity Z_{DR}, respectively.

Based on the benefits of this type of radar, in the West African tropical environment characterized by heavy convective rainfall, an X-band radar (X-port) has been used during various campaigns such as the African Multidisciplinary Monsoon Analysis (AMMA) [6, 7], the Megha-Tropiques mission [8, 9], as mentioned above. Several theoretical [16] and experimental studies [8, 9] have demonstrated its ability to estimate rainfall in this rainy tropical environment. The challenge now is to design more efficient algorithms for the quantitative estimation of rainfall, particularly for intense rainfall that is often not included, or only partially included, in the samples used to determine these estimators [17]. To go beyond traditional algorithms using the $Z - R$ relationship for the quantitative estimation of rainfall by radar, Ryzhkov et al. [18] and Bringi et al. [19] have argued that the differential reflectivity Z_{DR} and the specific differential phase shift K_{DP} provide additional information on the shape and distribution of drops, so an algorithm based on the horizontal reflectivity Z_H and the differential reflectivity Z_{DR}, $R(Z_H,Z_{DR})$ or $R(K_{DP}, Z_{DR})$ would be a better choice, provided of course that these radar observables are corrected for calibration and attenuation issues. Specifically, Zhang et al. [20] noted that because Z_{DR} and K_{DP} can be used to retrieve the raindrop size distribution (DSD), their utilization enables understanding of the physical processing in precipitation and so improvements in QPE. Furthermore, due to its independence with regard to the calibration and attenuation problem, the $R(K_{DP})$ algorithm appears more suitable to produce accurate rainfall maps if a robust relationship can be established according to the rainfall climatology of the study area [4] and especially for heavy rainfall [7] since K_{DP} is affected by the noise signals that characterize low-intensity rainfall. It is easy to imagine, if we disregard the problems associated with measurement, that the combined use of polarimetric observables could be an advantage likely to improve the performance of multiparametric algorithms compared with single-parameter ones [21]. However, the question of designing high-performance algorithms is still open for investigation, especially for efficient estimation of all types of rainfall including intense rainfall. Due to the spatiotemporal variability in DSDs [5, 20], it appears difficult to present the parametric functional relation in a simple form. Analyzing the observed DSD from typhoons and squall lines in Southern China, Zhang et al. [20] indicated the importance of fitting the rainfall estimator according these precipitation types to enhance estimation. In this way, they also proposed a piecewise fitting method using $R(Z_H,Z_{DR})$, $R(K_{DP}, Z_{DR})$ and $R(K_{DP})$ estimators according to the rainfall rate ($R< 6$ mm/h, 6 mm/h $< R <$ 50 mm/h and $R > 50$ mm/h classes) by dividing DSD data into three parts as corresponding to rainfall rate classes.

Zahiri et al. [17] conducted a study that revealed the lack of reliability in algorithms used for modelling the relationship between variables and rainfall rate. They found that the commonly used Gaussian framework is not effective in capturing the non-linear relationships between these variables, especially for extreme rainfall events. Similarly, a previous study by Tokay et al. [22] noted a significant discrepancy between radar variables and rainfall rate in the tails of the distribution, indicating a weak linear correlation in the upper tail for higher rainfall rate values. Furthermore, the traditional parametric algorithms heavily rely on the range of rainfall rates used to determine the coefficients [6, 7]. Since precipitation events often exhibit a scarcity of heavy precipitation events, this can result in the model's tendency to exhibit limited capability in estimating these types of precipitation [23]. Consequently, the resulting parametric estimators lead to poor accuracy for heavy precipitation events. Overall, the lack of robustness in these parametric algorithms is attributed to the difficulty in modelling variable dependence and non-linear relationships due to their sensitivity to variability in the raindrop size distribution, as well as the inadequate representation of intense rainfall rates in the calibration samples.

In recent years, there has been growing interest in the use of artificial neural networks, as an alternative to parametric methods for estimating rainfall rates [23–26]. This is part of the rapid development of deep learning techniques, that have been successfully implemented in many applications [27, 28]. Non-parametric approaches, such as artificial neural networks, attempt to directly capture the relationship between the radar parameters (as input data) and the rain gauge measurements (target variable) by exploring the complex functional relation based on the training method that reduce the need for any physical assumption. The performance of these deep learning methods has boosted the use of single-polarization radar using radar reflectivity to estimate rainfall. However, since conventional relationships between reflectivity and rainfall rate are not sufficient to capture the complex space - time variability in precipitation microphysics that impacts the QPE based on single-polarization radar, some authors have developed more sophisticated non-parametric methods based on machine learning techniques. Thus, two independent variables, that are radar reflectivity and radar echo-top height observed data are related to the rainfall rate on the ground from a Gate Recurrent Unit (GRU) neural network [24]. The assessment of this algorithm based on 200 rainfall events indicated that it performs better than optimal Z-R relationship and the GRU neural network with only reflectivity data. Zhang et al. [26] proposed an offline spatiotemporal deep fusion model that uses the reflectivity data, the precipitation data from national and automatic weather stations and capturing the time dependence of the precipitation from the long short-term memory network, whereas the spatial features of radar data from multi-elevation and multiscale are extracted and merge using the feature fusion network. Chen et al. [27] constructed a hybrid deep neural network that links point-wise rain gauge measurements, ground-based, and spaceborne radar reflectivity data. In this case, ground radar was used to overcome scale discrepancies between the space-based precipitation radar (PR) and rain gauges and for subsequent training of the second multi-layer perceptron (MLP) model matching PR and ground radar observations. Although these approaches produced better rainfall estimates compared to the standard

PR and conventional QPE algorithms, substantial or dense gauge networks and ground-based radar data are necessary; that is not available in West African countries where the state of the in situ hydrometeorological networks is inadequate for this kind of approach [3] and radar measurements are scarce and limited to specific experiments of AMMA and Megha-Tropiques. Furthermore, the performance of radar-derived QPE strongly depends on the physical model of the raindrop size distribution (DSD) and the relation between the physical model and radar observables [27] so that only the reflectivity parameter is not able to catch. Thus, Huangfu et al. [29] proposed a deep learning method based on two deep learning-based QPE networks including a single-parameter network and a multi-parameter network. They integrated a self-defined loss function during the training of the networks and distinguish specific precipitation types (heavy and light rain) setting K_{DP} to 0.5 °/km as a threshold value, to subsequently design 12 deep learning-based QPE models. This could be considered as a way to mitigate the effects DSD variability according to rainfall regimes. Better estimates were found using estimators that involve a specific type of precipitation and the self-defined loss function during modeling, compared with models that do not distinguish precipitation intensity and use the root mean square error as the loss function. Moreover, the deep learning-based models are found to outperform traditional empirical methods considering Z-R relationship and Z_H-K_{DP}-R method.

Vulpiani et al. [30] conducted a prior study using neural networks (NN) to estimate automatically DSD parameters based on polarimetric radar measurements. They employed simulations of polarimetric radar variables through the T-matrix scattering model [31] and subsequently computed the corresponding more reliable rainfall rates by deriving them from the NN-based technique estimated DSD parameters. Vulpiani et al. [32] assessed this NN-based technique alongside experimental weather radar dataset jointly comparing the indirect and direct method used for estimating rainfall from two or three polarimetric observables as input of the neural network model. Because the NN approach requires a large amount of data including a wide range of rainfall types and concurrent radar variables for the training process, this crucial step was achieved using radar polarimetric variables computed through the T-matrix method assuming assumptions regarding the raindrop microphysical parametrization (axis ratio of drops, temperature range of the medium, raindrop size distribution, and the canting angle). However, a framework of simulated data for the learning process of the neural network technique would deviate from that trained by observational data rarely characterized by various types of precipitation, with the consequence of significantly affecting experimental validation. This is particularly true since, in the training process, the dataset from the calibration sample is repeatedly used with different configurations until the network reaches a stable state where the synaptic weights no longer change [32]. To minimize this impact, simulations should be as realistic as possible. Recently, Zhang et al. [25] studied polarimetric radar QPE focusing on landfalling typhoon events in Southern China and using deep learning methods. They designed a convolutional neural network (CNN) characterized by three hybrid dataset of volume scan data of Z_H, Z_{DR} and K_{DP} considering 13 × 13, 25 × 25, and 41 × 41 radar range bins surrounding each rain gauge location, to better involve the spatial characteristics of precipitation from radar measurements and thus map the link between multidimensional radar observations and ground rainfall. Dual-polarization radar QPE based on deep learning leads to better performance than traditional parametric DSD-based nonlinear fitting algorithms, for moderate and heavy rainfall, whereas for light rainfall (R < 5 mm/h) performances are comparable. Comparison of the three versions of the deep learning shows the model trained with radar dataset binned 25 × 25 as having the best global performance. But, for training their deep-learning models, 18 national and 1041 regional automatic weather stations were used as target labels. To enhance the mapping capability from radar observations to precipitation, Li et al. [23] proposed a 3-D star neural network (StarNet) for polarimetric radar QPEs including two main aspects of improvement: (i) the above-mentioned spatiotemporal feature extraction of successive radar volume scanning data derived from recurrent neural networks (RNN) to better handle the dynamic characteristics of precipitation regimes, (ii) a reweighted loss function was designed to efficiently attenuate the problem of unbalanced distribution between heavy, moderate and light precipitation. Based on the mean absolute error (MAE) and considering all samples, their model was 34% lower than the best conventional QPE method R(K_{DP}). Recently, the same authors developed a more extensive assessment of the deep learning model's precipitation estimation performance by focusing on diverse rainfall amounts [33]. To do this, they proposed an improved deep learning method using a second module in the model that introduces an additional explanation method for quantitative precipitation estimation to assess the influence of each radar observable on model estimation for various rainfall intensities. All the methods mentioned agree on the fact that the use of neural networks is still faced with a number of challenges such as the availability, representativeness, and sufficiency of the training data set and learning the relationship between radar observations and surface precipitation.

Zahiri et al. [17] proposed the use of copulas for two purposes: (i) to determine the relationship between the K_{DP} specific differential phase shift and rain intensity, and (ii) to extend the algorithm calibration samples to all ranges of rainfall, particularly heavy rainfall. They achieve this through simulations based on copulas and the establishment of $R(K_{DP})$ algorithms using the quantile method applied to synthetic data. Their results show that the low quantiles of the synthetic data derived from the simulations of the Student, Gumbel, and HRT copulas provided better algorithms for estimating heavy rainfall (R sup 30 mm/h) compared to conventional methods. The normal copula required considering the 0.8 quantile to achieve comparable results. The copula approach was valuable as the algorithms based on the realistic synthetic data outperformed those based on observed data, regardless of the fitting methods used. But the highest scores came from the quantile method, which offers the possibility of sampling the entire distribution, unlike the traditional use of least mean squares method, which targets the mean distribution close to the 0.5 quantile. Although the authors' work was limited to a single-parameter algorithm, it opens a way for examining the performance of other multiparameter algorithms such as $R(Z_H,K_{DP})$, $R(Z_{DR},K_{DP})$, $R(Z_H,Z_{DR})$, and $R(Z_H,K_{DP},Z_{DR})$, using a similar approach.

Furthermore, copulas are increasingly being used in hydrometeorology for numerous applications such as multivariate frequency analysis, geostatistical interpolation, drought analysis, modelling extreme precipitations and risk assessment [34–42]. In short, this interest in copulas is motivated by its ability to find the whole bivariate or multivariate distribution, understand the relationship between variables and how they interact with each other [43] by estimating their marginal distributions and the copula functions separately. However, one challenge is choosing a suitable copula for the problem at hand. Zahiri et al. [17] propose a method that considers and tests different families of copulas, such as elliptical, Archimedean, and survival copulas, to address the issue of selecting the most appropriate copula for their given problem. They specifically focus on copulas that can accurately reproduce null-tail and non-null-tail distributions, as well as extreme values, in order to simulate a wide range of rainfall characteristics realistically.

In this present study, we extend the copula method from Zahiri et al. [17] drawing a framework to simulate sufficient representativeness, and realistic rain rate and polarimetric radar observables dataset. The goal of this work is to assess polarimetric multivariate algorithms for rainfall estimation based on neural networks (NN) that are trained with the large synthetic dataset involving the link between rain rate and the radar polarimetric parameters designed from copula method. For comparison, multiparametric power laws determined in an optimized manner using a non-linear approach are also assessed. To achieve these ends, the simulations of radar observables and rainfall rates are based on observation data gathered during the AMMA intensive campaigns in northern Benin (2006–2007) and Megha-Tropiques (2010) in Niger, respectively. For comparison, the experimental training process for NN-based technique and non-linear optimization of parametric algorithms are also based on the above-mentioned observed dataset. Neural network-based and parametric polarimetric radar rainfall estimations are validated using observational data from the validation sample.

