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ARTICLE

Empirical Wavelet Transform; Stationary and Nonstationary Signals

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ABSTRACT

Signal decomposition into the frequency components is one of the oldest challenges in the digital signal processing. In early nineteenth century, Fourier transform (FT) showed that any applicable signal can be decomposed by unlimited sinusoids. However, the relationship between time and frequency is lost under using FT. According to many researches for appropriate time-frequency representation, in early twentieth century, wavelet transform (WT) was proposed. WT is a well-known method which developed in order to decompose a signal into frequency components. In contrast with original WT which is not adaptive according to the input signal, empirical wavelet transform (EWT) was proposed. In this paper, the performance of discrete WT (DWT) and EWT in terms of signal decomposing into basic components are compared. For this purpose, a stationary signal including five sinusoids and ECG as biomedical and nonstationary signal are used. Due to being non-adaptive, DWT may remove signal components but EWT because of being adaptive is appropriate. EWT can also extract the baseline of ECG signal easier than DWT.

1. Introduction

Signal decomposition into the frequency components is one of the oldest challenges in the digital signal processing. In early nineteenth century, Fourier transform (FT) showed that any applicable signal can be decomposed by unlimited sinusoids. Moreover, the relationship between time and frequency is lost in FT. In order to overcome the mentioned problem, short time Fourier transform (STFT) was proposed, where a signal is windowed in time domain and the FT is individually computed for each window. Through this, the signal spectrum corresponding to every window is obtained separately. Although using STFT preserves the time-frequency relationship, and it is known as a time-frequency representation, increasing the width of used window is equivalent to decrease the time resolution [1]. Since basis functions of both FT and the STFT are in exponential form, under no similarity between the signal and the exponential element function, the resultant frequency spectrum cannot offer an appropriate representation about the signal frequency components. In early twentieth century, according to many researches for appropriate time-frequency representation, wavelet transform (WT) was proposed [2]. As, the mother wavelet is not necessarily exponential, it can be used for time-frequency analysis of those signals which are not combinations of exponential functions. The first basis function proposed for the WT called Haar [3], different basis functions as Little-Paley [4], Meyer [5], and Daubechies [1] were proposed.

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Although, many advantages of WT as a time-frequency decomposition method is known, the bottle neck of using wavelet is nonadaptivity to the input signal. So empirical mode decomposition (EMD) which operates adaptively according to the input signal was proposed [6]. In general, EMD decomposes a signal to different intrinsic mode functions (IMFs). It is a reversible operation that means sum of obtained IMFs and the residual signal synthesize the original input signal. Although, EMD algorithm has primarily been considered in several signal processing applications, lack of closed-form mathematical expression, time consuming, and also sensitivity to the noise are always known as its limitation factors.

In 2013, an approach called empirical wavelet transform (EWT) was proposed to overcome the mentioned drawbacks [7]. EWT is adaptive similar to EMD but instead of EMD, it is not noise sensitive. Also, having a mathematical expression, capable EWT to analyze signals faster than EMD. Comparison among EMD, EWT and discrete WT (DWT) as a well-known non adaptive time-frequency signal representation were reported [8-10]. In this paper, as a case study of processing a stationary signal and also ECG as a nonstationary signal, we compare the performance of the DWT and EWT as well.

The paper is organized as follows. In Section 2, the theory of original WT and EWT are explained. Then in Section 3, two signals are decomposed by EWT and DWT. Finally, both decomposition algorithms are evaluated. The paper conclusion is given in Section 4.

2. WT And EWT

In general, WT by using the filter bank decomposes a signal into specified frequency sub-bands. The cut-off frequency of the filter bank at the first and the second decomposition level, are $\pi/2$ and $\pi/4$ in order, so it is $\pi/2^n$ at the n[th] decomposing level. In other word, for the n[th] decomposition level, the bandwidth of low-pass filter is $[0, \pi/2^n]$ and the bandwidth of high-pass filter is [$\pi/2^n$ false, $\pi/2^{n-1}$]. Two functions called Φ as scaling function (SF) and Ψ as wavelet function (WF) have key roles in signal decomposition,

$$\Phi(t) = \begin{cases} 1 & \text{f} \quad 0 \le t < 1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$\Psi(t) = \begin{cases} 1 & \text{f} \quad 0 \le t < 1/2 \\ -1 & \text{f} \quad 1/2 \le t < 1 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

