BILINGUAL PUBLISHING GROUP

Journal of Electronic & Information Systems
https://journals.bilpubgroup.com/index.php/jeis

ARTICLE

Sliding Mode-Based Distributed Trajectory Tracking Control of Four-body Train Systems

Yueheng Ding[*], *Xinggang Yan*

School of Engineering, University of Kent, Canterbury, CT2 7NT, United Kingdom

ABSTRACT

This paper considers the speed tracking of a four-body train system modelled mathematically based on Newton's second law, which is described by a large-scale interconnected system with four subsystems. Uncertainties are included in the systems to represent the potential impacts on system performance caused by mechanical wear and external environmental changes. An adaptive sliding mode technique is employed to design a distributed control scheme to guarantee tracking accuracy. Coordinate transformations are introduced to transfer the model of train systems to a system in regular form to facilitate the design of the hyperplane and controllers. The *Barbashin-Krasovskii* theorem is employed to show the reachability of the hyperplane. In simulations, the Gaussian function is chosen as the desired signal, representing time-varying characteristics relevant to real-world situations, and the result demonstrates the feasibility of the proposed control strategy.

Keywords: Adaptive control; Distributed control; Large-scale interconnected systems; Sliding mode control; Trajectory tracking

1. Introduction

The train system, an integral part of the global transportation infrastructure, has played a pivotal role in shaping the socio-economic landscapes of nations worldwide. Offering a blend of efficiency, environmental sustainability, and unparalleled connectivity, rail transportation has not only bridged distant geographies but has also fostered economic growth, mitigated urban congestion, and introduced a greener mode of transit. Consequently, the study of such systems has received significant attention,

*CORRESPONDING AUTHOR:
Yueheng Ding, School of Engineering, University of Kent, Canterbury, CT2 7NT, United Kingdom; Email: ingyueheng@126.com

ARTICLE INFO
Received: 11 November 2023 | Revised: 2 January 2024 | Accepted: 16 January 2024 | Published Online: 25 January 2024
DOI: https://doi.org/10.30564/jeis.v6i1.6077

CITATION
Ding, Y.H., Yan, X.G., 2024. Sliding Mode Based Distributed Trajectory Tracking Control of Four-body Train Systems. Journal of Electronic & Information Systems. 6(1): 1–9. DOI: https://doi.org/10.30564/jeis.v6i1.6077

leading to numerous research findings [1–3].

Due to its unique construction, train systems can be described as a type of large-scale system that consists of a collection of interconnected lower-dimensional subsystems, where the behaviour of each subsystem is influenced by the interactions with other adjacent subsystems. Decentralised control, as a popular approach for interconnected systems, involves decomposing, if required, the system into smaller subsystems and designing local controllers for each subsystem independently [4,5]. In this approach, each subsystem's local controller is responsible for regulating its own behaviour while interacting and collaborating with neighbouring subsystems to achieve a global control objective. However, the train systems considered in this paper exhibit a distinctive chain structure, where the interconnections between subsystems can be modelled as functions of their own states and adjacent system states only. In such cases, the distributed scheme allows the local sub-controller to utilise not only information from its own subsystem but also information from its neighbouring subsystems, like information from adjacent subsystems. This characteristic aligns well with the interconnected nature of the train system studied in this paper. Therefore, the use of distributed control is a natural choice [6,7] and serves as the motivation for employing distributed control techniques in this paper.

Tracking control is a crucial subject in both control theory and control engineering, and significant progress has been made in this field (refer to the works [8,9]). In the work [10], an adaptive fuzzy technique-based tracking control approach for interconnected systems is investigated, while the work [11] focuses on decentralised tracking control for large-scale systems, exploring model reference control. However, it is important to note that the findings, obtained in works [10,11], impose a relatively strong limitation on the structural characteristics of the studied system. This specific system structure deviates somewhat from real-world scenarios. Therefore, investigating a more general and realistic train model is a research direction that holds significant value, and it aligns with the problem addressed in this paper. Furthermore, the sliding mode technique is often employed to enhance the robustness of interconnected systems with uncertainties, as the sliding mode dynamics typically govern system performance without uncertainties [12]. Hence, sliding mode control-based methods have been extensively applied in system tracking control. In the work [13], a tracking problem for a class of large-scale systems with interconnections is addressed using sliding mode techniques, which require that the desired signals are constant.

