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ARTICLE

Three Median Relations of Target Azimuth in one Dimensional Equidistant Double Array

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ABSTRACT

On the basis of the linear positioning solution of one-dimensional equidistant double-base linear array, by proper approximate treatment of the strict solution, and by using the direction finding solution of single base path difference, the sinusoidal median relation of azimuth angle at three stations of the linear array is obtained. By using the sinusoidal median relation, the arithmetic mean solution of azimuth angle at three stations is obtained. All these results reveal the intrinsic correlation between the azimuth angles of one-dimensional linear array.

1. Introduction

If based on strict mathematical derivation, there will be a tangent median relationship between the three sites of one-dimensional equal-spaced line array [1]. Strictly speaking, the tangent median relation may not be the original result given in the literature [1], because the author seems to have seen the same or similar results in a paper, but the provenance was not immediately available.

In this paper, based on the linear positioning solution of one-dimensional equidistant double-base linear array [1], the sinusoidal median relationship of azimuth angle at three stations of the array is given by proper approximate processing of the strict linear solution and by using the direction finding solution of single-base path difference. By using the sinusoidal median relation, the arithmetic mean solution of azimuth angle at three stations is obtained. For the convenience of reading and correction of the original errors, the derivation process of linear positioning solution of one-dimensional equidistant double base array and the derivation process of tangent median relation are given in this paper.

The results of this paper show that there are not only strict tangent median relation, but also approximate sinusoidal median relation and approximate azimuth angle median relation. Up to now, most of the researches on passive positioning systems only arrange mathematical equations from the perspective of solving unknown quantities, but do not study the internal correlation among various parameters in depth. In fact, one-dimensional linear arrays

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have many interesting intrinsic properties. In order to dig out these intrinsic characteristics, it is necessary to carry out relevant deduction, and further deduction needs to rely on the known intrinsic characteristics. The analysis of this paper is not only to explore the intrinsic characteristics, but also to provide the intrinsic characteristics for further reasoning.

2. Linear Solution of One-dimensional Equidistant Double-base Linear Array

Literature [1] gives the derivation process of the linear positioning solution based on the array midpoint, and gives the basic results of the linear positioning solution based on the left and right end of the array. There are some errors due to careless proofreading for the basic results of the linear positioning solutions based on the left and right ends of the array. Based on the needs of derivation in this paper and in order to correct the errors in literature [1], this section gives the derivation results of linear positioning solutions at all sites.

From the perspective of engineering design, the linear solution of one-dimensional equidistant double-base linear array can directly provide an accurate direction finding calculation method for the phase interference array [2], thus providing a more correct analysis result for the design of measurement error of phase interferometer. In fact, for a long period of time, people can only use the approximate phase interference direction finding formula. And although the analysis is known to be only approximate, it seems difficult to give a mathematical representation that is both relatively accurate and relatively simple.

2.1 Take the Midpoint of the Array as the Reference Point

For the one-dimensional double-base array shown in figure 1, the path difference between the two adjacent baselines is:

$$\Delta r_{12} = r_1 - r_2 \tag{1}$$

$$\Delta r_{23} = r_2 - r_3 \tag{2}$$

If the midpoint of the entire array is taken as the coordinate origin, the following two geometric auxiliary equations can be listed by the cosine theorem:

$$\begin{aligned} r_1^2 &= r_2^2 + d^2 - 2r_2 d\cos\left(90^0 + \theta_2\right) \\ &= r_2^2 + d^2 + 2r_2 d\sin\theta_2 \end{aligned} \tag{3}$$

$$\begin{aligned} r_3^2 &= r_2^2 + d^2 - 2r_2 d\cos\left(90^0 - \theta_2\right) \\ &= r_2^2 + d^2 - 2r_2 d\sin\theta_2 \end{aligned} \tag{4}$$

Because of $x = r_2\sin\theta_2$, the geometric auxiliary equation can be rewritten as:

$$r_1^2 = r_2^2 + d^2 + 2d\cdot x \tag{5}$$

$$r_3^2 = r_2^2 + d^2 - 2d\cdot x \tag{6}$$

Where: d is the length of a single baseline; x the x-coordinate of the rectangular coordinate system.