For every decomposition level, the signal projection with low-pass filter and high-pass filter are called approximation and detail. However, the cut-off frequencies in WT for all decomposition levels are constant that means the WT is not adaptive to the input signal. In contrast, for EWT, the filter bank cut-off frequencies are not constant and vary according to the input signal components [7]. Using FT, the frequency spectrum of the input signal is obtained in $[0, \pi]$ and the local maxima of frequency spectrum are marked, and then midpoints of every pair maximum are used as the filter bank cut-off frequency. It should be noted that the number of required local maxima depends on the number of decomposition levels. In other words n largest local maximums are required for n decomposition levels, also the first cut-off frequency falls between zero and the maximum at the lowest frequency. After specifying the cut-off frequencies, the filter bank is formed according to the idea of Littlewood–Paley and Meyers wavelets [11]. For EWT, the SF and the WF functions are defined in Fourier domain as [7],

$$\phi(\omega_f) = \begin{cases} 1 & \text{if} |\omega_f| \le (1\text{-}\lambda)\,\omega_1 \\ \cos(\frac{\pi\beta(\lambda,\omega_1)}{2}) & \text{if } (1\text{-}\lambda)\,\omega_1 \le |\omega_f| \le (1+\lambda)\,\omega_1 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

$$\psi_{i=2,..,n}(\omega_f) = \begin{cases} 1 & \text{if } (1+\lambda)\,\omega_i \le |\omega_f| \le (1\text{-}\lambda)\,\omega_{i+1} \\ \cos(\frac{\pi\beta(\lambda,\omega_{i+1})}{2}) & \text{if } (1-\lambda)\,\omega_{i+1} \le |\omega_f| \le (1\text{-}\lambda)\,\omega_{i+1} \\ \sin(\frac{\pi\beta(\lambda,\omega_i)}{2}) & \text{if } (1+\lambda)\,\omega_i \le |\omega_f| \le (1+\lambda)\,\omega_i \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where,

$$\beta(\lambda,\omega_i) = \beta(\frac{|\omega_f| - (1-\lambda)\omega_i}{2\lambda\omega_i}) \tag{5}$$

where $\omega_{i=1,2,..n} = \{f_{cut_1}, f_{cut_2}, ..., f_{cut_n}\}$ and ë < $\min(\frac{\omega_{i+1} - \omega_i}{\omega_{i-1} + \omega_i})$ which make sure the EWT coefficient are in $L^2(\Re)$ space, and $\beta(y)$ is,

$$\beta(y) = \begin{cases} 0 & \text{if } y \le 0 \\ \beta(y) + \beta(1-y) = 1 & \forall\ y \in [0,1] \\ 1 & \text{if } y \ge 1 \end{cases} \tag{6}$$

Similar to WT, approximation and detail coefficients are obtained by using the inner product between the input signal and SF and WF, respectively.