Therefore, this paper focuses on a distributed control approach using sliding mode techniques to tackle the speed tracking challenge of a four-body train system where some extensions and improvements of the time-varying desired signals and unknown interconnections are explored. According to the prior works [14,15], the train system is modelled as an interconnected system with unknown uncertainties and disturbances. Then, a sliding surface is synthesised based on the tracking and the *Barbashin-Krasovskii* theorem is introduced to guarantee the occurrence of a reaching phase and sliding motion with the proposed distributed control. The main contributions of this paper are listed as follows.

- Through the application of sliding mode techniques, the strong robustness of the four-body train system can be guaranteed, due to the sliding motion is insensitive [12] to matched uncertainty and disturbance.
- In comparison to the existing result [16], the desired reference signal is permitted to have a more general form, specifically, a smooth function, and is no longer restricted to be constant.
- Asymptotically tracking the performance of the system with unknown uncertainties is achieved with the proposed control scheme, which involves the use of adaptive techniques.

Lastly, a simulation is conducted to demonstrate the effectiveness of the proposed approaches.

2. System description

According to the work [17], the train system, as depicted in **Figure 1**, can be represented mathematically as:

$$M_i \ddot{z}_i(t) = F_i(t) + F_{i-1}(t) - F_{i+1}(t) - F_{r_i}(t) \tag{1}$$

for $i = 1, 2, 3, 4$, where M_i represents the mass of the ith body, $\ddot{z}_i$ *is* the corresponding acceleration, and F_i *is* the traction force. Additionally, F_{i-1} and F_{i+1} are the restoring forces caused by adjacent bodies. $F_r i$ denotes the resistive force.

Remark 1: Equation (1) describes the most general situation (the middle bodies). For the special parts, like the locomotive and caboose, some terms in (1) will be omitted due to the real situation, which will be discussed later.

Due to the relatively small displacements between bodies, the restoring force can be modelled approximately as the following linear function:

$$F_{i+1}(t) = k_i(z_i - z_{i+1}) + d_i(\dot{z}_i - \dot{z}_{i+1}) \tag{2}$$

where k_i and d_i represent the spring and damping parameters, respectively. z_i, z_{i+1}, $\dot{z}_i$, and $\dot{z}_i$+1 correspond to the ith and $(i + 1)$th displacements and corresponding speeds, respectively for $i = 1, 2, 3, 4$. The general resistance $F_r i(t)$ can be modelled by (see the works [18–20]):

$$F_{r_i}(t) = \begin{cases} b_{1o} + b_{1v}\dot{z}_1 + b_{1a}\dot{z}_1^2, & i = 1 \\ b_{io} + b_{iv}\dot{z}_i, & i = 2,3,4 \end{cases} \tag{3}$$

where b_{io} , b_{iv} and b_{1a} are the resistance coefficients. $b_{1a}\dot{z}_1^2$ denotes aerodynamic drag, while b_{io} and $b_{iv}\dot{z}_i$ are rolling mechanical resistances.

From (1)–(3), the four-body train system in **Figure 1** is given by:

$$\begin{aligned} M_1\ddot{z}_1 = F_1 &- (k_1 + \Delta k_1)(z_1 - z_2) - (d_1 + \Delta d_1)(\dot{z}_1 - \dot{z}_2) \\ &- b_{1o} - b_{1v}\dot{z}_1 - b_{1a}\dot{z}_1^2, \end{aligned} \tag{4}$$

$$\begin{aligned} M_i\ddot{z}_i = F_i &- (k_i + \Delta k_i)(z_i - z_{i+1}) - (k_{i-1} + \Delta k_{i-1})(z_i - z_{i-1}) \\ &- (d_i + \Delta d_i)(\dot{z}_i - \dot{z}_{i+1}) - (d_{i-1} + \Delta d_{i-1})(\dot{z}_i - \dot{z}_{i-1}) \\ &- b_{io} - b_{iv}\dot{z}_i, \ i = 2,3 \end{aligned} \tag{5}$$

