BILINGUAL PUBLISHING GROUP

Journal of Electronic & Information Systems
https://journals.bilpubgroup.com/index.php/jeis

SHORT COMMUNICATION

A Doppler Location Method Based on Virtual Path Difference

Yu Tao

China Academy of Management Science, Beijing, 101100, China

ABSTRACT

This paper presents a Doppler passive location method for moving targets with fixed single station using the Doppler frequency shift and time difference information. First, based on the relationship between frequency shift and path difference, the virtual path difference is calculated from the measured value of Doppler frequency shift by means of mean value correction. Then, under the assumption that the target is moving at a constant speed along a straight line, two coaxial virtual double base arrays are constructed by using the moving track of the moving target based on the method of fixed period time difference. On this basis, the moving distance of the moving target can be calculated by using the ratio relationship between the frequency difference and the radial distance between the two adjacent detection points in the middle of the array, and the linear solution of the two double base path difference positioning equations. At this point, the relative coordinate position of the moving target can be obtained by directly using the linear solution of the double base path difference positioning equation again.

Keywords: Fixed single station; Passive location; Doppler frequency; Doppler frequency change rate; Frequency shift-path difference equation; Virtual path difference

1. Introduction

In electronic reconnaissance, when the signal bandwidth is insufficient, resulting in the ambiguity of time difference, or the phase calibration and time synchronization are limited by the system complexity, the location technology based on Doppler frequency shift information is preferred for target position estimation [1-7]. The above defects can be remedied by using this technology, and the requirements for receiving equipment are low. In addition, with the rapid development of frequency measurement technology and the continuous improvement of frequency measurement accuracy, the

*CORRESPONDING AUTHOR:
Yu Tao, China Academy of Management Science, Beijing, 101100, China; Email: 18001648180@163.com

ARTICLE INFO
Received: 24 February 2023 | Revised: 29 March 2023 | Accepted: 31 March 2023 | Published Online: 11 April 2023
DOI: https://doi.org/10.30564/jeis.v5i1.5496

CITATION
Tao, Y., 2023. A Doppler Location Method Based on Virtual Path Difference. Journal of Electronic & Information Systems. 5(1): 20-25. DOI: https://doi.org/10.30564/jeis.v5i1.5496

passive location technology based on the Doppler frequency shift has great development potential. However, due to the higher complexity of the nonlinear equation formed by Doppler frequency shift and target state compared with other positioning methods, this will not only lead to difficulties in analyzing the mathematical model, but also lead to the complexity of system design in practical engineering, so that the original advantages may be lost at last. At the same time, because the positioning system based on Doppler measurement is a nonlinear system, it is usually necessary to study the observability of positioning [8-10].

The author recently proposed a method that can directly convert the nonlinear Doppler frequency shift function into a linear solution in the research of the Doppler passive location of fixed targets by the motion detection station [11]. The new method obtains the relationship between the Doppler frequency shift and the radial path difference by differential processing of the radial velocity, so that the virtual path difference can be constructed according to the detected value of the Doppler frequency shift, and then the position of the target can be directly obtained by using the linear solution of the double-base path difference positioning equation. The new method not only has unique results, but also has very simple forms. The analysis complexity of passive location based on the Doppler frequency shift is effectively reduced.

Different from the application scenario of document [11], which is a way to detect fixed targets using motion detection stations, this paper studies the detection of moving target by fixed single station. Its difficulty is that the moving speed or moving distance of moving target is unknown. Obviously, its analysis is relatively more difficult. In this regard, the innovative method proposed by the author is: On the one hand, two coaxial virtual dual-basis arrays are constructed by using the moving track of the moving target, on the other hand, the moving distance of the moving target is calculated by using the linear solution of the two dual-basis path difference positioning equations according to the ratio relationship between the frequency difference and the radial distance between the adjacent detection points.

2. Path difference-frequency shift equation

2.1 Differential processing

According to the relationship between Doppler frequency shift and radial range change rate:

$$\frac{\partial r(t)}{\partial t} = v_r = v\cos\beta = \lambda f_d \tag{1}$$

where $r(t)$ is the radial distance; v_r is the radial velocity; v is the moving speed of the target; f_d is the Doppler frequency shift; λ is the wavelength; β is the leading angle.