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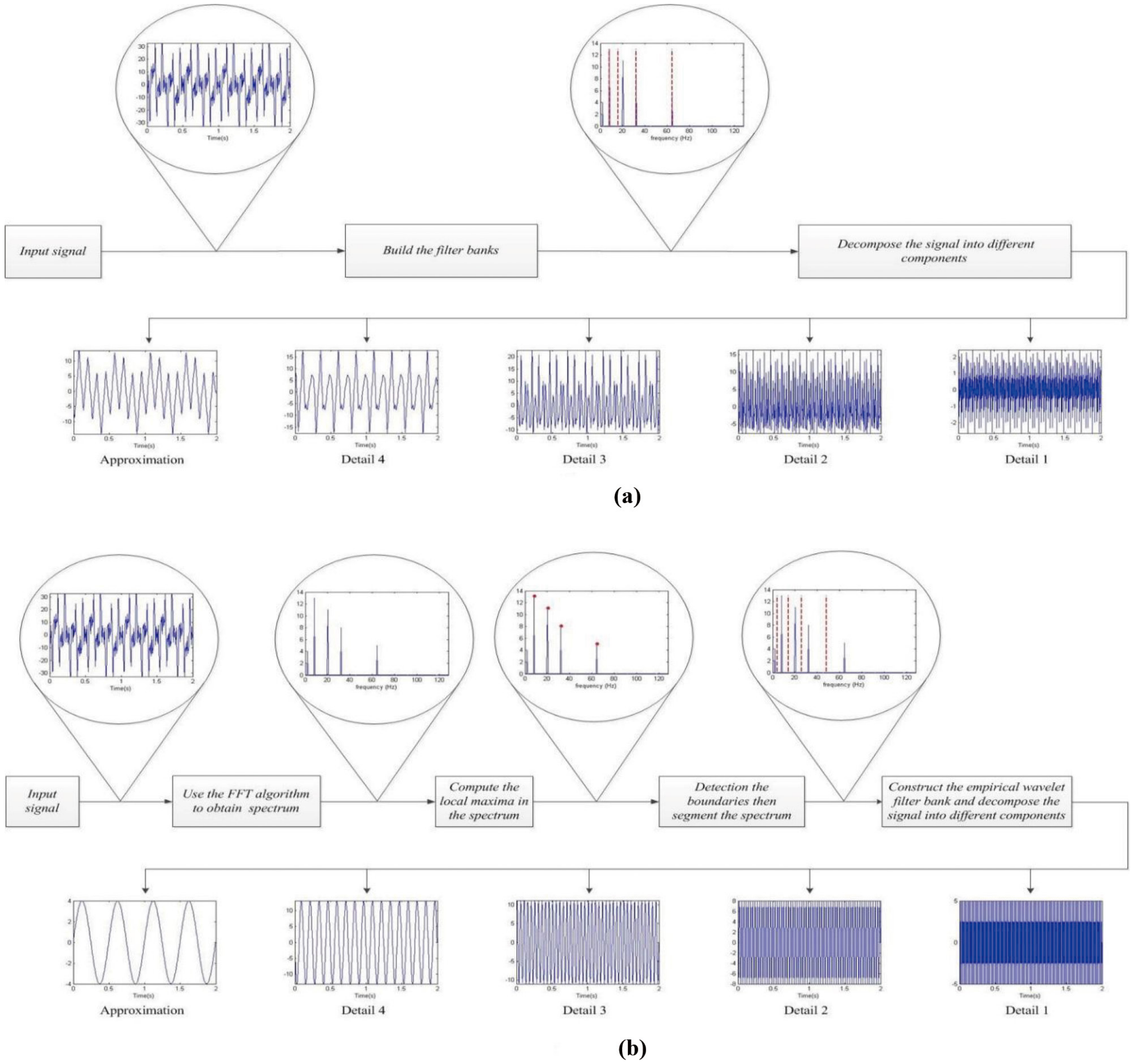