$$\begin{aligned} M_4\ddot{z}_4 = F_4 &- (k_3 + \Delta k_3)(z_4 - z_3) - (d_3 + \Delta d_3)(\dot{z}_4 - \dot{z}_3) \\ &- b_{4o} - b_{4v}\dot{z}_4, \end{aligned} \tag{6}$$

where Δk_i and Δd_i for $i = 1, 2, 3$ are unknown constants. $\dot{z}_1$, $\dot{z}_2$, $\dot{z}_3$ and $\dot{z}_4$ are taken as the system's outputs.

Remark 2: In this paper, a more realistic situation is taken into account. It is assumed that there are variations in the spring and damping parameters, denoted by Δk_i and Δd_i, respectively, in the interconnections compared with their nominal values. These variations may occur due to aging of components and external factors such as temperature changes, external disturbances, and other similar influences.

3. System structure analysis

For the four-body train described in system (4)–(6), choose the following coordinate transformation:

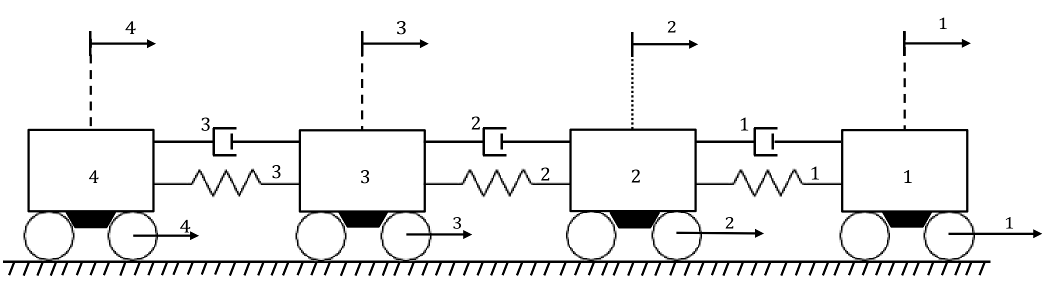

Figure 1. Sketch of a four-body train system.

$$\begin{aligned}&\left[x_{11}\, x_{12}\, x_{21}\, x_{22}\, x_{31}\, x_{32}\, x_{41}\, x_{42}\right]^T \\ &=\left[z_1\, \dot{z}_1\, z_2\, \dot{z}_2\, z_3\, \dot{z}_3\, z_4\, \dot{z}_4\right]^T\end{aligned} \tag{7}$$

Furthermore, an additional feedback transformation is introduced:

$$\begin{aligned}F_1 = {} & k_1(x_{11}-x_{21})+d_1(x_{12}-x_{22})+b_{1o}+b_{1v}x_{12} \\ & +b_{1a}x_{12}^2+M_1v_1,\end{aligned} \tag{8}$$

$$\begin{aligned}F_i = {} & k_i(x_{i,1}-x_{i+1,1})+k_{i-1}(x_{i,1}-x_{i-1,1})+d_i(x_{i,2}-x_{i+1,2}) \\ & +d_{i-1}(x_{i,2}-x_{i-1,2})+b_{io}+b_{iv}x_{i,2}+M_iv_i,\quad i=2,3\end{aligned} \tag{9}$$

$$\begin{aligned}F_4 = {} & k_3(x_{41}-x_{31})+d_3(x_{42}-x_{32})+b_{4o} \\ & +b_{4v}x_{42}+M_4v_4,\end{aligned} \tag{10}$$

where v_1, v_2, v_3 and v_4 are the new control inputs which will be designed later. In the new coordinates $x = col(x_{11}, x_{12}, \ldots, x_{42})$, system (4)–(6) can be described by:

$$\dot{x}_{i1} = x_{i2} \tag{11}$$

$$\dot{x}_{i2} = v_i + H_i(x),\quad i=1,2,3,4 \tag{12}$$

where

$$H_1(x) = -\frac{\Delta k_1}{M_1}(x_{11}-x_{21})-\frac{\Delta d_1}{M_1}(x_{12}-x_{22}), \tag{13}$$