The remainder of this paper is organized as follows: a description of the basic rainfall and radar data, the methodology for simulating synthetic data using copulas and the various methods for estimating rainfall using polarimetric radar is provided in Section 2. The Results and Discussion of the validation of the realistic nature of the simulated variables by selected copulas, and the rainfall estimates by the various multiparametric and neural network-based algorithms, are the subject of Section 3. The relevant conclusions are provided in the final Section 4.

2. Materials and Methods

2.1. Description of Basic Radar and Rainfall Data

In the West African region (**Figure 1**, top panel), most of the rainfall-producing weather systems are mesoscale convective systems (MCS) moving typically westward and are often strengthened by deep convection. According to Atiah et al.[44], these MCS contribute approximately 80%–90% of the Sahel's and about half of the Sudan Savannah's annual rainfall. The most recent experiments investigating these rain-fall systems date back to the intensive measurement campaigns of the AMMA program in northern Benin (2006–2007) and the Megha-Tropiques mission in Niger (2010 experiment) that included X-band radar experiments. In Niger, the observation site is located in the square degree of Niamey, in the Sahel region. The rainfall in this area varies between 450 and 600 mm per year, with mesoscale convective systems being the primary carriers of rain. In northern Benin, the data was collected from the Observatoire Hydrométéorologique de la Haute Vallée de L'Ouémé (OHHVO), which covers the Donga basin (**Figure 1**, bottom panel). This region has a Soudanian climate[2], with rainfall occurring from March to October and peaking in July and August. The average annual rainfall in this area is approximately 1,500 mm. Radar measurements data used in this study are obtained from the X-band polarimetric radar (X-port) which operated at Djougou during the 2006–2007 experiments and at Niamey in 2010. The main observables measured at a time step of 5 minutes are the horizontal reflectivity (Z_H) and differential reflectivity (Z_{DR}), as well as the differential phase shift (ϕ_{DP}). The specific differential phase shift (K_{DP}) was calculated from ϕ_{DP} and used in combination with the other variables to determine radar rainfall estimation algorithms. The data underwent attenuation correction and ground echo removal using methods previously established by Koffi et al.[7]. The attenuation correction involved using a linear relationship between the total path horizontal attenuation (PIA_H) and differential attenuation (PIA_{DR}) and the total differential phase shift ϕ_{DP}, which was not affected by attenuation or calibration issues. Additionally, coherence analysis was performed to detect and correct radar calibration issues[10, 45]. The radar variables were compared to each other and checked against theoretical reference curves to ensure their consistency. The latter are obtained from numerical simulations of polarimetric variable pairs of interest based on real DSDs as carried out by Koffi et al.[7] and Zhang et al.[46]. To take advantage of the fact that K_{DP} is calibration-independent, the relations used are mainly K_{DP}-Z_H and Z_H-Z_{DR} for reference.

For the ground-based rainfall taken as a reference or 'ground truth', we considered rainfall data for North Benin from a network of 54 rain gauges installed as part of AMMA-CATCH and stretching over the Donga basin covered by the X-port radar. The rainfall data for Niger comes from a network of 54 rain gauges installed as part of the EPSAT-NIGER experiment and spread over the square degree of Niamey. The data used are sampled at a time step of 5 minutes using the method described by Russell et al.[47].

All these devices provided us with a fairly large dataset, consisting of radar data, namely the observables Z_H, Z_{DR} and K_{DP}, which are respectively horizontal reflectivity, differential reflectivity and specific differential phase shift, and rainfall data providing the rain rate. All the data collected is used and divided into two parts, one of which is used to calibrate the copulas and rainfall estimators (two-thirds of the dataset) and the second used to assess quantitative rainfall estimation methods (one-third of the dataset). For the validation study, we compare 5-minutes gauge and radar rainfall rates over gauge locations from both experimental areas.

2.2. Simulated Synthetic Rainfall and Radar Data Using Copulas

As previously stated, the methodology outlined in this study is based on the generation of realistic synthetic data reflecting diverse rainfall rates and associated radar variables, including those that are sparsely observed. These datasets will be used to determine the polarimetric rainfall algorithms at a later stage. In order to achieve this objective, the present section provides a description of the principles of copulas and the resulting simulation methods. For this purpose, we may refer to the detailed description of bivariate copulas provided by Zahiri et al.[17] and De Luca and Rivieccio[43].

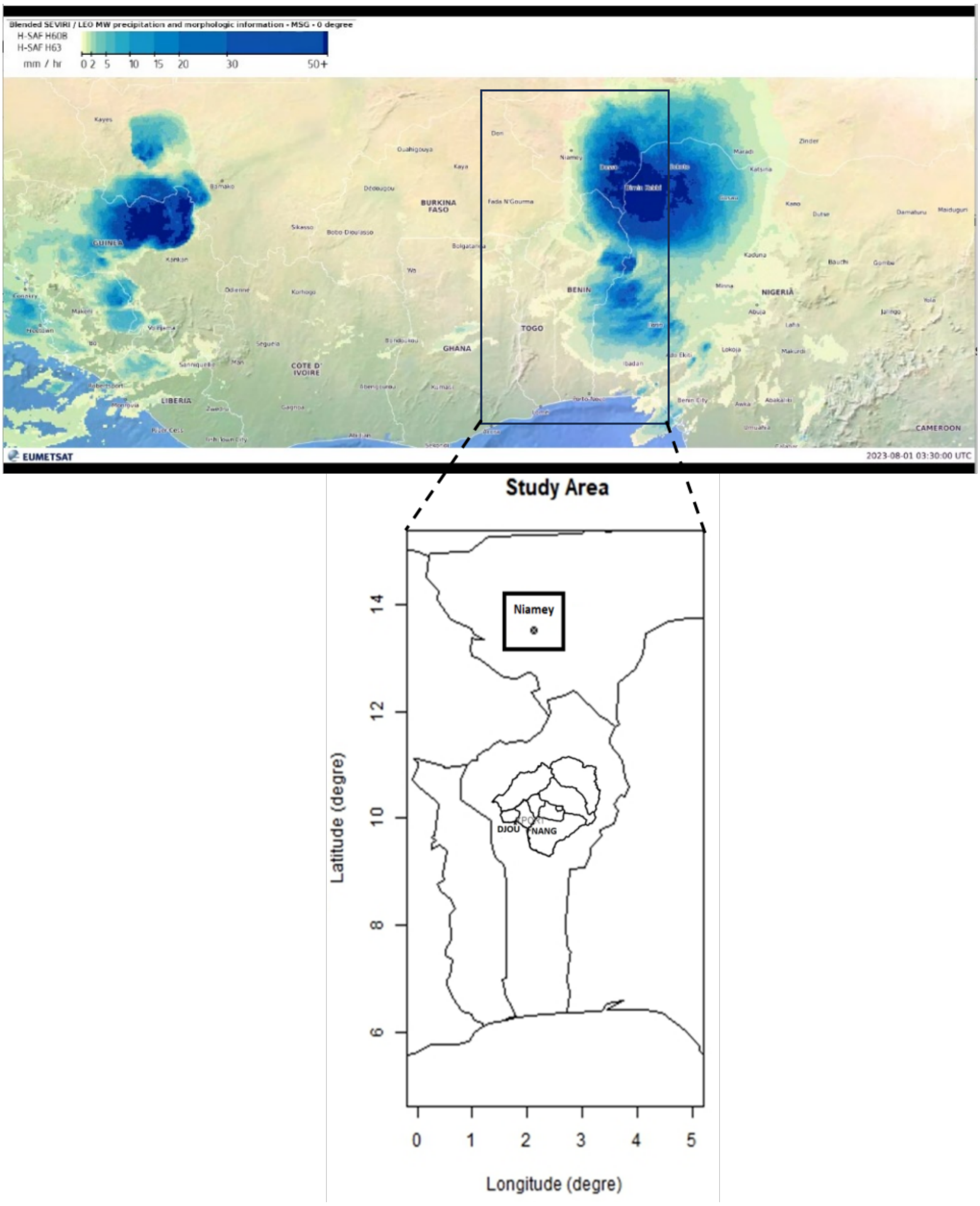


Figure 1. Global map showing the West Africa region swept by mesoscale convective systems as show MSG image (top panel) and geographical location of measurement sites and position of Xport at Djougou (OHHVO), in northern Benin during the AMMA intensive campaigns (2006–2007), Niamey (Squared degree) during the Megha-Tropiques mission (2010) (bottom panel). In this bottom panel, NANG is the position of the rain gauge whose measurements are used for the times series rainfall shown in this study.

A copula is a multivariate distribution function that links the probability distribution functions (PDFs) of two or more random variables, in particular their one-dimensional marginal distributions [48]. The basic theory of copulas was first introduced by Sklar in 1959 in order to solve a probability problem identified by Schweizer and Sklar (1958) in their research on random metric spaces [49]. Subsequently, many studies have focused on this statistical method [50–52]. The copula is used to develop non-Gaussian models and is defined as a powerful statistical tool for extracting the dependence structure of a joint distribution and for separating the notion of dependence from marginal behavior. Technically, copula is defined as joint distribution of unit uniform variates. Let us consider, for example, p-uniform random variables (which we will assume to be non-independent) $U_1,...,U_p$ in the set $\cup(0,1)$. The relationship between these variates is described through their joint distribution function such as [34, 43]:

$$C(u_1, \ldots, u_p) = \Pr(U_1 \leq u_1, \ldots, U_p \leq u_p) \quad (1)$$

In this expression, we call the function C a copula. To complete the construction, we arbitrarily select marginal distribution functions $F_1(x_1),..., F_p(x_p)$. The copula is therefore a multivariate distribution function, estimated for the variables $x_1,..., x_p$:

$$C(F_1(x_1), \ldots, F_p(x_p)) = F(x_1, \ldots, x_p) \quad (2)$$

The difficulty of finding an appropriate copula for the problem at hand has led us to test different families of theoretical copulas, as Zahiri et al. [17] have done:

- Elliptic copulas (Gauss and Student copulas) because they are symmetrical and have the advantage of being easy to simulate. In addition, the Student copula, depending on the value of the degree of freedom, can admit tail dependencies;
- Archimedean copulas (Frank, Clayton and Gumbel copulas) constructed from generating functions [17, 53] and having the ability to better simulate tail dependencies. In particular, Frank's copula is symmetrical in the lower and upper tails, and therefore tends to correlate with both small and large events. Thus, it is stronger with regard to the mode. The Clayton copula is suitable for studying the dependency between low-intensity events and is therefore useful in the lower tail, while the Gumbel copula is a good representation of events with a more accentuated dependency structure in the upper tail.
- A copula of extreme HRT (Heavy Right Tail) values, better known as Clayton's survival copula [53].

This study focuses on using selected copulas to determine the relationship between three and four variables. The use of the copula allows the researcher to exert a considerable degree of control over the specific aspects of the distributions that are more strongly associated with the variables in accordance with the chosen copula. One key focus of this study is the issue of controlling the strength of the relationship in the upper tails of the distributions which are rarely targeted.

The process of generating synthetic data for these variables is explained in four steps. Firstly, the optimum theoretical marginal distributions are determined for the variables by testing around twenty different distribution functions and fitting them using the maximum likelihood method. The reliability of these theoretical functions is then assessed using the Akaike criterion, as suggested by Frees and Valdez [54]. Secondly, the copula parameters are estimated using the Marginal Inference Method (MIM), and their reliability is assessed by comparing them with the parameters of the empirical copulas derived from observed data using the Canonical Likelihood Method (CLM). Next, the p-uniform random variates U_p associated with the variables of interest are simulated using the determined copulas. Finally, the simulated uniform random variables are transformed into values of the variables of interest (X_p) using the inverses of the calibrated marginal distribution functions. The simulation methods used in this study are similar to those used by Zahiri et al. [17] and are applicable to multivariate copulas with three or four related variables. The methods involve using the method of distributions to simulate variables based on the Gauss and Student copulas. The conditional distribution method is used to generate variables using Frank's, Clayton's, and HRT copulas, which are used to link the rainfall variable to two radar variables. When simulating more than three variables, the Marshall and Olkin [55] method is used in combination with Frank and Clayton copulas to produce synthetic data. Regardless of the number of variables, the same method is used for simulating data based on the Gumbel copula. To create a substantial database of synthetic data with a sufficient number of strong values, 10,000 realizations of the copula's joint laws are simulated.

2.3. Polarimetric Rainfall Rate Retrieval Algorithms

The present study is concerned with the use of multiparametric rainfall rate retrieval algorithms, which are mathematical equations employed for the estimation of rainfall based on radar data. The general form of these algorithms is given by the equation $R = aZ_H^b Z_{DR}^c K_{DP}^d$ [6, 56]. The study explores different variations of the algorithm based on the cancellation of exponents for different variables. For example, the R(Z_H,Z_{DR}) algorithm proposed by many authors[5, 20, 32, 57] uses two radar polarimetric variables assuming a relationship between differential reflectivity and the size parameter of the raindrop size distribution (DSD), i.e., the median volume diameter D_0. Many other researchers[5, 57] have suggested using the combination of K_{DP} and Z_{DR} to compensate for DSD variability[6, 58]. Huangfu et al.[29] used R(Z_H,K_{DP}) as traditional empirical formula method to compare with deep learning-based QPE. Additionally, a three-parameter algorithm has been proposed, which takes into account DSD variability in terms of size and concentration[15, 56, 57]. All these authors argued that the use of two or three radar observables has been shown to reduce errors in rainfall estimation caused by DSD and drop shape variability.