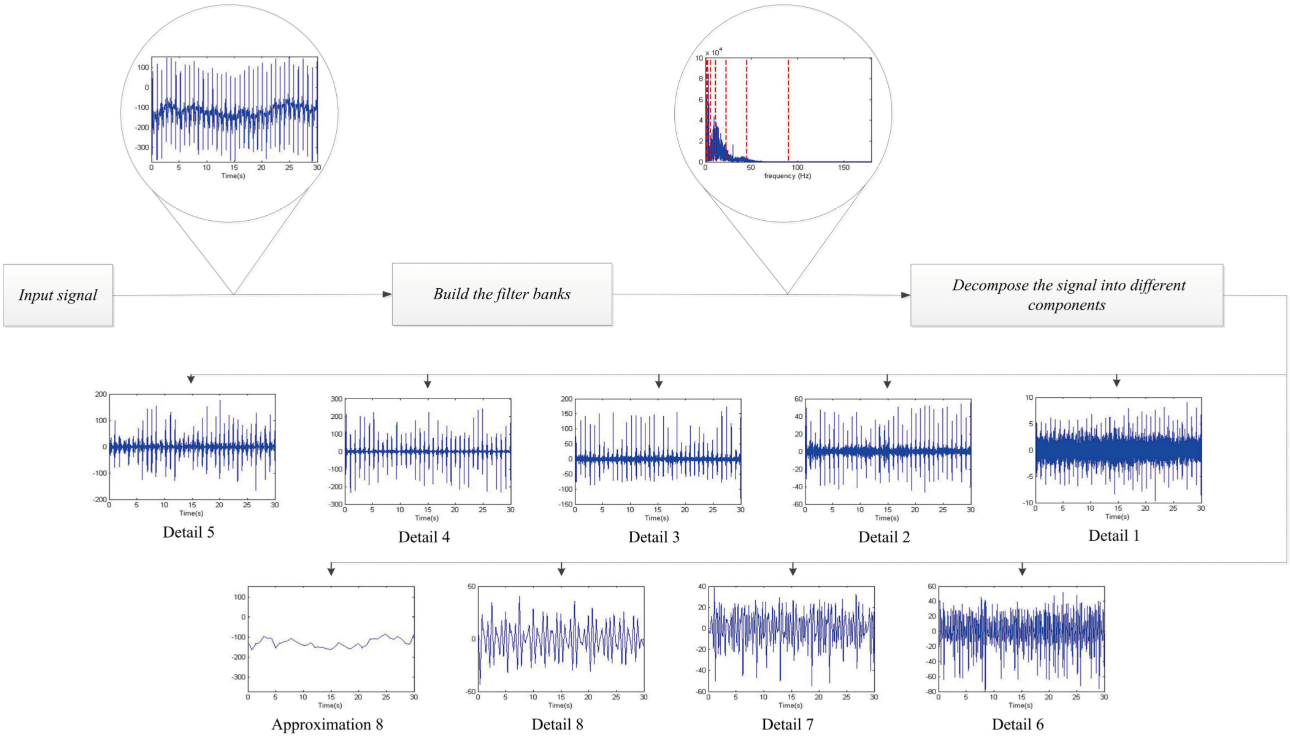
Figure 1. Decomposing the signal x(t), Eq(7), by (a) WT, (b) EWT

3. Simulation Result

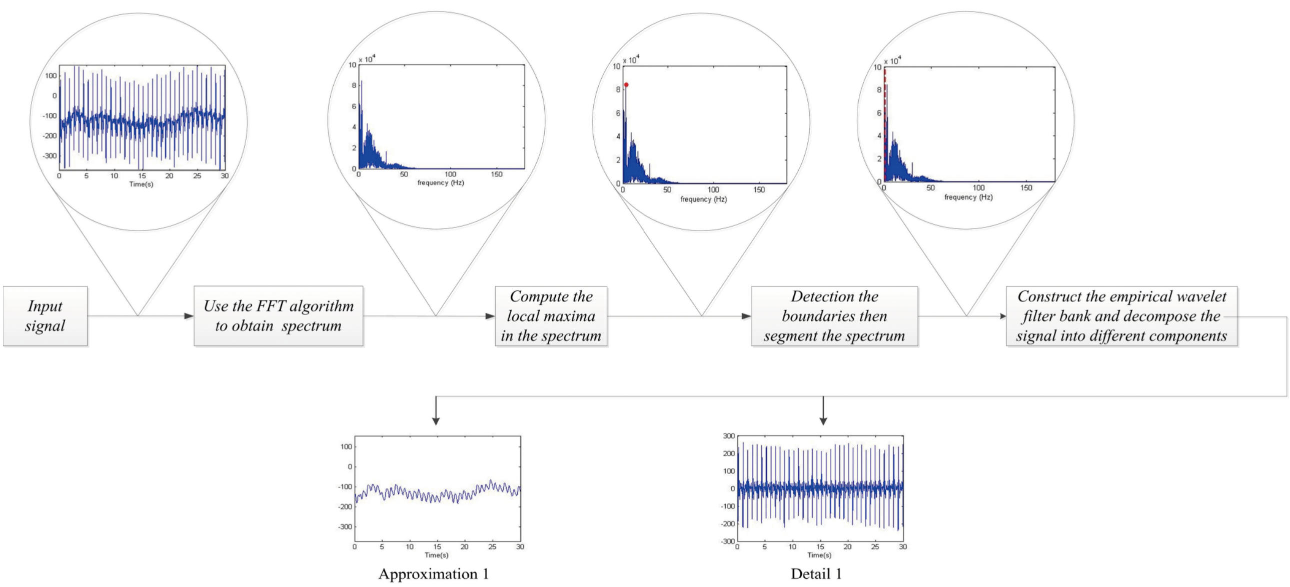
In order to demonstrate the capability of EWT, in comparison with DWT based on Dubeches, a stationary signal and ECG signal are used. The first signal $x(t)$ is an stationary and multicomponent consists of five sinusoids with different amplitudes and frequencies as,