For radial velocity, assuming that the change of time is short, the differential of distance to time can be converted into the ratio of path difference and time difference by using the difference calculation method:

$$\frac{\partial r(t)}{\partial t} \approx \frac{\Delta r}{\Delta t} \tag{2}$$

where Δr is the path difference; Δt is the time difference.

The geometric model corresponding to the mathematical model is shown in **Figure 1**. Substitute formula (2) for formula (1) to obtain the virtual path difference expression based on Doppler frequency shift measurement:

$$\Delta r = \lambda f_d \Delta t \tag{3}$$

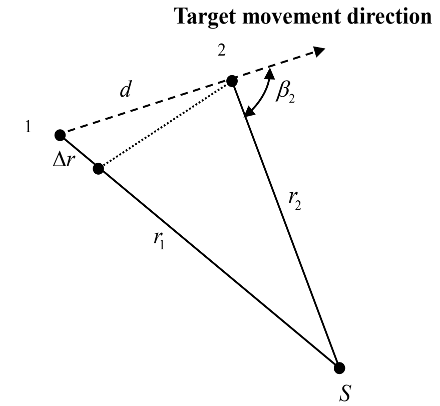

Figure 1. Geometric model of single motion station.

2.2 Mean value correction

If the Doppler frequency shift measurement value of the moving target at position 1 is used, the expression of virtual path difference is:

$$\Delta r_{f1} = \lambda f_{d1} \Delta t \quad (4)$$

If the theoretical value of path difference is compared with the virtual path difference obtained based on frequency shift measurement:

$$\varepsilon = 100 \times \frac{\left| \Delta r - \Delta r_f \right|}{\Delta r} \% \quad (5)$$

where $\Delta r = r_1 - r_2$ is the theoretical value of path difference; Δr_f is the virtual path difference calculated according to equation (3).

The simulation calculation shows that the virtual path difference obtained has a large calculation error. However, further simulation calculation shows that if detection is also carried out at position 2 according to the geometric model in **Figure 1**, the Doppler frequency shift is obtained and used to calculate the path difference from position 1 to position 2, the virtual path difference obtained is:

$$\Delta r_{f2} = \lambda f_{d2} \Delta t \quad (6)$$

The subsequent simulation calculation shows that its calculation error is exactly opposite to the calculation error obtained by using the Doppler frequency shift at position 1. The upper and lower curves in **Figure 2** describe the relative calculation error curves of the virtual path difference obtained based on the Doppler frequency shift measurements at two locations.

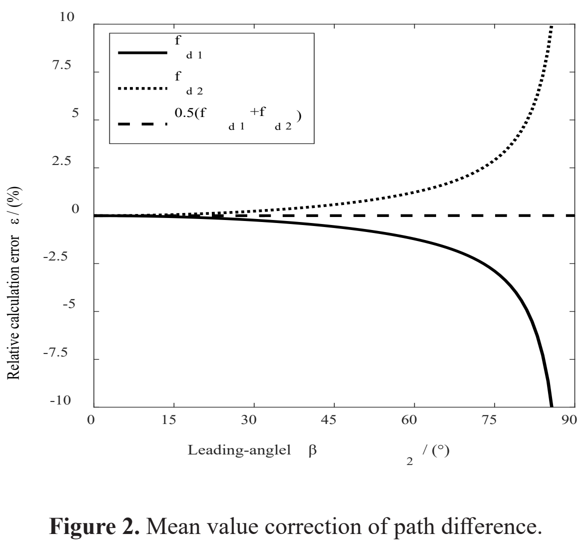

Figure 2. Mean value correction of path difference.

On this basis, if the average value of Doppler frequency shifts at two positions is used to calculate the virtual path difference:

$$\Delta r = 0.5\lambda\left(f_{d1} + f_{d2}\right)\Delta t \quad (7)$$

Then the calculation error is just offset. The middle curve in **Figure 2** shows that the relative calculation error of the virtual path difference obtained based on the Doppler average method tends to zero.

The parameters used in the simulation calculation are: $r_2 = 600\ km,\ d = 10\ km$.

3. Detection model

As shown in **Figure 3**, the target moves approximately at a uniform speed in a straight line, from position 1, through positions 2 and 3, to position 4. The fixed single station detects and receives the radiation signal from the target at a fixed time. If the fixed station detects four times continuously, then according to the linear solution of the double base array, two coaxial virtual double base arrays can be constructed simultaneously from the target's motion trajectory.