Thus, the parametric rainfall rate algorithms R(Z_H,K_{DP}), R(Z_{DR},K_{DP}), R(Z_H,Z_{DR}) and R(Z_H,Z_{DR},K_{DP}) were selected. The coefficients for these algorithms were determined through non-linear multiple regression, which involved complex iterative numerical procedures. The convergence of the estimators towards optimal values was used to assess the effectiveness of this approach. The subspace algorithm, based on the Gauss-Newton method[59, 60], was used to determine the coefficients of the algorithms. The coefficients were optimized through iterative calculation, using the coefficients obtained from multiple linear regression as the starting point for the iteration procedure.

Mindful of the fact that in the quantitative rainfall estimation by radar the assumption of linearity between the variables of interest is very quickly undermined when it comes to integrating heavy values, we thought it would be useful to free ourselves from this parametric approach through the neural network technique. Vulpiani et al.[32] judged this NN-method to be a powerful approach for designing a more flexible and robust algorithm than the linear regression parametric methods commonly used.

As they did, the multilayer perceptron neural network (MLP), organized in several layers including an input layer, several hidden layers facilitating the modelling of non-linear links between the input variables (Z_H, Z_{DR}, K_{DP}) and the output variable (R), are used. For the crucial learning process, we apply calibration or training samples from the synthetic data of the copula-based simulations. A similar exercise is also carried out on part of the real data (i.e., calibration sample from observational data) used for training the network, as a comparison. This will enable us to assess the contribution of the copula-based simulations to the rainfall estimations using the neural network method, drawing on the validations carried out on the real data. Formally, we can write these algorithms as follows:

$$R_{NN1} = NN_R\left(Z_H, K_{DP}\right), \tag{3}$$

for the neural network tested by using the radar variables Z_H and K_{DP} as inputs, and

$$R_{NN2} = NN_R\left(Z_{DR}, K_{DP}\right), \tag{4}$$

that corresponds to the algorithm based on the neural network with Z_{DR} and K_{DP} as inputs. NN_R indicates the neural network operator for direct rainfall rates retrieval.

2.4. Assessment Metrics

The performance of these estimators derived from copula-based simulation method is assessed using polarimetric radar data and ground rainfall measurements from northern Benin and Niamey. Similar assessments are also done with observed data from the calibration sample, for comparison. The assessments are mainly based on scatter plots comparing estimated and measured rainfall, as well as on a specific rainfall event that occurred in northern Benin during a certain period of time. Statistical scores such as the Kling and Gupta efficiency coefficient (KGE)[61] and root mean square error (RMSE) are used to assess the accuracy of radar-based rainfall estimations. The KGE is employed for overall assessment using the entire validation dataset sampled at 5-min time step, while the RMSE is used to assess the accuracy of the estimations during the specific 25–26 September 2007 rainfall event observed in northern Benin. These metrics are defined as follows:

$$KGE = 1 - \sqrt{(1-r)^2 + (1-\alpha_e)^2 + (1-\beta_e)^2}, \tag{5}$$

where r is the correlation coefficient, α_e is the ratio of standard deviations of estimated and observed rainfall and β_e is the ratio of their respective means; and

$$RMSE = \sqrt{\frac{1}{N}\sum_{k=1}^{N}(R_r - R_g)^2} \tag{6}$$

where R_r and R_g are the values estimated by the radar and observed by the rain gauge, respectively. For these assessments we consider 5-min QPE estimates to keep the quasi-operational setup, as done during radar measurements.

3. Results and Discussion

3.1. Assessment of Copula Simulations

As copulas provide a convenient way to model and simulate correlated variates, in order to test its efficiency, simulated fields of pseudo-observations are compared to the original radar and rain gauge measurements. For illustrative purposes, **Figure 2** shows the 3-D scatterplot of the simulated values (grey dots) of the triplet (Z_H, K_{DP}, R) for the different copulas considered. Black dots stand for observation data set from the calibration sample. Analysis of this figure highlights the good agreement between the simulated and observed cluster, at least for the well-fitted copulas that match those aspects of the data. This agreement is more obvious for the Gumbel, Frank and Student copulas, where we note a good superposition of the black and grey point clouds, and to a lesser extent for the HRT copula. In addition, the synthetic samples provided are richer in strong extreme values than in the observations. Such samples are important for determining efficient rainfall estimation algorithms since they include a wider range of rainfall types [17, 20].

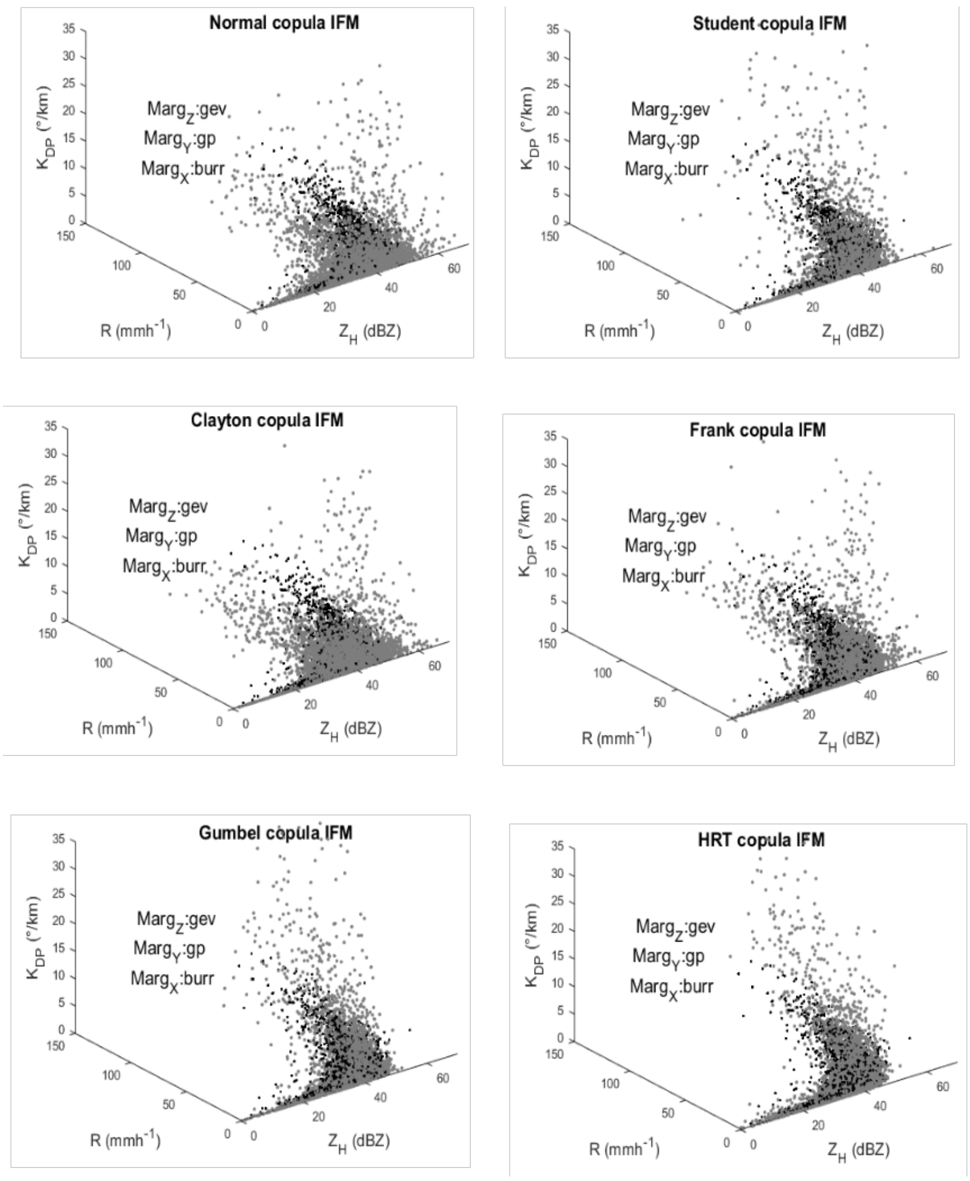


Figure 2. 3-D scatterplots of the triplet (Z_H, K_{DP}, R) for 10,000 simulated values (gray dots) for the different copulas considered and reference data (black dots): (**a**) Normal copula; (**b**) Student copula; (**c**) Clayton copula; (**d**); (**e**) Gumbel copula; (**f**) HRT copula.

However, another objective the Goodness of Fit (GoF) test performed to select a specific copula is the K-function test, which is a non-parametric method. The best-fitting copulas are determined based on the mean square distance between the empirical and theoretical K-function values. The Kendall diagram (also called K-K plot) is a tool used to assess visually the accuracy of copula matching on original data, i.e., it allows for a direct comparison between the empirical copula (based on reference data) and the simulated theoretical copulas. **Figure 3** shows the K-K plot diagrams for each of group of variables considered, which is a plot of the empirical K percentiles as function of Copula K percentiles. A closer curve to a straight line, specifically the first bisector, indicates a better fit between the dependency structure of the sample and the empirical copula estimated from the same sample. As mentioned above, the mean standard error or distance (MSE) is used to measure the distance between the empirical copula and the simulated copula for each copula and the smallest MSE score means the best fit. From **Figure 3**, it can be observed that the copulas closest to the empirical copula are Student's, Gumbel's, HRT's, and Frank's, regardless of the triplets or quadruplets of variates used for the simulations. The Gumbel, HRT, and Student copulas also effectively replicate the dependence in the upper tails, as indicated by the relative overlap of their Kendall curves with the first bisector.

Thus, by better determining the relationships between radar polarimetric observables and ground rain rates, for rainfall estimators based on synthetic data from these copulas, we would expect better estimates of intense rainfall. Zhang et al.[25] demonstrated that this could better learn the relationship between multidimensional radar observations and corresponding surface rainfall to improve QPE. Conversely, since the Normal and Clayton copulas are systematically characterized by higher mean square deviations, we exclude them because of their relatively poor ability to reflect dependencies between variables, in favor of the other four copulas, namely the Student, Frank, Gumbel and HRT copulas, which have the lowest mean square deviations.

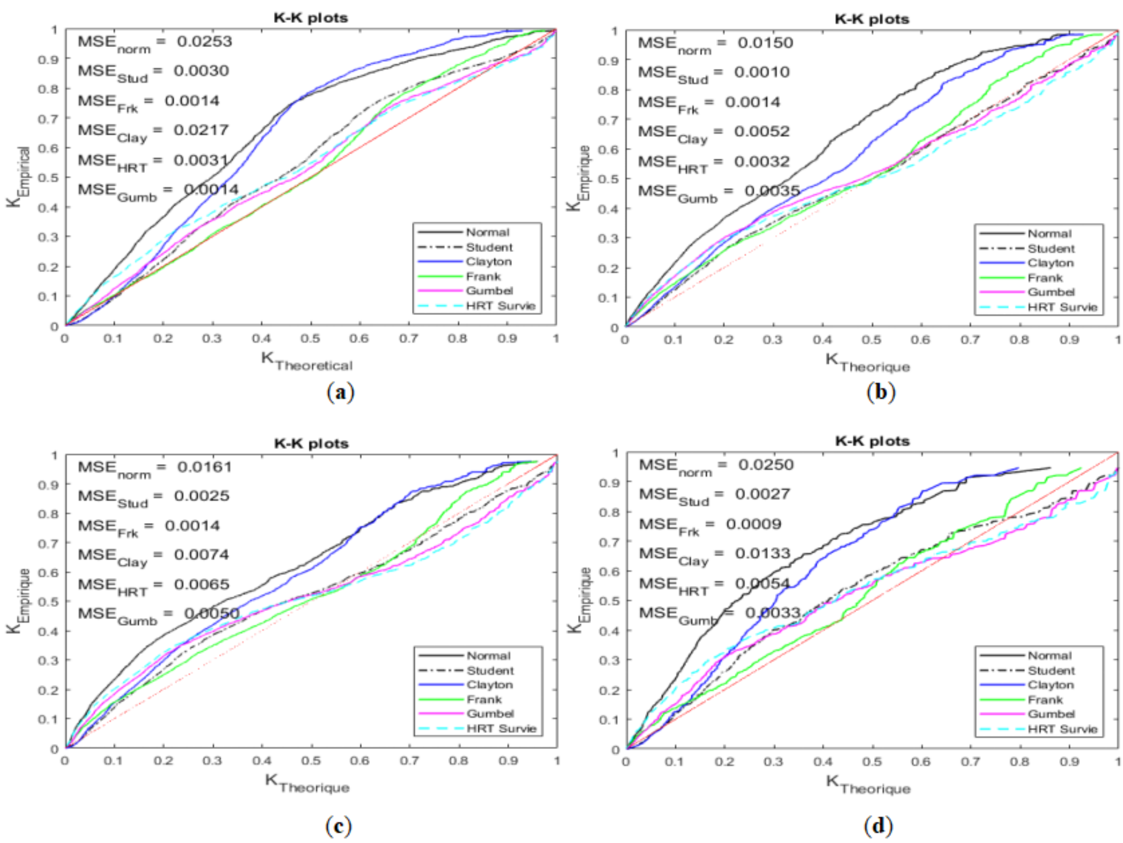


Figure 3. K-K plot of the Kendall function K(u) for: (**a**) (Z_H,K_{DP},R); (**b**) (Z_{DR},K_{DP},R), (**c**) (Z_H,Z_{DR},R) and (**d**) (Z_H,Z_{DR},K_{DP},R) variables considered.