At this point, if equation (1) and (2) of the path difference between two adjacent baselines are substituted into geometric auxiliary equations (5) and (6), the following binary linear equations can be obtained after the term transition:

$$2d\cdot x - 2\Delta r_{12} r_2 = -d^2 + \Delta r_{12}^2 \tag{7}$$

$$2d\cdot x - 2\Delta r_{23} r_2 = d^2 - \Delta r_{23}^2 \tag{8}$$

From this, the transverse distance of the target can be directly solved:

$$x = \frac{\left(d^2 - \Delta r_{12}^2\right)\Delta r_{23} + \left(d^2 - \Delta r_{23}^2\right)\Delta r_{12}}{2d\left(\Delta r_{12} - \Delta r_{23}\right)} \tag{9}$$

And the radial distance of the target:

$$r_2 = \frac{2d^2 - \Delta r_{12}^2 - \Delta r_{23}^2}{2\left(\Delta r_{12} - \Delta r_{23}\right)} \tag{10}$$

Thus, the arrival angle of the target can be obtained:

$$\sin\theta_2 = \frac{x}{r_2} = \frac{\left(d^2 - \Delta r_{12}^2\right)\Delta r_{23} + \left(d^2 - \Delta r_{23}^2\right)\Delta r_{12}}{d\left(2d^2 - \Delta r_{12}^2 - \Delta r_{23}^2\right)} \tag{11}$$

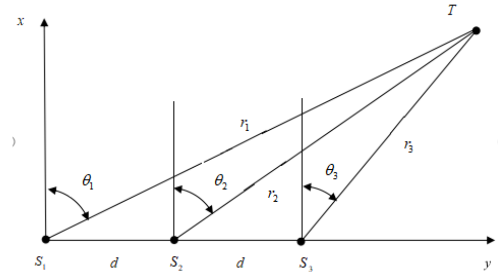

Figure 1. One-dimensional double base linear array

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2.2 Take the Left End of the Array as the Reference

Since the distance and orientation between the target and the leftmost site of the detection array are solved, the positioning equation is established by taking the leftmost position of the array as the coordinate origin. At this time, the geometric auxiliary equation listed by the cosine theorem is:

$$r_2^2 = r_1^2 + d^2 - 2r_1 d\cos(90 - \theta_1) \tag{12}$$

$$r_3^2 = r_1^2 + 4d^2 - 4r_1 d\cos(90 - \theta_1) \tag{13}$$

The path difference equation adopted should be:

$$\Delta r_{12} = r_1 - r_2 \tag{14}$$

$$\Delta r_{13} = r_1 - r_3 \tag{15}$$

The above two equations are substituted into equations (12) and (13) respectively, and the following binary linear equations are obtained after the term transfer:

$$2d \cdot x - 2\Delta r_{12} r_1 = d^2 - \Delta r_{12}^2 \tag{16}$$

$$4d \cdot x - 2\Delta r_{13} r_1 = 4d^2 - \Delta r_{13}^2 \tag{17}$$

From this, we can directly solve:

$$x = \frac{(4d^2 - \Delta r_{13}^2)\Delta r_{12} - (d^2 - \Delta r_{12}^2)\Delta r_{13}}{2d(2\Delta r_{12} - \Delta r_{13})} \tag{18}$$

$$r_1 = \frac{2d^2 + 2\Delta r_{12}^2 - \Delta r_{13}^2}{2(2\Delta r_{12} - \Delta r_{13})} \tag{19}$$

From the target position parameter obtained, the target arrival angle at the left end of the array can be obtained:

$$\sin\theta_1 = \frac{(4d^2 - \Delta r_{13}^2)\Delta r_{12} - (d^2 - \Delta r_{12}^2)\Delta r_{13}}{d(2d^2 + 2\Delta r_{12}^2 - \Delta r_{13}^2)} \tag{20}$$