$$\begin{aligned}H_i(x) = {} & -\frac{\Delta k_i}{M_i}(x_{i,1}-x_{i+1,1})-\frac{\Delta k_{i-1}}{M_i}(x_{i,1}-x_{i-1,1})-\frac{\Delta d_i}{M_i}(x_{i,2}-x_{i+1,2}) \\ & -\frac{\Delta d_{i-1}}{M_i}(x_{i,2}-x_{i-1,2}),\quad i=2,3\end{aligned} \tag{14}$$

$$H_4(x) = -\frac{\Delta k_3}{M_4}(x_{41}-x_{31})-\frac{\Delta d_3}{M_4}(x_{42}-x_{32}), \tag{15}$$

where $H_i(x)$ represents uncertainties in the interconnections of the ith subsystems for $i = 1, 2, 3, 4$ with the state $x = col(x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32}, x_{41}, x_{42}) \in R^8$. The inputs $v_i \in R$.

Remark 3: According to (13)–(15), the interconnected uncertainties $H_i(x)$ in the ith subsystem are functions of the unknown coefficients, state x_i and its adjacent states x_{i-1} and x_{i+1}. The state $x_i = col(x_{i1}, x_{i2})$ for $i = 1, 2, 3, 4$. It is clear to see that the interconnected structure in (13)–(15) reflects the practical train system shown in **Figure 1**. Therefore, distributed control is naturally considered to cope with the tracking problem of the system above.

Consider the interconnected system (11)–(12) with interconnections in (13)–(15). The desired displacement of each body in the train system is assumed as $S_d(t)$. Its first derivate is thus the desired signal $y_d(t)$ (speed signal). Then, the objective of this paper is to design an adaptive-based distributed sliding mode control that allows the speed of each body to track the desired signal $y_d(t)$. In other words, the objective of this paper is to achieve $\lim_{t\to\infty} |y_d(t) - x_{i2}(t)| = 0$ for $i = 1, 2, 3, 4$. Additionally, the displacement errors between the desired displacement $S_d(t)$ and the actual displacement of the four bodies should remain to be bounded, despite the presence of unknown uncertainties in the interconnections.

Remark 4: It is important to note that the displacement states z_1, z_2, z_3, and z_4 may tend towards infinity as time t approaches infinity, especially when the speeds are non-zero. However, from a practical perspective, it is essential to ensure that the displacement errors $S_d(t) - z_1$, $S_d(t) - z_2$, $S_d(t) - z_3$, and $S_d(t) - z_4$ remain bounded. Failure to do so may result in the connections between adjacent bodies being broken.

Assumption 3.1: The desired signal $y_d(t)$ and its first derivate $\dot{y}_d(t)$ are assumed to be smooth for all $t \in [0, \infty)$. In this case, a proper transformation $T = diag\ \{T_i\}$ for $i = 1, 2, 3, 4$ with T_i is defined by:

$$T_i \triangleq \begin{bmatrix}\delta_i(t)\\ e_i(t)\end{bmatrix} = \begin{bmatrix}S_d(t)-x_{i1}(t)\\ y_d(t)-x_{i2}(t)\end{bmatrix} \tag{16}$$

where $S_d(t)$ and $y_d(t)$ satisfy the Assumption 3.1.

Then, system (11)–(12) in the new coordinates $col(\delta_i, e_i)$ can be described by:

$$\dot{\delta}_i(t) = e_i(t), \tag{17}$$

$$\dot{e}_i(t) = \dot{y}_d(t) - v_i + \Gamma_i(\delta, e), \quad i = 1,2,3,4 \tag{18}$$

where $\delta = col(\delta_1, \delta_2, \delta_3, \delta_4)$, $e = col(e1, e2, e3, e4)$ and

$$\Gamma_1(\delta, e) = T_1 H_1(x)|_{x=T^{-1}col(\delta,e)} = \alpha_1(\delta_2 - \delta_1) + \beta_1(e_2 - e_1), \tag{19}$$