3.2. Comparison of Different Radar Multiparametric QPE Approaches

The parametric algorithms for rainfall retrieval are based on empirical non-linear least squares regression (known as optimal regression) and utilize both measured data from rain gauges and radar, as well as synthetic data generated through copula simulations. As many studies [6, 7, 20, 26, 29] highlighted that the accuracy of these estimators is heavily influenced by the range of rainfall rate considered for their determination, in this study we decided to use the entire calibration dataset without assuming a specific rainfall range. This approach has the potential to enhance the design of quantitative functional relations between the surface rainfall and aloft radar observations, which are influenced by the complex spatio-temporal variability in DSD [25] across precipitation types and regions [20]. The coefficients of these estimators are provided in **Table 1**. For comparison purposes, the coefficients of certain algorithms determined by Koffi et al. [7] using simulated data from T-matrix are also presented. These coefficients were calculated based on raindrop distribution data collected in Northern Benin from 2005 to 2007 [62], using optical disdrometers. Unlike in our study, these authors only considered rainfall values greater than 5 mm/h when calculating their coefficients.

This table exhibits a relatively wide difference between the coefficients deduced from the copulas and those from the observation sample or the T-matrix simulation for all the algorithms selected in this present study. Specifically, the weights given to the various radar parameters (Z_H, Z_{DR}, K_{DP}) in the algorithms by the adjustments to the synthetic data from the copulas are greater than in the case of the observations or the T-matrix simulations. These discrepancies are even more striking for the Z_{DR} variable, which is often poorly weighted in the adjustments on observations affected by 'noise' or random uncertainties on measured and attenuation-corrected values [7]. In the specific case of the $R(Z_{DR},K_{DP})$ algorithm, the differential reflectivity Z_{DR} and the specific differential phase shift K_{DP} are identically weighted (almost similar exponents) for the copulas admitting tail dependence as the association between extreme values in the same tail. In their deep learning model incorporating polarimetric radar variables, Pan et al. [28] demonstrated that K_{DP} and Z_{DR} provided crucial microphysics and dynamic structure information of storms that improve significantly the skills of the nowcasting model of convective storms, by acting synergistically. Since these polarimetric observables directly reflect the microphysical properties of storms, including DSD, their result probably suggests, as far as rainfall retrieval is concerned, that equal weight should be given to these two variables with a bid to improve estimator performance, especially by targeting all rainfall types including intense rainfall. A recent study noted that K_{DP} is more important for heavy precipitation estimates [33]. However, this conclusion assumes Z_{DR} is biased toward larger hydrometeors in presence of large non-rain hydrometeors in precipitation, and the ambiguity of the shape information of hydrometeors provided by the Z_{DR} for those that are bigger than the radar wavelength. This means that, apart from these two limiting factors, Z_{DR} can be considered as an essential contributor to the estimation of heavy rainfall, like K_{DP}. Unlike adjustments on the basis of copula-based datasets, by fitting $R(Z_{DR},K_{DP})$ with T-matrix simulations and observations datasets, the contribution of Z_{DR} and K_{DP} are in ratios of 3 and 8 in favor of K_{DP} , respectively. According to Koffi et al. [7], the weight given to Z_{DR} is even lower when the relationships are adjusted with observations because of high noise level for light rainfall or random uncertainties in the areal radar data associated with low signal-noise-ratios (SNRs) and attenuation corrected values of this observable for heavy precipitation [20]. Finally, fitting done with Frank copula-based dataset focusing on light to moderate values leads to a greater weight assigned to Z_{DR} than to K_{DP} to compensate the lack of heavy values. Li et al. [33] described a more comprehensive understanding of the microphysical processes guiding precipitation by analyzing and quantifying the correlation between polarimetric radar variables at different radar elevation angles and according to various precipitation types. They concluded that for light to moderate rainfall, the contribution of K_{DP} is less important than Z_{DR} to the estimates of precipitation. This is probably why Z_{DR} is more important in light to moderate precipitation estimates for the model using Frank's copula-based dataset.

Table 1. Coefficients of the algorithms obtained by the non-linear least squares method applied to the synthetic data from the copula simulations and the calibration observation data.

	$R = aZ_H^b K_{DP}^c$			$R = aZ_{DR}^b K_{DP}^c$			$R = aZ_{DR}^b Z_H^c$			$R = aZ_H^b K_{DP}^c Z_{DR}^d$			
	a	**b**	**c**	**a**	**b**	**c**	**a**	**b**	**c**	**a**	**b**	**c**	**d**
Student	1.075	0.412	0.332	21.918	1.000	1.000	1.028	−0.372	0.710	0.028	0.693	0.138	2.000
Frank	1.659	0.222	0.319	22.182	0.678	0.250	1.427	1.125	0.189	3.680	0.098	0.189	1.945
Gumbel	1.084	0.397	0.502	27.359	0.987	0.989	1.059	2.416	0.305	0.171	0.375	0.133	1.885
HRT	1.076	0.407	0.536	24.877	0.988	0.992	1.002	3.000	0.573	0.628	0.259	0.124	2.000
Obs.	3.748	0.182	0.602	51.274	−0.086	0.752	1.008	−0.210	0.710	2.869	0.188	0.564	0.161
Tmatrix*				15.13	−0.29	0.9				9.42	0.05	0.89	−0.34

* Rainfall rates above 5 mm/h have been considered by Koffi et al. [7] to determine these coefficients.

Consequently, when theoretical values of polarimetric radar observables are used as input, the orders of magnitude of the theoretical rainfall calculated for these different estimators give significant relative deviations from the algorithms calibrated on observations, with values of up to 800% for high values of polarimetric observables, as shown in **Figure 4**. We thus encounter two groups of algorithms of which the first is composed of, R(Z_H,K_{DP}) and R(Z_H,Z_{DR}) estimators for which the differences between estimators based on copulas and those calibrated on the basis of observations (calibration sample) quickly become significant from light rainfall characterized by weak radar observables. For both R(Z_{DR},K_{DP}) and R(Z_H,Z_{DR},K_{DP}), the comparison of estimators derived from copulas with those based on T-matrix simulation and thresholded observations [7] from the same database as the one used in this study yields wider discrepancies for heavy rainfall of more than 30 to 50 mm/h. These results highlight the significant impact of the variability of estimators on the accuracy of rainfall retrievals, as indicated by Zhang et al. [20]. It demonstrates the challenge of agreeing on consistent climatological relationships, particularly when using different rainfall measurement methods and simulation approaches. The differences in estimations are particularly pronounced for intense rainfall events. For this type of precipitation characterizing typhoon and squall lines, Zhang et al. [20] exhibit the importance of fitting the rainfall estimator for different precipitation types. They found enhanced estimation when the dedicated estimators are used for specific precipitation types. The variation in results for polarimetric parametric algorithms is mainly attributed to the difficulty in determining the variability of the Drop Size Distributions [5, 20] and characterizing the dependencies between the variables used in these estimators [17]. The copula approach, which measures dependence between variables, is thus confirmed as a suitable method. It offers the advantage of representative sample sizes and diversity across various rainfall categories and radar variable configurations, including uncommon values rarely observed. This approach would enhance the accuracy of rainfall retrieval models.

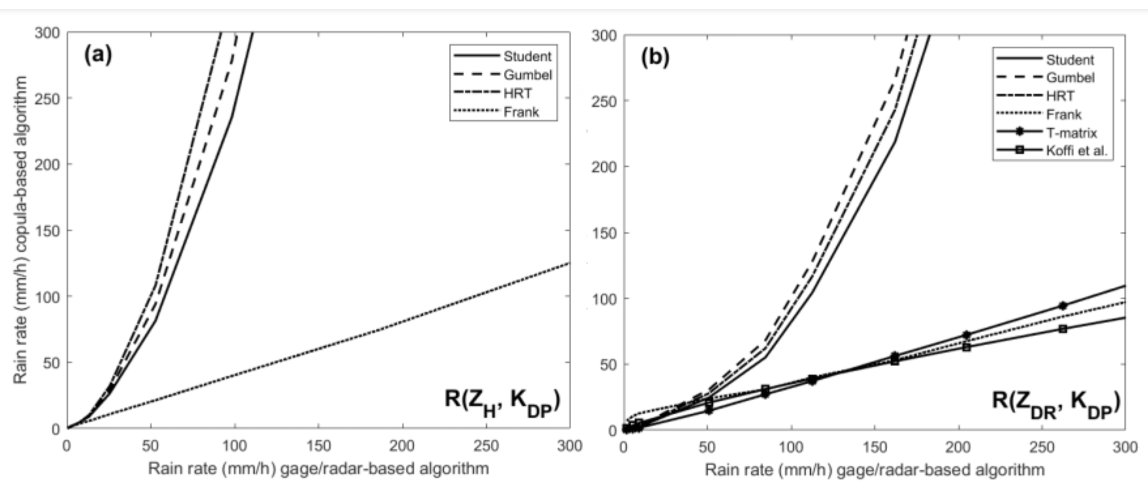


Figure 4. *Cont.*

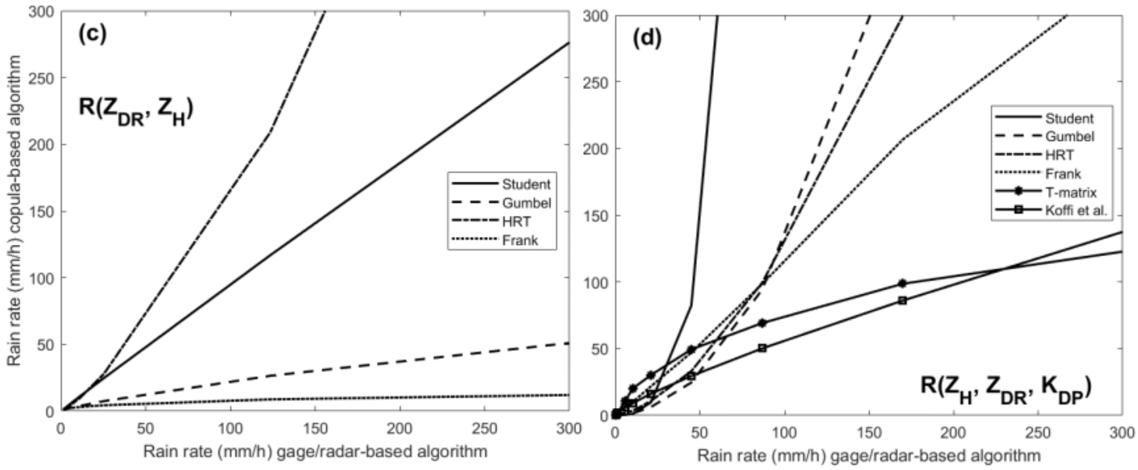


Figure 4. Comparison of theoretical rainfall rate as calculated from copula-based and gage/radar-based algorithms: (**a**) R(Z_H, K_{DP}); (**b**) R(Z_{DR}, K_{DP}); (**c**) R(Z_{DR}, Z_H); and (**d**) R(Z_H, Z_{DR}, K_{DP}). For panels b and d, theoretical values of rainfall rate from T-matrix and gage/radar-based algorithms from Koffi et al. [9] are also shown.