$$x(t) = x_1(t) + x_2(t) + x_3(t) + x_4(t) + x_5(t) \tag{7}$$

where $x_1(t) = 4\sin(4\pi\ t)$, $x_2(t) = -13\sin(16\pi\ t)$, $x_3(t) = 11\sin(40\pi\ t)$, $x_4(t) = -8\sin(64\pi\ t)$ and $x_5(t) = 5\sin(128\pi t)$. According to the frequency components of signal $x(t)$, the sampling frequency is considered 256 Hz. The signal $x(t)$ is decomposed by wavelet with 4 decomposition levels and EWT considering 5 subbands, see Figure1. As mentioned before and observed in Figure1-a, the bandwidth of filter banks for WT are fixed; that means if any frequency component of the input signal lay on the cut-off frequency of filter bank, it is removed. For the signal $x(t)$, it happened for the second, fourth and fifth components with frequencies equal 8, 32, and 64 Hz. As shown in Figure1-b, signal decomposition by EWT, at first the frequency spectrum of $x(t)$ is obtained in [0,], then local maximums are specified, and accordingly the



(a)



(b)

Figure 2. Decomposing ECG signal and extracting the baseline by (a) WT, (b) EWT

cut-off frequencies of filter bank are determined. It should be noted that the first cut-off frequency lies between zero and the first maximum with the lowest frequency. All signals in nature have higher amplitude in lower frequencies compared to high frequencies, in addition a large amount of information exists in lower frequencies where high frequencies include noise. According to the explained EWT methodology, the most of EWT sub bands are chosen in low frequencies

In WT, as the sampling frequency increases, the number of decomposition level is increased in order to be capable of investigating the low frequency components

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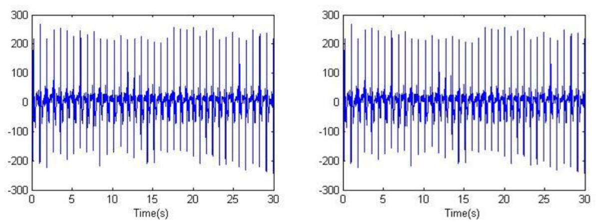
accurately. The EWT decomposes the signal regarding to local maximums of the frequency spectrum which usually exist in low frequency bands, therefore the sampling frequency increment does not directly affect the number of EWT decomposition levels. In other words, the EWT can investigate low frequency components of the signal with few decomposition levels compared to the WT. In order to better understand the issue, ECG signal with the baseline noise existing in the data base MIT-BIH [12] is considered. Generally, the baseline noise has a frequency lower than 0.7 Hz [13]. The ECG signal sampled with 360 Hz and investigated during 30 seconds. According to Figure2-a, in the WT, 8 levels are required in order to extract the baseline. According to Figure2-b, employing the EWT, the baseline noise is extracted only by one decomposing level. Generally by removing the baseline noise (approximation), the clean ECG signal is achieved. Figure3 shows the clean ECG where the baseline noise extracted by EWT and WT as well.



(a) **(b)**

Figure 3. Removing baseline noise and showing the clean ECG signal by (a) WT, (b) EWT

4. Conclusion

In this paper, the performance of WT and EWT as signal decomposition methods are compared. Due to the fixed filter bank cut-off frequency in WT, some signal components may remove. Although, the frequency sampling effects on the number of decomposition level in WT, for EWT selected local maximum obligate the number of decomposition levels. So, the baseline in ECG is extracted by EWT with only one level decomposition in compared with WT which requires 8 decomposition levels. It seems that EWT can extract low frequency components by less levels compared to WT. Anyway, based on EWT methodology and the simulation results, it is recommended for multi-component signals modeled as but it is not advise for linear frequency modulation or chirp signal with , where the instantaneous frequency is increasing or decreasing linearly in time domain.

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