2.3 Take the Right End of the Array as the Reference

To solve the distance and azimuth between the target and the site at the rightmost end of the detection array, the positioning equation is established by taking the position at the rightmost end of the array as the coordinate origin. At this time, the geometric auxiliary equation listed by the cosine theorem is:

$$r_1^2 = r_3^2 + 4d^2 - 4r_3 d\cos(90 + \theta_3) \tag{21}$$

$$r_2^2 = r_3^2 + d^2 - 2r_3 d\cos(90 + \theta_3) \tag{22}$$

The path difference equation adopted should be:

$$\Delta r_{23} = r_2 - r_3 \tag{23}$$

$$\Delta r_{13} = r_1 - r_3 \tag{24}$$

By substituting the above two equations into the geometric auxiliary equations (21) and (22), the following binary linear equations can be obtained after the term transfer:

$$4d \cdot x - 2\Delta r_{13} r_3 = \Delta r_{13}^2 - 4d^2 \tag{25}$$

$$2d \cdot x - 2\Delta r_{23} r_3 = \Delta r_{23}^2 - d^2 \tag{26}$$

From this, we can directly solve:

$$x = \frac{(4d^2 - \Delta r_{13}^2)\Delta r_{23} - (d^2 - \Delta r_{23}^2)\Delta r_{13}}{2d(\Delta r_{13} - 2\Delta r_{23})} \tag{27}$$

$$r_3 = \frac{2d^2 + 2\Delta r_{23}^2 - \Delta r_{13}^2}{2(\Delta r_{13} - 2\Delta r_{23})} \tag{28}$$

$$\sin\theta_3 = \frac{(4d^2 - \Delta r_{13}^2)\Delta r_{23} - (d^2 - \Delta r_{23}^2)\Delta r_{13}}{d(2d^2 + 2\Delta r_{23}^2 - \Delta r_{13}^2)} \tag{29}$$

3. Three Median Relationships

3.1 The Direction Finding Solution of Single base Path Difference Obtained by Approximate Simplification

For the direction-finding equation (11) derived by taking the midpoint of the whole array as the coordinate origin, if the higher-order terms of the path difference contained in the equation are simplified to be approximately equal $\Delta r_{12}^2 \approx \Delta r_{23}^2$, then:

$$\sin\theta \approx \frac{(d^2 - \Delta r_2^2)(\Delta r_1 + \Delta r_2)}{2d(d^2 - \Delta r_1^2)} = \frac{(\Delta r_1 + \Delta r_2)}{2d} \tag{30}$$

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Due to:

$$\Delta r_{13} = r_1 - r_3 = (r_1 - r_2) + (r_2 - r_3) = \Delta r_1 + \Delta r_2 \quad (31)$$

Therefore, there is a single-base direction finding solution:

$$\sin\theta = \frac{\Delta r_{13}}{2d} \quad (32)$$

It is verified by simulation that the reference datum of single base path difference direction finding is at the midpoint of the whole baseline length. As far as the expression form of the obtained single-base direction finding formula is concerned, the analysis result given by the author seems to be only a shift correction to the measurement datum of the existing approximate direction finding formula. And it is through this simple shift that the single - base direction finding formula is obtained.

The practical engineering significance of this result is that it provides a method for direction finding with long baseline time difference [3]. Theoretically, the exact solution can be obtained based on multi-station time difference measurement, but in the actual engineering design process, multi-station is often difficult to be located on a straight line. In this way, it is necessary to consider the direction finding problem of non-straight line, and the measurement error of angle between baselines will be generated. According to the existing analysis [4], the included angle error has a relatively large impact on the positioning accuracy. Therefore, two-station high precision direction finding is more worthy of consideration.

3.2 Tangent Median Relationship

For the one-dimensional equidistant double-base array shown in figure 1, the radial distance of the three stations is projected onto the x axis and y axis respectively, and the following identities can be obtained:

$$r_2 \sin\theta_2 = r_1 \sin\theta_1 - d \quad (33)$$

$$r_2 \sin\theta_2 = r_3 \sin\theta_3 + d \quad (34)$$

$$r_2 \cos\theta_2 = r_1 \cos\theta_1 \quad (35)$$

$$r_2 \cos\theta_2 = r_3 \cos\theta_3 \quad (36)$$

If equations (33) and (34) are added, we can get:

$$2r_2 \sin\theta_2 = r_1 \sin\theta_1 + r_3 \sin\theta_3 \quad (37)$$

Then, substitute equation (35) and equation (36) into the above equation to eliminate the radial distance r_1 and r_3, and get:

$$2r_2 \sin\theta_2 = \frac{r_2 \sin\theta_1 \cos\theta_2}{\cos\theta_1} + \frac{r_2 \sin\theta_3 \cos\theta_2}{\cos\theta_3} \quad (38)$$