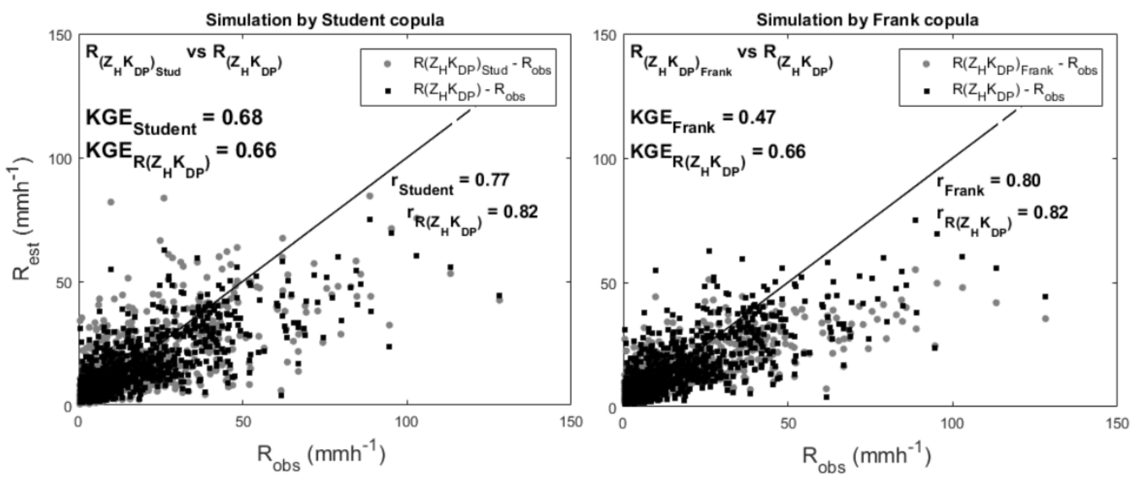
Table 2 summarizes, in terms of KGE for the entire validation sample and RMSE calculated on the specific rainfall event case, the performance of the various estimators used in this study to estimate rainfall rates at the 5-minute time step. For comparison with the work of Koffi et al. [7], who used the same validation database, we show in this table for certain algorithms their results obtained by calibrating the algorithms using synthetic data simulated by the T-matrix method based on DSD measurements. Among the polarimetric multiparametric algorithms, R(Z_H,K_{DP}) and R(Z_{DR},K_{DP}), which were calibrated and optimized using synthetic data from copulas' simulations, outperformed both the other algorithms R(Z_H,Z_{DR}) and R(Z_H,Z_{DR},K_{DP}). These former algorithms, especially when using copulas with higher tail distributions, showed significant improvements in the Kling-Gupta Efficiency (KGE) around 3% to 15% for R(Z_H,K_{DP}) and 30% to 40% for the R(Z_{DR},K_{DP}) algorithms, compared to estimators calibrated on observations. The reliability of the 5-minute time-step rainfall estimations was demonstrated by the better alignment of the data dots around the first bisector, as shown in **Figures 5** and **6** showing scatterplots of rain rates versus their estimates from R(Z_H,K_{DP}) and R(Z_{DR},K_{DP}) optimal relations based on the copulas' synthetic database and the calibration sample of the observation dataset. Previous research had achieved similar performances but at hourly time steps, although with lower KGEs. The results suggest that including intense rainfall rates in the calibration samples is important for accurate rainfall estimation. Thus, estimators based on Frank's copula, which poorly reproduces upper-tail dependence, performed worse than those based on the observational calibration data sample. The rate of performance decline is around 30% for R(Z_H,K_{DP}) and increased to around 80% for R(Z_{DR},K_{DP}). **Figures 7** and **8** display time series of instantaneous radar-based rainfall rates for the 25–26 September 2007 event over the Nangatchori gauge. For comparison, the 5-min rainfall rates measurements from the gauge are included. Considering data from rain gauge as reference, estimators based on Frank's copula synthetic data provide poor performance for the peak retrieval of the rainy event. The same rate of decrease in performance compared to estimators calibrated on observations, although less impressive extent, is observed in terms of RMSE with 8% and 60% for the R(Z_H,K_{DP}) and R(Z_{DR},K_{DP}) algorithms, respectively. It is also noted that estimators based on copulas with upper-tail dependence such as Gumbel and HRT failed to estimate lower rainfall rates.

In any case, it is clear from this analysis that a combination of differential reflectivity and differential specific phase shift, and the weight given to them in rainfall variability, is of great benefit. We note that for the R(Z_{DR},K_{DP}) algorithm, the best estimators are those in which the weights of Z_{DR}, and K_{DP} are both identical and close to 1. The advantage of this algorithm lies in the fact that it capitalizes on two radar parameters that have been found to be insensitive to the variability of raindrop distributions [17, 32, 64]. In contrast, the high number of radar variables (three) does not guarantee better performance, as is the case with the R(Z_H,Z_{DR},K_{DP}) algorithm, which, together with R(Z_H,Z_{DR}), seems to give the worst scores of the four algorithms assessed in this study, irrespective of the copulas considered. For R(Z_H,Z_{DR}), its poor performance (lower KGE) in relation to ground-measured rainfall can be linked to the large values of the Z_{DR} exponent, as explained by Matrosov [65] when they obtained similar results in comparisons of several polarimetric algorithms. In this study, estimators derived from the Student copula and those calibrated by observation data, giving the lowest Z_{DR} exponents, demonstrated the best performance in terms of KGE, compared to estimators based on the HRT, Gumbel, and Frank copulas, which provide coefficients that were 3 to 15 times higher. In addition, uncertainties in attenuation and calibration corrections, as well as the bright-band problem, though to a lesser extent in this study due to the use of a low radar elevation angle, may also contribute to the underperformance of the R(Z_H,Z_{DR}) algorithm. These factors acting collectively more influence the quality of its performance. Ryzhkov et al. [63] argued that it is not possible to use a combination of Z_H and Z_{DR} to estimate precipitation from X-band radar measurements, as their biases due to horizontal and differential attenuation are too great and difficult to take into account. For all these reasons, in the following, we consider R(Z_H,K_{DP}) and R(Z_{DR},K_{DP}) to be the best performing multiparametric algorithms among those tested in this study. Comparing the performances of these four estimators, Zhang et al. [20] found that R(Z_H,K_{DP}) and R(Z_{DR},K_{DP}) generated the best performances. R(Z_{DR},K_{DP}) had the optimal performance because K_{DP} can adapt to the DSDs' variability and includes additional information of raindrop diameters across Z_{DR}.

Table 2. Performance of copula and observed retrieval-based rainfall algorithms relative to validation sample observed in north Benin (from 2006–2007) and Niamey (Niger) in 2010. In bold, KGE values similar to or greater than those of estimators calibrated from T-matrix simulations or the calibration sample of actual observations.

Algorithms	**R (Z_H, K_{DP})**	**R (Z_{DR}, K_{DP})**	**R (Z_H, Z_{DR})**	**R (Z_H, Z_{DR}, K_{DP})**
Scores	**KGE**	**KGE**	**KGE**	**KGE**
Student	**0.68**	**0.82**	**0.64**	0.20
Gumbel	**0.75**	**0.79**	0.39	**0.51**
HRT	**0.76**	**0.81**	0.40	0.46
Frank	0.47	0.13	0.21	0.13
Observations	0.66	0.60	0.54	0.68
T-matrix simulation1		0.50		0.52

1 From Koffi et al. [7] using the same data as in the present study.



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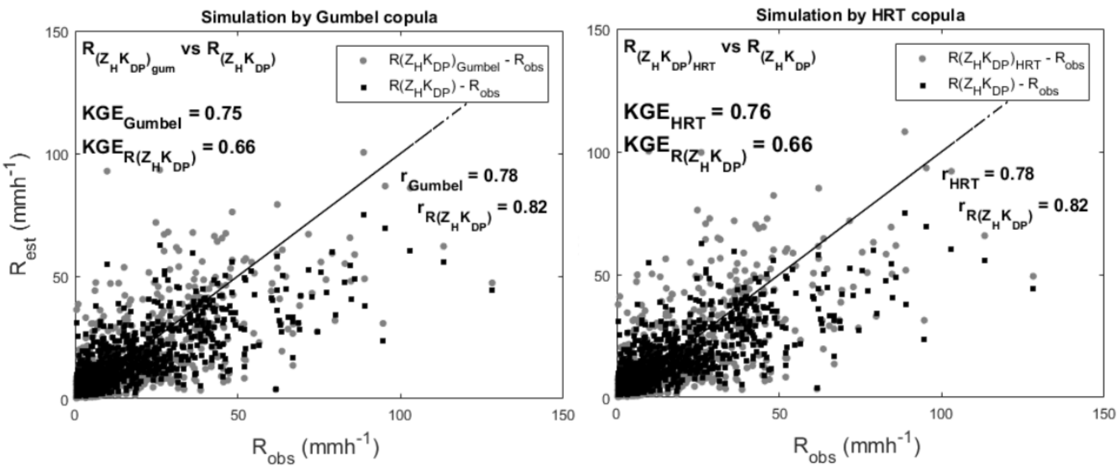


Figure 5. Performance of the R(Z_H,K_{DP}) algorithm through rainfall rate retrieval from estimators calibrated on synthetic data from copula simulations (grey dots) and on observed calibration data (black dot) vs. "ground truth" rain rate measured by rain gauge (validation data sample). The statistics scores in the scatterplots are KGE and correlation coefficient *r*.

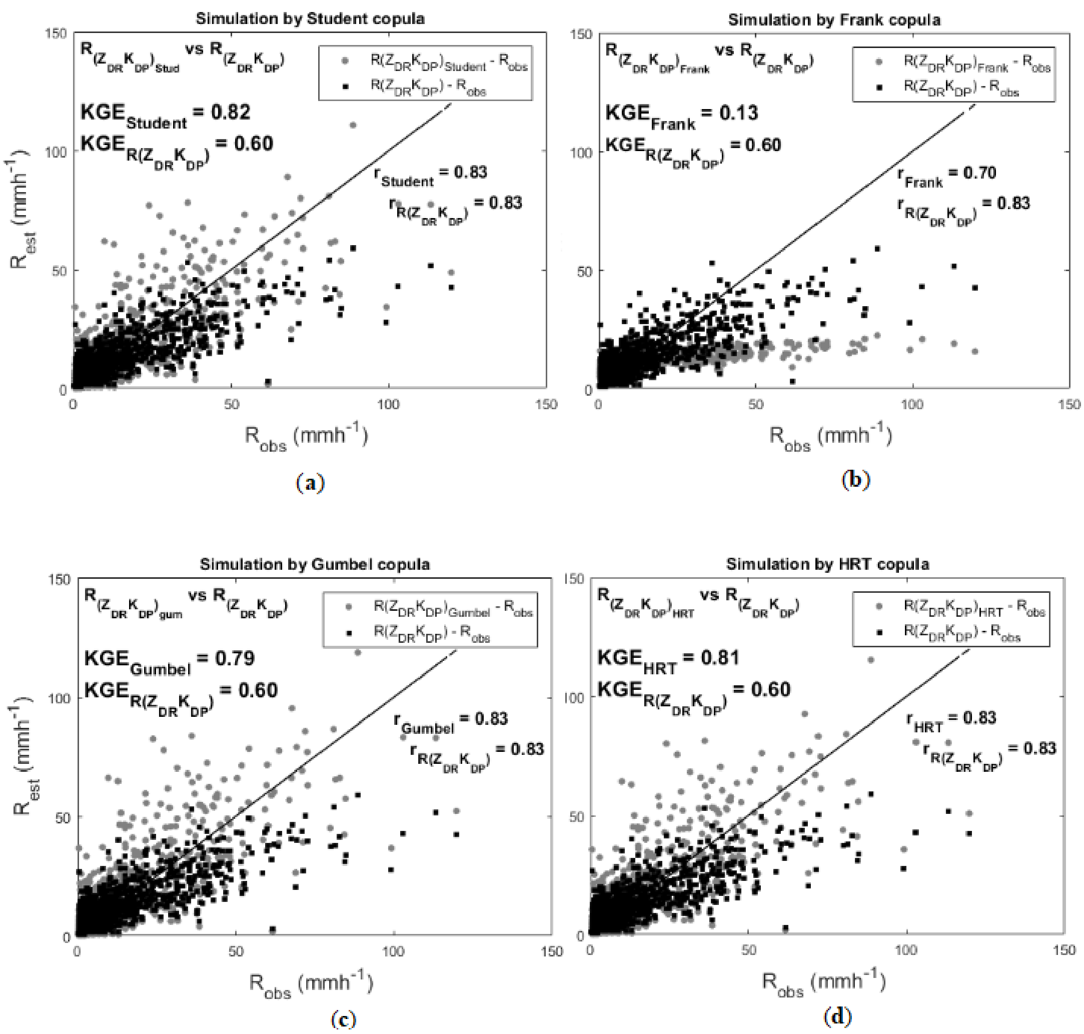


Figure 6. As in **Figure 5**, but for the R(Z_{DR}, K_{DP}) algorithm calibrated on synthetic data from copula simulations (grey dots): (**a**) Student copula, (**b**) Frank copula, (**c**) Gumbel copula, and (**d**) HRT copula.