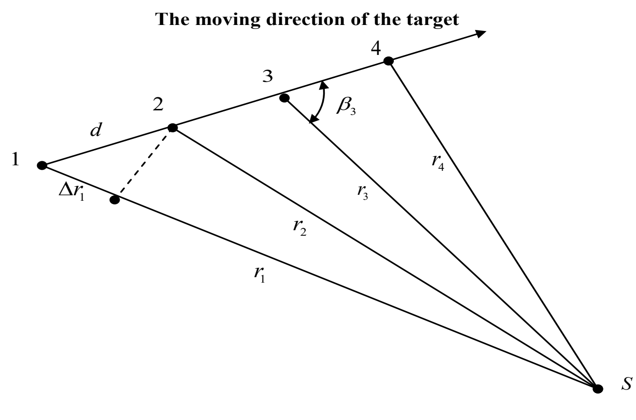

Figure 3. Geometric structure of virtual double-base array.

Based on the Doppler frequency shift measurement, three virtual path differences can be obtained:

$$\Delta r_1 = 0.5\lambda\left(f_{d1} + f_{d2}\right)\Delta t \quad (8)$$

$$\Delta r_2 = 0.5\lambda\left(f_{d2} + f_{d3}\right)\Delta t \quad (9)$$

$$\Delta r_3 = 0.5\lambda\left(f_{d3} + f_{d4}\right)\Delta t \quad (10)$$

Then, two ranging solutions are directly given by the double base path difference ranging solution [12]:

$$r_2 = \frac{2d^2 - \Delta r_1^2 - \Delta r_2^2}{2(\Delta r_1 - \Delta r_2)} \tag{11}$$

$$r_3 = \frac{2d^2 - \Delta r_2^2 - \Delta r_3^2}{2(\Delta r_2 - \Delta r_3)} \tag{12}$$

4. Moving distance of target

4.1 Ratio of Doppler change rate of adjacent nodes

According to the definition of Doppler frequency shift change rate and the target motion trajectory, the following two equations can be listed at two adjacent detection points in the middle of the virtual array:

$$\dot{f}_{d2} = \frac{v_{t2}^2}{\lambda r_2} \tag{13}$$

$$\dot{f}_{d3} = \frac{v_{t3}^2}{\lambda r_3} \tag{14}$$

where $\dot{f}_{d2}$ and $\dot{f}_{d3}$ are Doppler change rates; λ is the wavelength. r_2 and r_3 are radial distances; v_{t2} and v_{t3} are tangential velocities.

The ratio of the Doppler frequency shift change rate between these two adjacent detection points is:

$$\frac{\dot{f}_{d3}}{\dot{f}_{d2}} = \frac{r_2}{r_3}\frac{v_{t3}^2}{v_{t2}^2} \tag{15}$$

According to the sine theorem, the ratio between the radial distances of the two adjacent detection points is:

$$\frac{r_2}{r_3} = \frac{\sin\beta_3}{\sin\beta_2} = \frac{v\sin\beta_3}{v\sin\beta_2} = \frac{v_{t3}}{v_{t2}} \tag{16}$$

where β_1 and β_2 are leading angles.

Replace it with formula (15) to get:

$$\frac{\dot{f}_{d3}}{\dot{f}_{d2}} = \frac{r_2^3}{r_3^3} \tag{17}$$

4.2 Frequency difference ratio

Use differential processing:

$$\dot{f}_d = \frac{\Delta f_d}{\Delta t} \tag{18}$$

where Δf_d is the Doppler frequency difference.

Moreover, due to the constant range or periodic detection, the adjacent time difference is nearly equal, so the ratio of Doppler change rate between adjacent nodes can be approximately expressed by the ratio of Doppler frequency difference:

$$q = \frac{\dot{f}_{d3}}{\dot{f}_{d2}} \approx \frac{\Delta f_{d32}}{\Delta f_{d21}} = \frac{f_{d3} - f_{d2}}{f_{d2} - f_{d1}} \tag{19}$$

where f_{d1}, f_{d2} and f_{d3} are Doppler frequency shifts.