$$\begin{aligned}\Gamma_i(\delta, e) = T_i H_i(x)|_{x=T^{-1}col(\delta,e)} &= \alpha_{i1}(\delta_{i+1} - \delta_i) + \alpha_{i2}(\delta_{i-1} - \delta_i) \\ &+ \beta_{i1}(e_{i+1} - e_i) + \beta_{i2}(e_{i-1} - e_i), \ i = 2,3\end{aligned} \tag{20}$$

$$\Gamma_4(\delta, e) = T_4 H_4(x)|_{x=T^{-1}col(\delta,e)} = \alpha_4(\delta_3 - \delta_4) + \beta_4(e_3 - e_4), \tag{21}$$

with

$$\alpha_1 = \frac{\Delta k_1}{M_1}, \beta_1 = \frac{\Delta d_1}{M_1}, \alpha_{i1} = \frac{\Delta k_i}{M_i}, \alpha_{i2} = \frac{\Delta k_{i-1}}{M_1}, \beta_{i1} = \frac{\Delta d_i}{M_i},$$
$$\beta_{i2} = \frac{\Delta d_{i-1}}{M_i}, (i = 2,3), \alpha_4 = \frac{\Delta k_3}{M_4} \quad \text{and} \quad \beta_4 = \frac{\Delta d_3}{M_4}.$$

4. Stability analysis and control law construction

4.1 Stability analysis of sliding motion

For system (17)–(18), consider the sliding surface defined by:

$$col(e_1, e_2, e_3, e_4) = 0. \tag{22}$$

From the sliding mode control theory, the sliding motion of the system (17)–(18) corresponding to the sliding surface (22) is given by:

$$\dot{\delta}_i(t) = 0. \ i = 1,2,3,4 \tag{23}$$

It is easy to see from (23) that $\delta_i(t)$ for $i = 1, 2, 3, 4$ are bounded when the sliding motion occurs, which is consistent with the objective of this paper.

4.2 Reachability problem and distributed control design

This section aims to design a distributed sliding mode control to drive the system into the sliding surface (22). Then, the controllers are proposed as:

$$v_1 = \dot{y}_d(t) + \hat{\alpha}_1(t)(\delta_2 - \delta_1) + \hat{\beta}_1(t)(e_2 - e_1) + k_1 e_1, \tag{24}$$

$$\begin{aligned}v_i = \dot{y}_d(t) &+ \hat{\alpha}_{i1}(t)(\delta_{i+1} - \delta_i) + \hat{\alpha}_{i2}(t)(\delta_{i-1} - \delta_i) + \hat{\beta}_{i1}(t)(e_{i+1} - e_i) \\ &+ \hat{\beta}_{i2}(t)(e_{i-1} - e_i) + k_i e_i, \ i = 2,3\end{aligned} \tag{25}$$

$$v_4 = \dot{y}_d(t) + \hat{\alpha}_4(t)(\delta_3 - \delta_4) + \hat{\beta}_4(t)(e_3 - e_4) + k_4 e_4, \tag{26}$$

where k_i for $i = 1, 2, 3, 4$ are positive constants. $\hat{\alpha}_1(t)$, $\hat{\beta}1(t)$, $\hat{\alpha}_{i1}(t)$, $\hat{\alpha}_{i2}(t)$, $\hat{\beta}i1(t)$, $\hat{\beta}i2(t)$, $(i = 2, 3)$, $\hat{\alpha}_4(t)$ and $\hat{\beta}4(t)$ are the approximation to the parameters α_1, β_1, α_{i1}, α_{i2}, β_{i1}, β_{i2}, $(i = 2, 3)$, α_4 and β_4 in (19)–(21) respectively, and the adaptive laws are given by:

$$\begin{aligned}\dot{\hat{\alpha}}_1(t) &= e_1(\delta_2 - \delta_1), \\ \dot{\hat{\beta}}1(t) &= e1(e2 - e1);\end{aligned} \tag{27}$$