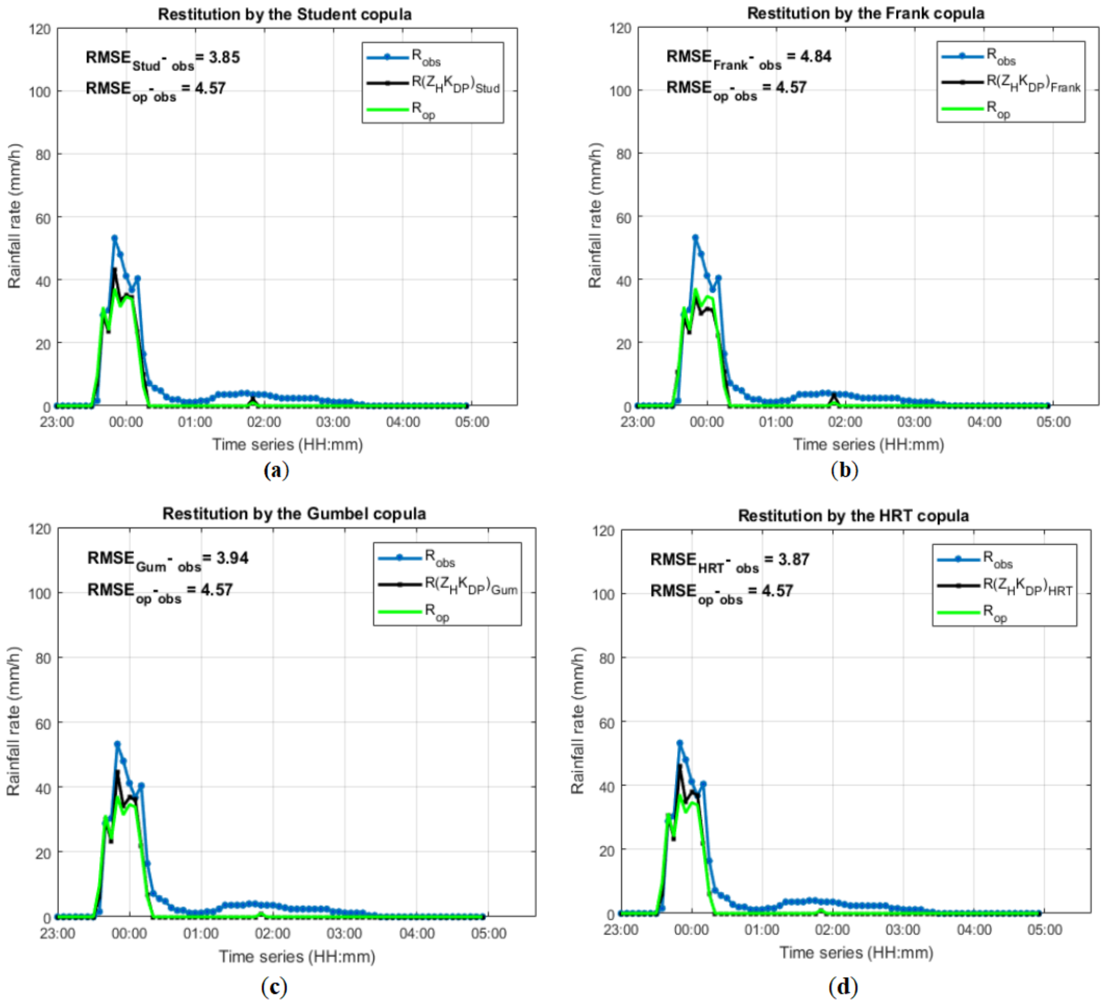
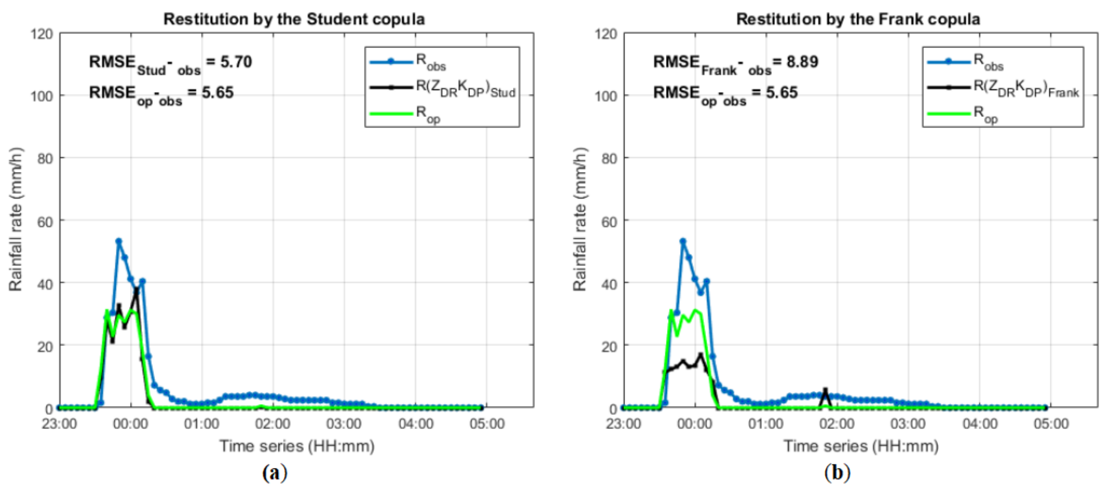


Figure 7. Time sequence of radar rainfall estimates over the Nangatchori gauges during the September 25–26, 2007 event through $R(Z_H, K_{DP})$ algorithm calibrated by copulas synthetic datasets: (**a**) Student copula, (**b**) Frank copula, (**c**) Gumbel copula, and (**d**) HRT copula. For comparison, the 5-min rainfall totals of the rain gauges, converted into rainfall rates, are also shown. R_{op} stands for estimates through optimization performed on observed calibration data.



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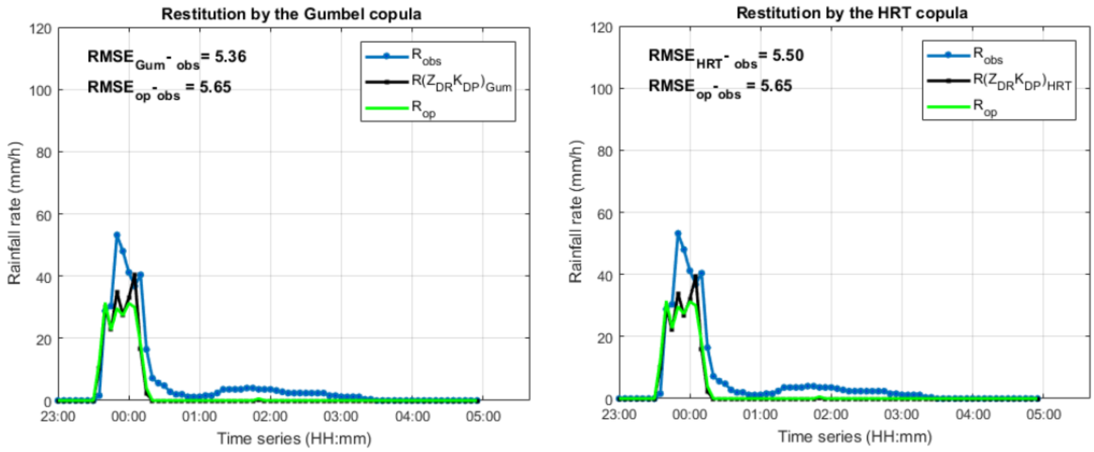


(c) (d)

Figure 8. As in **Figure 7**, but for radar rainfall estimates using R(Z_{DR}, K_{DP}) algorithm calibrated by copulas synthetic datasets: (**a**) Student copula, (**b**) Frank copula, (**c**) Gumbel copula, and (**d**) HRT copula.

Nevertheless, all these results clearly illustrate the difficulties encountered in applying parametric algorithms for estimating precipitation rates, due to their sensitivity to the variability in raindrop size illustrated by scatterplots that are particularly wide. For this reason, the following subsection assesses the performance of the neural network method using polarimetric radar variables as input, as an alternative to parametric methods.

3.3. Comparison of NN-Based Retrieval of Rainfall Rate with Optimal Multiparametric Estimators

In this section, the assessment of rainfall estimates by the neural network method is carried out by comparing them with the best parametric algorithms R(Z_H,K_{DP}) and R(Z_{DR},K_{DP}), optimized by nonlinear regression on data simulated by copulas and observations. The training process of the artificial neural network is also carried out on the same data simulated by copulas and those of the calibration sample derived from observations, for comparison purposes.

A list of the overall (KGE) and event-specific (RMSE) scores for the tested relationships can be found in **Table 3**. Overall, the global performances (KGE) obtained by the direct application of the R_{NN}(Z_H,K_{DP}) and R_{NN}(Z_{DR},K_{DP}) neural networks are not significantly different from those previously obtained by the parametric R(Z_H,K_{DP}) and R(Z_{DR},K_{DP}) polarimetric algorithms when the training data come from simulations of the HRT, Gumbel, and Student copulas. For these copulas, the KGE scores of R_{NN}(Z_H,K_{DP}) are slightly improved by 5 to 8% compared to R(Z_H,K_{DP}), while those of R_{NN}(Z_{DR},K_{DP}) are reduced by 1 to 5% compared to the performance of the optimized parametric R(Z_{DR},K_{DP}) algorithm. When the training data for the neural networks are obtained from calibration observations, the advantages of this method are obvious, especially for the algorithm using differential reflectivity and specific differential phase shift, that gives an improvement rate of about 18% compared to the parametric algorithm using the same combination of radar parameters. For the combination of radar observations (Z_H, K_{DP}), the improvement (6%), although moderate, remains in favor of R_{NN}(Z_H,K_{DP}) for the network trained by the observational data for designing algorithms.

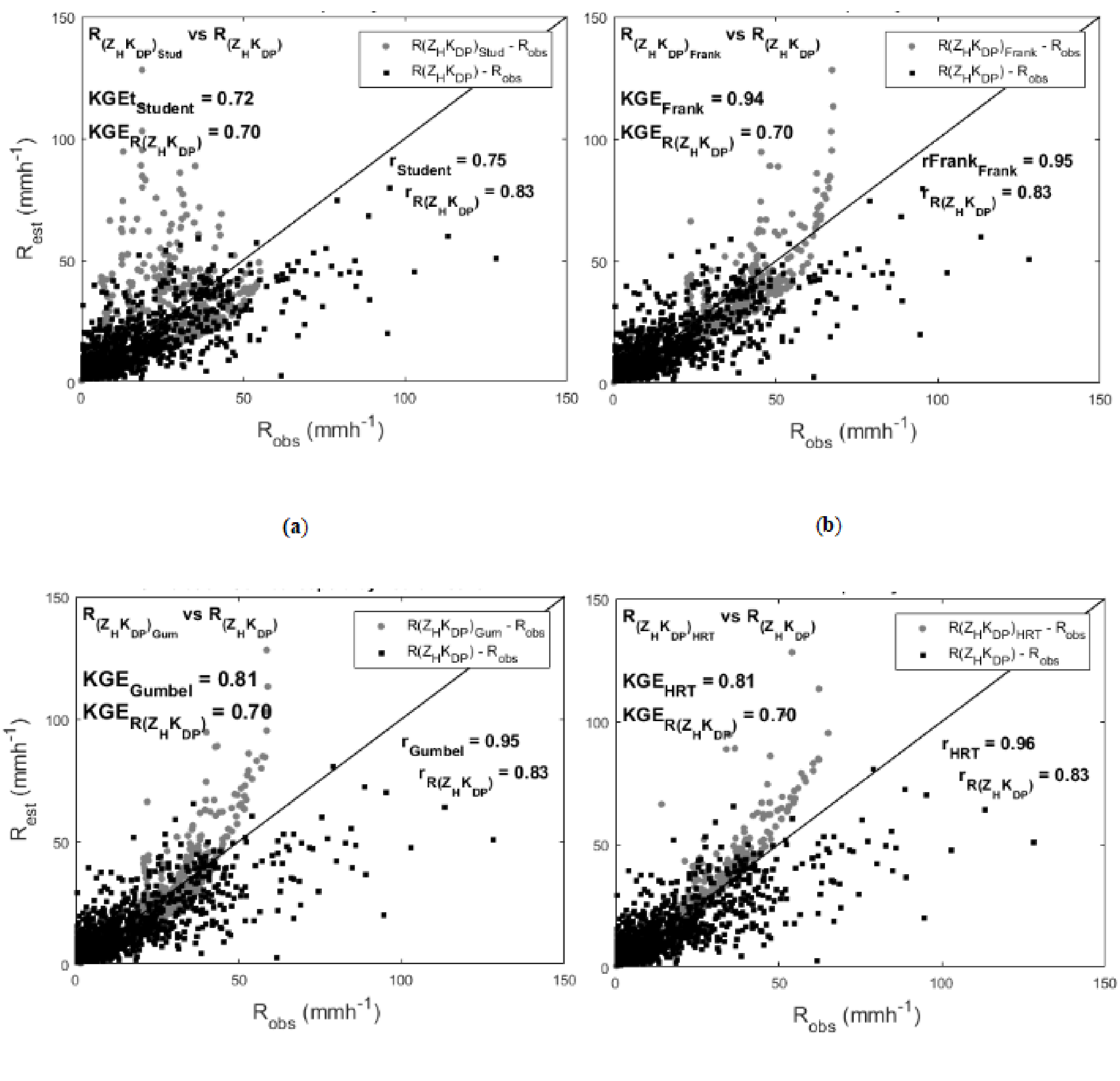
From the comparison of the KGE statistical values in **Table 3**, the R_{NN}(Z_H,K_{DP}) algorithm calibrated with data simulated by Frank's copula yields the best overall results (0.94 in terms of KGE, corresponding to a 100% increase over its parametric counterpart) among all the quantitative radar rainfall estimators tested. Similarly, comparing the performance of R_{NN}(Z_{DR},K_{DP}) with that of the corresponding parametric algorithm calibrated with the same data from Frank's copula, it achieves an improvement ratio of 5, although its KGE remains lower than that of the other estimators trained with synthetic data from the other copulas and calibration observations (**Figure 9**). As can be seen in the K-K plot (**Figure 3**), the results are not surprising given that Frank's copula appeared to have the most successful transcription of the dependencies between the variables of interest (lowest MSE). However, it should be noted that this copula shows limitations for the upper tail distributions for (R, Z_H, K_{DP}) and the lower and upper tail distributions for the triplet (R, Z_{DR}, K_{DP}). As the parametric algorithms, determined by the non-linear least-squares method, are sensitive to these upper-tail values [22], and as no thresholding was applied to the data, but rather all simulated samples were used for fitting, it is clear that the algorithms are unable to estimate these types of rainfall, which explains why the KGE scores are so low. In contrast, for the neural network method, the learning process is repeated for numerous examples making up the synthetic dataset until the network reaches a stable state where there are no further changes in the synaptic weights of the neural network [32]. This would explain the superiority of the $R_{NN}(Z_H,K_{DP})$ algorithm trained using Frank's copula synthetic data. In the case of the $R_{NN}(Z_{DR},K_{DP})$ algorithm, despite the clear improvement in KGE over the corresponding parametric estimator $R(Z_{DR},K_{DP})$, it fails to outperform the neural network trained on observational data (**Figure 10**) due to the poor reproduction of dependencies between variables of interest for the lower and upper tail distributions. To explain the causes of the precipitation estimates from double modules deep learning models under different rainfall amounts, Li et al. [33] quantified the influence of each radar observable on their model estimations by analyzing various precipitation intensities. Their results show that Z_{DR} is more critical in light rainfall, whereas K_{DP} becomes more important in heavy rainfall estimates. Thus, the compromised performance of $R_{NN}(Z_{DR},K_{DP})$ in the case of the Frank copula-based dataset may be explained by the limitation of the functional relation between R, Z_{DR}, and K_{DP} determined across that copula, for light and heavy rainfall.