4.3 Solution of the equation

Substitute the two ranging solutions (11) and (12) into the frequency difference ratio:

$$\sqrt[3]{q} = \frac{(2d^2 - \Delta r_1^2 - \Delta r_2^2)}{(\Delta r_1 - \Delta r_2)}\frac{(\Delta r_2 - \Delta r_3)}{(2d^2 - \Delta r_2^2 - \Delta r_3^2)} \tag{20}$$

After sorting, there are:

$$\sqrt[3]{q}\frac{(\Delta r_1 - \Delta r_2)}{(\Delta r_2 - \Delta r_3)}(2d^2 - \Delta r_2^2 - \Delta r_3^2) = (2d^2 - \Delta r_1^2 - \Delta r_2^2) \tag{21}$$

Finally, it can be solved as follows:

$$d = \sqrt{\frac{(\Delta r_1^2 + \Delta r_2^2) - A(\Delta r_2^2 + \Delta r_3^2)}{2(1 - A)}} \tag{22}$$

including:

$$A = \sqrt[3]{q}\frac{(\Delta r_1 - \Delta r_2)}{(\Delta r_2 - \Delta r_3)}$$

5. Location of the target

5.1 Distance

After the moving distance of the moving target is obtained, the distance between the detection station and the moving target can be obtained from the double base path difference solution again:

$$r_2 = \frac{2d^2 - \Delta r_1^2 - \Delta r_2^2}{2(\Delta r_1 - \Delta r_2)} \tag{23}$$

Figure 4 shows the relative calculation error of the ranging solution at a different radial distances and different motion distances. It can be seen that the longer the radial distance, the shorter the moving distance of the target, and the smaller the relative calculation error. Divergence will occur when it approaches 90 degrees.

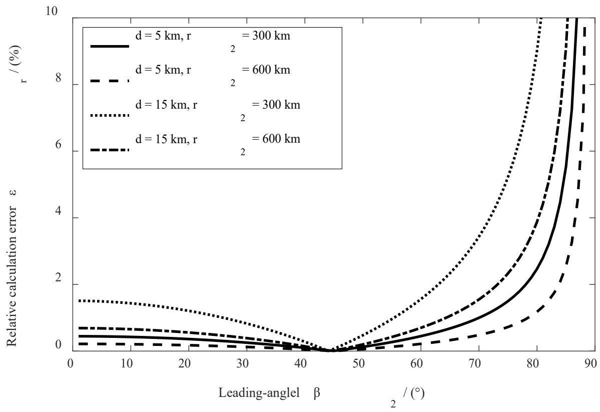

Figure 4. Relative calculation error of ranging solution.

5.2 Orientation of the target

Using two virtual path differences, the included angle between the moving direction of the target and the radial distance can be directly obtained from the double base path difference DF solution [12]:

$$\cos\beta_2 = \frac{\left(d^2 - \Delta r_1^2\right)\Delta r_2 + \left(d^2 - \Delta r_2^2\right)\Delta r_1}{d\left(2d^2 - \Delta r_1^2 - \Delta r_2^2\right)} \tag{24}$$

Figure 5 shows the relative calculation error of the DF solution at different radial distances and different motion distances.

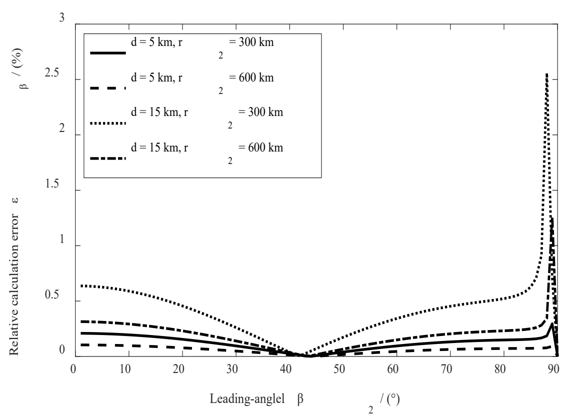

Figure 5. Relative calculation error of DF solution.

5.3 Target's movement speed

After the azimuth angle is obtained, the moving speed of the target can be directly solved by Doppler frequency shift:

$$v = \frac{\lambda f_{d2}}{\cos\beta_2} \tag{25}$$

where v is the moving speed of the target.

Or the moving speed of the target in the detection time can be approximately obtained from the moving distance and detection time of the target:

$$v = \frac{1}{\Delta t}\sqrt{\frac{\left(\Delta r_1^2 + \Delta r_2^2\right) - A\left(\Delta r_2^2 + \Delta r_3^2\right)}{2\left(1 - A\right)}} \tag{26}$$

6. Conclusions

Based on the innovative results of the basic application theory of passive location technology [3], the Doppler location method using virtual path difference presented in this paper greatly simplifies the system design. In fact, the author's research has shown that the path difference-frequency shift equation will help to construct new passive location methods.