$$\begin{aligned}\dot{\hat{\alpha}}_{i1}(t) = e_i(\delta_{i+1} - \delta_i), \ \dot{\hat{\alpha}}i2(t) = e_i(\delta_{i-1} - \delta_i), \\ \dot{\hat{\beta}}i1(t) = ei(ei{+}1 - ei), \ \dot{\hat{\beta}}i2(t) = ei(ei{-}1 - ei); i = 2, 3\end{aligned} \tag{28}$$

$$\begin{aligned}\dot{\hat{\alpha}}_4(t) &= e_4(\delta_3 - \delta_4), \\ \dot{\hat{\beta}}4(t) &= e4(e3 - e4).\end{aligned} \tag{29}$$

Remark 5: From (8)–(10), the final distributed controller in the original coordinate is given by:

$$\begin{aligned}F_1 = (k_1 + M_1\hat{\alpha}_1(t))(x_{11} - x_{21}) &+ (d_1 + M_1\hat{\beta}_1(t))(x_{12} - x_{22}) \\ &+ b_{1o} + (b_{1v} - M_1 k_1)x_{12} + b_{1a}x_{12}^2 \\ &+ M_1(\dot{y}_d(t) + k_1 y_d),\end{aligned} \tag{30}$$

$$\begin{aligned}F_i = (k_i + M_i\hat{\alpha}_{i1}(t))(x_{i,1} - x_{i+1,1}) &+ (k_{i-1} + M_i\hat{\alpha}_{i2}(t)) \\ (x_{i,1} - x_{i-1,1}) &+ (d_i + M_i\hat{\beta}_{i1}(t))(x_{i,2} - x_{i+1,2}) \\ &+ (d_{i-1} + M_i\hat{\beta}_{i2}(t))(x_{i,2} - x_{i-1,2}) \\ &+ b_{io} + (b_{iv} - M_i k_i)x_{i,2} + M_i(\dot{y}_d(t) + k_i y_d), \ i = 2,3\end{aligned} \tag{31}$$

$$F_4 = (k_3 + M_4\hat{\alpha}_4(t))(x_{41} - x_{31}) + (d_3 + M_4\hat{\beta}_4(t))(x_{42} - x_{32}) + b_{4o} + (b_{4v} - M_4 k_4)x_{42} + M_4(\dot{y}_d(t) + k_4 y_d). \quad (32)$$

with the parameters $\hat{\alpha}_1(t)$, $\hat{\beta}1(t)$, $\hat{\alpha}_{i1}(t)$, $\hat{\alpha}_{i2}(t)$, $\hat{\beta}i1(t)$, $\hat{\beta}i2$, (i = 2, 3), $\hat{\alpha}_4(t)$ and $\hat{\beta}4(t)$ satisfies (27)–(29).

Theorem 1: For the interconnected system (17)–(18) with the adaptive laws in (27)–(29), under Assumption 3.1, the controller (24)–(26) can drive the considered system to the sliding surface (22) and maintains a sliding motion on it thereafter.

Proof 1: Define the adaptive errors as:

$$\tilde{\alpha}_1(t) = \alpha_1 - \hat{\alpha}_1(t), \;\; \tilde{\beta}_1(t) = \beta_1 - \hat{\beta}_1(t); \quad (33)$$

$$\tilde{\alpha}_{i1}(t) = \alpha_{i1} - \hat{\alpha}_{i1}(t), \;\; \tilde{\alpha}_{i2}(t) = \alpha_{i2} - \hat{\alpha}_{i2}(t),$$
$$\tilde{\beta}_{i1}(t) = \beta_{i1} - \hat{\beta}_{i1}(t), \;\; \tilde{\beta}_{i2}(t) = \beta_{i2} - \hat{\beta}_{i2}(t); \quad i = 2,3 \quad (34)$$

$$\tilde{\alpha}_4(t) = \alpha_4 - \hat{\alpha}_4(t), \;\; \tilde{\beta}_4(t) = \beta_4 - \hat{\beta}_4(t). \quad (35)$$