Table 3. Comparison between neural network-based algorithms and parametric ones calibrated from copula-simulated and observed data (calibration sample). In bold, KGE values similar to or greater than those of estimators calibrated using actual observations data.

Algorithms	**R (Z_H, K_{DP})**	**R_{NN} (Z_H, K_{DP})**	**R (Z_{DR}, K_{DP})**	**R_{NN} (Z_{DR}, K_{DP})**
Scores	**KGE (RMSE*)**	**KGE (RMSE*)**	**KGE (RMSE*)**	**KGE (RMSE*)**
Student	**0.68** (3.85)	**0.72** (4.91)	**0.82** (5.70)	**0.81** (3.08)
Gumbel	**0.75** (3.94)	**0.81** (2.92)	**0.79** (5.36)	**0.75** (3.67)
HRT	**0.76** (3.87)	**0.81** (3.02)	**0.81** (5.50)	**0.77** (5.03)
Frank	0.47 (4.84)	**0.94** (3.82)	0.13 (8.89)	**0.64** (2.94)
Observations	0.66 (4.50)	0.70 (1.83)	0.60 (5.65)	0.71 (3.87)

* RMSE (mm/h) at rainfall event scale considering the 25–26 September 2007 event over Nangatchori (Northern Benin).

At last, the scatterplots of rainfall rates from gauges measurements versus their estimates from neural network method with learning process using copula-based synthetic dataset, regardless of the combination of polarimetric radar observables, are narrower than NN-method trained with observational data and parametric algorithms scatterplots. The correlation coefficients close to 1 indicate that the scatter points are linearly distributed neural networks trained by copula-based datasets and corroborate the good agreement between the values estimated by the neural network method and ground rainfall observations (**Figures 9** and **10**). In terms of correlation coefficients, the quantitative evaluation also demonstrates the superiority $R_{NN}(Z_{DR},K_{DP})$. Except for NN-method training with Student's copula dataset, in the case of $R_{NN}(Z_H,K_{DP})$, our results go beyond those from Zhang et al. [25] based on a sophisticated deep-learning convolutional neural network (CNN) algorithm using hybrid volume scan data of Z_H, Z_{DR}, and K_{DP}. Their method was composed of multiscale convolutional operations designed to achieve the complex nonlinear mapping from radar measurements to rainfall rate and showed performance varying according to different segments of Z_H, Z_{DR}, and K_{DP}, denoting dependence on DSD variability, as exhibited in comparison with DSD-based nonlinear fitting algorithm. Assessment results of hourly rainfall retrieval from quantitative precipitation estimates deep-learning network proposed by Li et al. [33] exhibit comparable RMSE of 3.06 mm/h, but their correlation coefficient remains lowest (0.89). Our findings are striking results which may indicate that NN-methods calibrated with copula-based data are insensitive to raindrop size variability, albeit some useful adjustments may be done to make tangible the intense quantitative precipitation estimations. In other words, taking into account joint dependencies between radar variables and rainfall rate leads to cancellation of the influence of raindrop size variability. In this respect, we hypothesize that multi-parameter copulas would be appropriate to better reflect the dependence between radar variables and rainfall rate, particularly for intense rainfall.



(a) (b) (c) (d)

Figure 9. Radar rainfall estimates versus gauges measurements using NN(Z_H, K_{DP}) algorithms with training dataset from copula (grey dots) (**a**) Student copula, (**b**) Frank copula, (**c**) Gumbel copula, (**d**) HRT copula, and observational calibration sample (black dots).

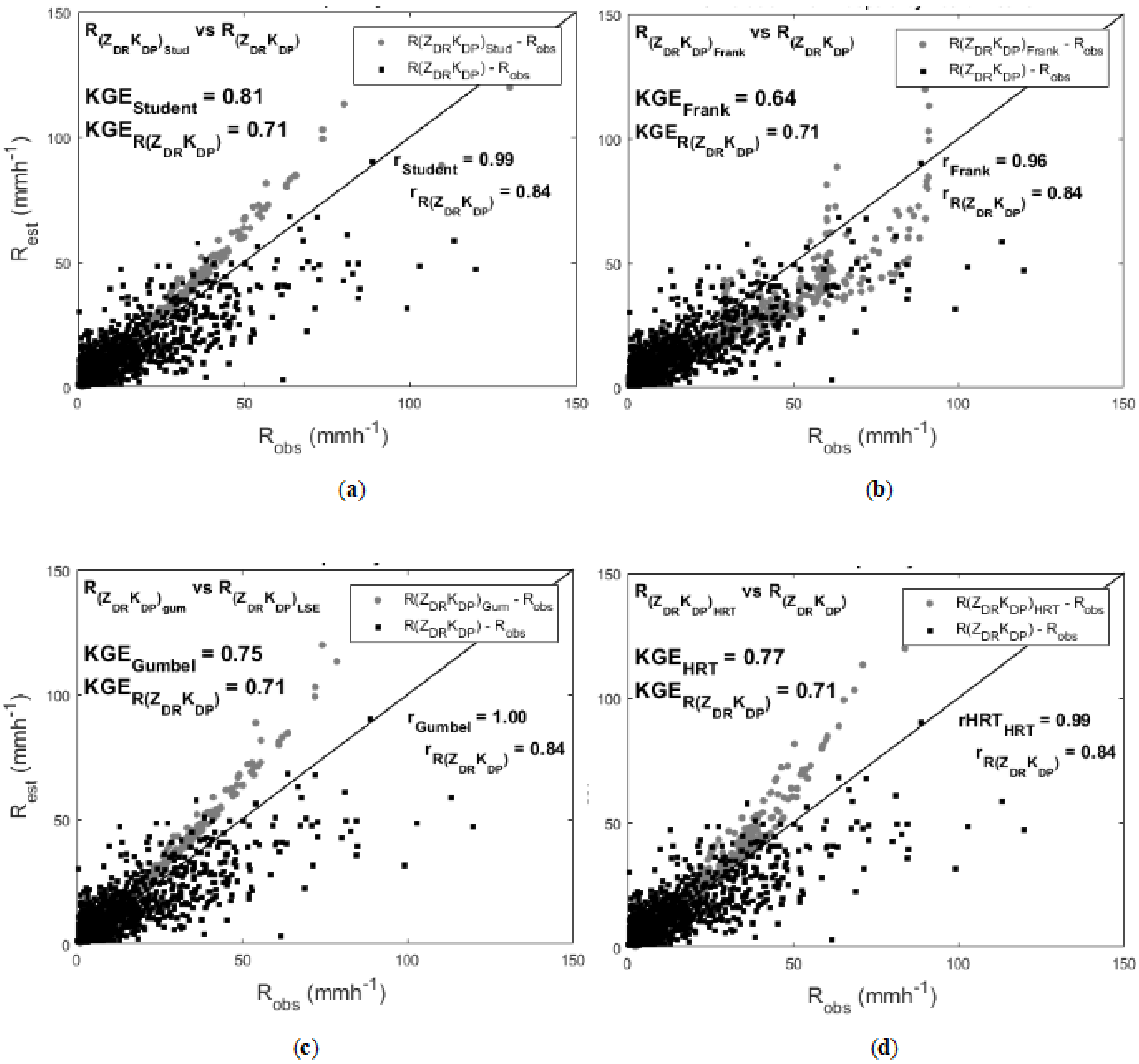


Figure 10. As in **Figure 9**, but radar rainfall estimates using NN(Z_{DR}, K_{DP}) algorithmswith training dataset from copula (grey dots): (**a**) Student copula, (**b**) Frank copula, (**c**) Gumbel copula, (**d**) HRT copula, and observational calibration sample (black dots).

An essential criterion for assessing the efficiency of a model is to assess its robustness, defined as its capacity to provide consistent estimates of precipitation for a given event. The root-mean-square error (RMSE) scores of the various estimators applied to the retrieval of the rainfall event that occurred on September 25–26, 2007 in Benin provide a means of assessing the aforementioned robustness (**Table 3**). Thus, **Figures 11** and **12** provide a summary of cumulative rainfall comparisons at 5-min intervals between ground measurements collected by rain gauges and radar for this event. These comparisons are made possible by parametric relationships and the neural network trained by synthetic copula data and calibration observations.

In comparison to parametric algorithms, the RMSE values demonstrate a notable decrease, irrespective of the algorithms utilizing the Z_H-K_{DP} and K_{DP}-Z_{DR} radar parameter combinations. This suggests that the neural network-based algorithms exhibit enhanced robustness. Therefore, for this particular instance of the event derived from the radar measurements, the root mean square errors (RMSEs) relative to NN(Z_H,K_{DP}) are reduced by between 20 and 60%, while those relative to NN(Z_{DR},K_{DP}) range between 8 and 70% reduction rate range when the neural networks are trained by the synthetic copula data. In the case of neural networks using observation data for training process, the reduction in RMSE is relatively important, with a rate of 60% and around

30% for the (Z_H, K_{DP}) and (Z_{DR}, K_{DP}) combinations, respectively, in comparison with parametric algorithms calibrated on the same calibration data. In other words, considering only the neural network approach, it can be observed that the learning process from copula-simulated data performs well for retrieval models. This is evidenced by the ability to reproduce the dynamics and rainfall rates on the ground (**Figures 11** and **12**), in comparison to improvement provided by real observation data which is characterized by a lack of representativeness and a diversified richness of the configurations tested during neural network training. Conversely, the likelihood of the synthetic configurations proposed for training the neural network appears as a critical factor with regard to the efficiency and robustness of the estimators for specific events considered individually. Indeed, the HRT and Gumbel copulas, which demonstrated the least reliability in reflecting the joint dependency between the variables R, Z_{DR}, and K_{DP} (**Figure 3**), in consequence provide the worst rainfall rate estimations. For these copulas, the neural network model leads to an increase of 30% and a small decrease of 5% in terms of RMSE score, compared to training the network with observed data.