From this, it can be concluded that there exists the following tangent median relationship among the arrival angles of three sites:

$$2tg\theta_2 = tg\theta_1 + tg\theta_3 \quad (39)$$

3.3 Sum of Sine Angles at the Two Endpoints of the Double Array

Add the direction-finding equations (20) and (29) at the left and right endpoints of the array:

$$\begin{aligned}\sin\theta_1 + \sin\theta_3 = &\frac{\left(4d^2 - \Delta r_{13}^2\right)\Delta r_{12} - \left(d^2 - \Delta r_{12}^2\right)\Delta r_{13}}{d\left(2d^2 + 2\Delta r_{12}^2 - \Delta r_{13}^2\right)} \\ &+ \frac{\left(4d^2 - \Delta r_{13}^2\right)\Delta r_{23} - \left(d^2 - \Delta r_{23}^2\right)\Delta r_{13}}{d\left(2d^2 + 2\Delta r_{23}^2 - \Delta r_{13}^2\right)}\end{aligned} \quad (40)$$

Make approximate processing for the path difference of single base array:

$$\Delta r_{12} \approx 0.5\Delta r_{13} \quad (41)$$

$$\Delta r_{23} \approx 0.5\Delta r_{13} \quad (42)$$

After sorting, on the right side of the equation (40):

$$\text{The right side of the equation} \approx 2\frac{\Delta r_{13}}{2d} \quad (43)$$

Therefore, the formula (32) in the previous section is quoted as follows:

$$\sin\theta_1 + \sin\theta_3 \approx 2\sin\theta_2 \quad (44)$$

That is to say, the sinusoidal angles of the three sites approximately meet the arithmetic mean, and the obtained sinusoidal median relation is very similar to the tangent median relation.

3.4 Sum of Azimuth Angles at Two Endpoints of the Double Array

Use the trig formula, the sum of two sinusoidal angles of formula (20) and formula (29) can be converted into:

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$$\sin\theta_1 + \sin\theta_3 = 2\sin\left(\frac{\theta_1+\theta_3}{2}\right)\cos\left(\frac{\theta_1-\theta_3}{2}\right) \tag{45}$$

Using equation (37), there is an approximate relationship:

$$\sin\left(\frac{\theta_1+\theta_3}{2}\right)\cos\left(\frac{\Delta\theta_{13}}{2}\right) \approx \sin\theta_2 \tag{46}$$

For remote direction finding, since the intersection angle $\Delta\theta_{13}$ is small, it can be approximated that $\cos(0.5\Delta\theta_{13})\approx 1$, so there are:

$$\left(\frac{\theta_1+\theta_3}{2}\right) \approx \theta_2 \tag{47}$$

That is, the arithmetic mean of azimuth at the two endpoints is approximately equal to the azimuth of the midpoint of the baseline. Previous studies by the author have shown that, the deviation characteristics of the approximate direction-finding errors of two adjacent single-base arrays are exactly opposite [5]. If the arithmetic average of the approximate direction-finding results of two adjacent single-base arrays in one dimension is taken, the arithmetic average of the two direction-finding values can be used to get the result that the deviation of the direction-finding error is close to zero by using an offsetting method.

4. Conclusion

The tangent mean relation is derived strictly, while the sine mean relation and the arithmetic mean of azimuth are approximate results. The results of this paper can be understood from two different perspectives: On the one hand, it seems that there are many magical features about one-dimensional linear array. The azimuth at three stations not only satisfies the tangent median relation, but also approximately satisfies the sinusoidal median relation, and also approximately satisfies the azimuth angle median relation. In fact, in reference [1], the author also studies that there exists the properties of arithmetic series between adjacent processes.

On the other hand, it is actually strictly proved that the arithmetic mean of azimuth of two endpoints is not equal to the angle value of the intermediate site. In fact, existing research and analysis show that, in most analytical analysis, the positioning solution of one-dimensional array has a high requirement for the accuracy of calculation of direction-finding angle, and a few differences will make it difficult to get correct results. Therefore, the results of this paper undoubtedly have guiding significance for the subsequent analysis.

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