The existing mathematical description of the Doppler frequency shift is basically carried out in one-dimensional space. The author's earlier research proved that when the azimuth between the wave source and the observer changes with time, the Doppler shift should be a function on the two-dimensional plane [13]. The Doppler frequency shift on the two-dimensional plane can always be decomposed into the sum of two terms, one of which is only related to the radial velocity, and the other is related to the radial acceleration. In addition, the given formula includes two Doppler frequency shifts at different times, which provides a mathematical method for obtaining the Doppler frequency shift value at the current time by using the Doppler frequency recurrence at the previous time.

The recent research results [11] show the relationship between path difference and frequency shift, which provides a new solution for determining the position of the target directly using Doppler frequency shift. The frequency shift-path difference equation obtained by mean value correction is the analysis basis of the new method proposed in this paper. It is based on the modified frequency shift-path difference equation with a small relative calculation error that can directly use the double base path difference positioning theory to obtain the results with good calculation accuracy.

With regard to the relationship between frequency shift and path difference, the author's current concern is: Can we derive a more rigorous mathematical formula? Can it rise to the height of a physical equation?

Conflict of Interest

There is no conflict of interest.

References

[1] Zhou, Ch., Ma, C.Sh., Ying, T., et al., 2022. Source localization using Doppler frequency shift with erroneous carrier frequency. Journal of University of Electronic Science and Technology of China. 51(4), 529-534.
DOI: https://doi.org/10.12178/1001-0548.2022050

[2] Tian, M.H., Ma, M., Zhang, W.Y., 2018. Target localization based on Doppler frequency and range sum. Journal of Terahertz Science and Electronic Information Technology. 16(6), 984-988.
DOI: https://doi.org/10.11805/TKYDA201806.0984

[3] Wang, D., Yin, J.X., et al., 2017. Direct iocalization method for constant modulus source based on Doppler frequency shifts. Acta Aeronautica ET Astronautica Sinica. 38(9), 284-297.
DOI: https://doi.org/10.7527/S1000-6893.2017.321084

[4] Tao, Y., 2011. Airborne Doppler direct ranging method based on angle change rate. Advances in Aeronautical Science and Engineering. 2(3), 335-338.
DOI: https://doi.org/10.3969/j.issn.1674-8190.2011.03.018

[5] Tao, Y., 2012. A moving single station Doppler passive location method. Guidance & Fuze. 33(1), 16-18,32.
DOI: https://doi.org/10.3969/j.issn.1671-0576.2012.01.004

[6] Zhou, Zh., Wang, G.Ch., 2008. Passive location and tracking of maneuvering targets by airborne single station. Electronics Optics & Control. 15(3), 60-63.
DOI: https://doi.org/10.3969/j.issn.1671-637X.2008.03.016

[7] Liu, C.F., 2011. Wu Yuan Ding Wei Yu Gen Zong (Chinese) [Passive Location and Tracking]. Xidian University Press: Xi'an.

[8] Xiang, F.H., Wang, J.G., 2021. Observability and simulation analysis of fixed single observer passive location. Modern Defense Technology. 49(04), 72-78.

[9] Zhan, R.H., Wang, L., Wan, J.W., 2020. Observable conditions for passive location and tracking of maneuvering targets based on azimuth and Doppler. Journal of National University of Defense Technology. 29(1), 54-58.

[10] Deng, X.P., 2007. Comments on observability of single observer passive location. Strategic Study of CAE. 11, 54-62.

[11] Tao, Y., 2020. An airborne passive positioning method based on angle and frequency difference measurement. Proceedings of the 2020 4th International Conference on Electronic Information Technology and Computer Engineering (EITCE 2020); 2020 Nov. p. 296-301.
DOI: https://doi.org/10.1145/3443467.3443771

[12] Tao, Y., 2017. Wu Yuan Tan Ce Ding Wei Ji Shu (Chinese) [Technology of passive detection location]. National Defense Industry Press: Beijing.

[13] Tao, Y., 2010. Expression for Doppler shift in two-dimensional plane. The 2010 International Conference on Information, Electronic and Computer Science. Scientific Research Publishing: USA. p. 1-4.