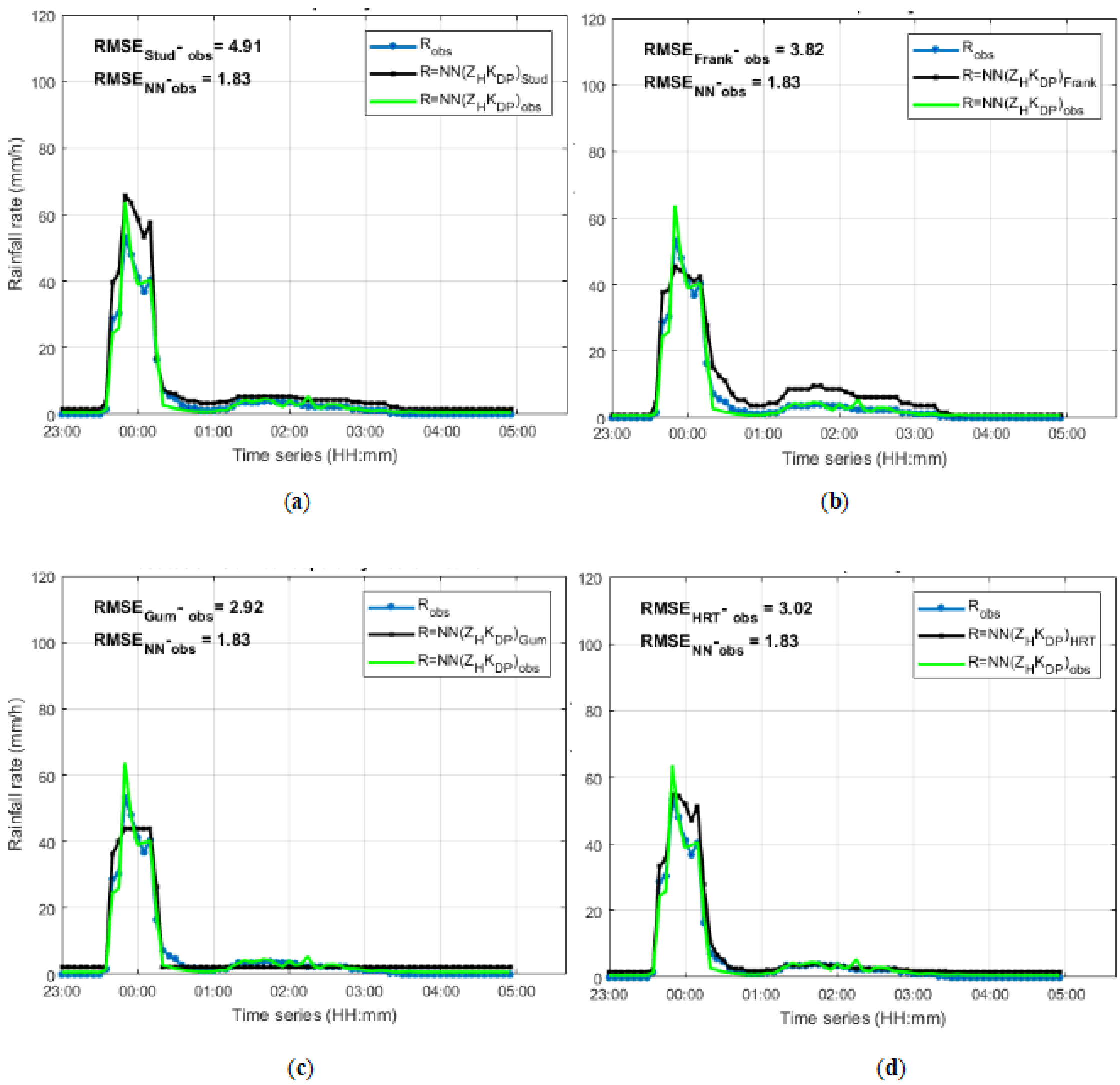


Figure 11. Time sequence of radar rainfall estimates over the Nangatchori gauges during the September 25–26, 2007 event through NN(Z_H, K_{DP}) algorithm calibrated by copulas synthetic datasets vs original dataset: (**a**) Student copula, (**b**) Frank copula, (**c**) Gumbel copula, and (**d**) HRT copula. For comparison, the 5-min rainfall totals of

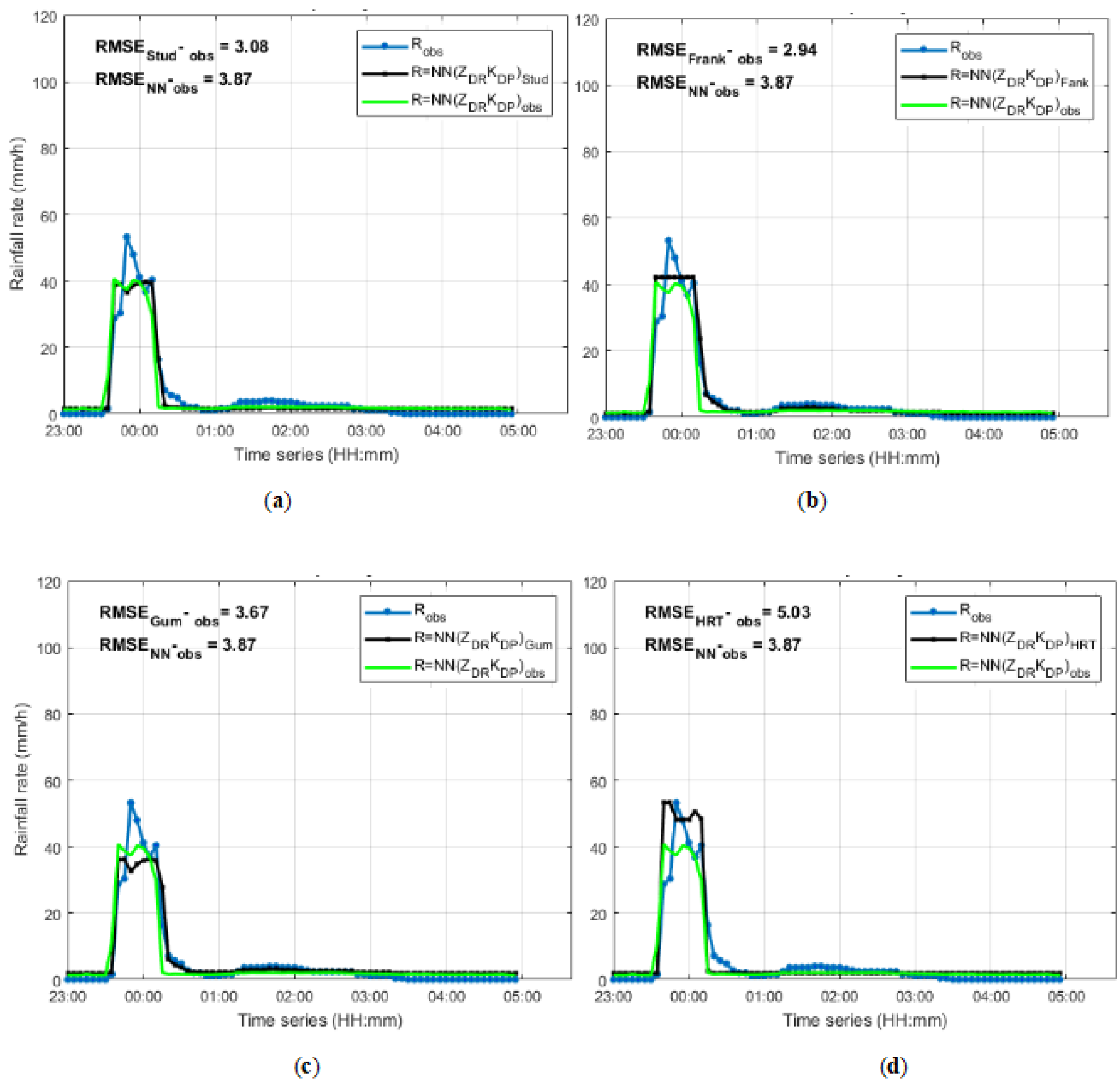


Figure 12. As in **Figure 11**, but for NN(Z_{DR}, K_{DP}) neural network method calibrated by copulas synthetic datasets vs original dataset: (**a**) Student copula, (**b**) Frank copula, (**c**) Gumbel copula, and (**d**) HRT copula.

4. Conclusions

The main challenge in quantitative rainfall estimation by radar currently lies in the design of efficient algorithms, particularly for intense rainfall, which is often excluded or only partially represented in the samples used to determine these estimators. In this study, we examine the applicability of the neural network in polarimetric radar-based quantitative precipitation estimation. Because this method is limited by the availability of training data in West Africa, leading to poor training performance, we designed a statistical nonlinear framework through copulas approach, to provide a wide range of realistic synthetic data samples including different polarimetric radar variables (Z_H, Z_{DR}, K_{DP}) and rainfall rate (R), high extreme values by choosing copulas with upper tail distributions such as Student, Gumbel, and HRT. This was done drawing on a database of radar and rainfall data collected during past AMMA intensive multi-year campaigns in northern Benin and during the Megha-Tropiques program aimed at validating satellite measurements, achieved in 2006–2007 and 2010, respectively. This copula-based approach appears as a method for modeling the dependence between polarimetric radar variables of interest (Z_H, Z_{DR}, K_{DP}) and rain rate (R), that is critical to take account of the complex spatiotemporal variability in DSDs that affects QPE algorithms. For comparison, we also designed multiparametric polarimetric estimators using nonlinear optimization basing on real observation data and synthetic datasets from copulas simulations. These estimators encompass Z_H-K_{DP}, Z_{DR}-K_{DP}, Z_H-Z_{DR}, and Z_H-Z_{DR}-K_{DP} combinations.

Based on the assessment results for all the rainfall intensities, the multi-parametric algorithms $R(Z_H,K_{DP})$ and $R(Z_{DR},K_{DP})$ demonstrated superior performance compared to $R(Z_H,Z_{DR})$ and $R(Z_H,Z_{DR},K_{DP})$ with KGE scores of approximately 0.8. For these algorithms, the assumption of embedding intense rainfall corresponding to high values of the variables of interest in the calibration samples through synthetic data simulated by copulas with upper-tail distributions plays a significant role. Given that higher values of the variables of interest are relatively rare or under-represented in real data, the usefulness of copula simulations is obvious, particularly for $R(Z_{DR},K_{DP})$, which is significantly influenced by the presence of high values. Rainfall estimates performed by the estimator based on extreme-value copulas are more accurate than those performed by the estimators based on T-matrix simulations. A comparison of the best parametric algorithms with their corresponding $R_{NN}(Z_H,K_{DP})$ and $R_{NN}(Z_{DR},K_{DP})$ derived from the neural network reveals that their copula-based design with tail distributions results in minimal performance differences in terms of KGE.

Conversely, the neural network-based algorithms demonstrated greater robustness than the parametric algorithms, particularly given that they were constructed on synthetic rather than observational data. The most striking result of our study shows that the neural network rainfall estimates method is virtually insensitive to raindrop size variability, given that their learning process using a synthetic dataset combines both the dependencies between the variables of interest and the influence of rainfall types that are built into the copula approach.

Although we have demonstrated the beneficial contribution of copulas and neural networks to radar rainfall estimation using multivariate estimators, the specific results of Frank, Gumbel and HRT copulas suggest potential avenues for enhancing synthetic data simulations using copulas that more accurately reflect the relationships between variables across all value distributions. In light of these considerations, it seems reasonable to posit that multi-parameter extreme value copulas, such as the Hüsler-Reiss copula or the Galambos copula families [66], may offer a more nuanced characterization of the dependencies between variables due to their multi-parametric nature. Another potential avenue for exploration in future work is the development of composite algorithms that leverage the capabilities of copulas in specific intensity classes. This approach could involve the integration of algorithms derived from Clayton's copula for light rainfall, Frank's copula for intermediate rainfall, and copulas of extreme values or upper tail distributions for heavy rainfall. In such a way, a comprehensive explanation should be developed to assess the contribution or the orders of importance of each radar observable used for model precipitation estimation according to rainfall types.

Author Contributions

Conceptualization, S.A.O. and E.-P.Z.; methodology, S.A.O. and E.-P.Z.; software, S.A.O, E.-P.Z., M.K., and K.A.K.; validation, A.D.O., E.-P.Z.; formal analysis, E.-P.Z., S.A.O., and M.K.; investigation, S.A.O. and E.-P.Z.; resources, S.A.O. and E.-P.Z.; data curation, M.K., and K.A.K.; writing—original draft preparation, E.-P.Z.; writing—review and editing, M.K., A.D.O, K.A.K.; visualization, M.K., A.D.O., K.A.K.; supervision, A.D.O. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement

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Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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