Choose a Lyapunov candidate function as:

$$V = \frac{1}{2}\sum_{i=1}^{4} e_i^2 + \frac{1}{2}\sum_{i=2}^{3}(\tilde{\alpha}_{i1}^2 + \tilde{\alpha}_{i2}^2 + \tilde{\beta}_{i1}^2 + \tilde{\beta}_{i2}^2) + \frac{1}{2}(\tilde{\alpha}_1^2 + \tilde{\beta}_1^2 + \tilde{\alpha}_4^2 + \tilde{\beta}_4^2). \quad (36)$$

Then, the time derivate of V along the trajectories of (18) is given by:

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{4} e_i\dot{e}_i - \sum_{i=2}^{3}(\tilde{\alpha}_{i1}\dot{\hat{\alpha}}_{i1} + \tilde{\alpha}_{i2}\dot{\hat{\alpha}}_{i2} + \tilde{\beta}_{i1}\dot{\hat{\beta}}_{i1} + \tilde{\beta}_{i2}\dot{\hat{\beta}}_{i2}) \\ &\quad - \tilde{\alpha}_1\dot{\hat{\alpha}}_1 - \tilde{\beta}_1\dot{\hat{\beta}}_1 - \tilde{\alpha}_4\dot{\hat{\alpha}}_4 - \tilde{\beta}_4\dot{\hat{\beta}}_4 \\ &= -\sum_{i=1}^{4} k_i e_i^2 + \sum_{i=2}^{3}(e_i(\tilde{\alpha}_{i1}(\delta_{i+1} - \delta_i) + \tilde{\alpha}_{i2}(\delta_{i-1} - \delta_i) \\ &\quad + \tilde{\beta}_{i1}(e_{i+1} - e_i) + \tilde{\beta}_{i2}(e_{i-1} - e_i)) - \tilde{\alpha}_{i1}\dot{\hat{\alpha}}_{i1} - \tilde{\alpha}_{i2}\dot{\hat{\alpha}}_{i2} - \tilde{\beta}_{i1}\dot{\hat{\beta}}_{i1} - \tilde{\beta}_{i2}\dot{\hat{\beta}}_{i2}) \\ &\quad + e_1(\tilde{\alpha}_1(\delta_2 - \delta_1) + \tilde{\beta}_1(e_2 - e_1)) - \tilde{\alpha}_1\dot{\hat{\alpha}}_1 - \tilde{\beta}_1\dot{\hat{\beta}}_1 + e_4(\tilde{\alpha}_4(\delta_3 - \delta_4) \\ &\quad + \tilde{\beta}_4(e_3 - e_4)) - \tilde{\alpha}_4\dot{\hat{\alpha}}_4 - \tilde{\beta}_4\dot{\hat{\beta}}_4 \\ &= -\sum_{i=1}^{4} k_i e_i^2.\end{aligned} \quad (37)$$

From the analysis above, $\dot{V}$ is a negative semi-definite function. Then, from the *Barbashin-Krasovskii* theorem (refer to section 4.2 in the work [21]), its solution $e_i(t) \to 0$ as $t \to \infty$ for i = 1, 2, 3, 4. Therefore, the proposed controller (24)–(26) can drive the system to the sliding surface. By integrating (27)–(29) and considering the results $\lim_{t\to\infty} e_i(t) = 0$, it is evident that the adaptive parameters $\hat{\alpha}_1(t)$, $\hat{\beta}1(t)$, $\hat{\alpha}_{i1}(t)$, $\hat{\alpha}_{i2}(t)$, $\hat{\beta}i1(t)$, $\hat{\beta}i2(t)$, (for i = 2, 3), $\hat{\alpha}_4(t)$ and $\hat{\beta}4(t)$ are bounded. Hence, the result is valid.

Remark 6: The boundedness of the sliding motion (23) is demonstrated. Theorem 1 illustrates that the control scheme (24)–(26) can drive system (17)–(18) to the sliding surface (22). According to sliding mode theory, this implies that the proposed distributed control approach (30)–(32) not only ensures that each body's speed asymptotically tracks the desired signal $y_d(t)$ but also guarantees that all displacement errors between adjacent bodies remain bounded.

Remark 7: The adaptive laws (27)–(29) proposed above ensure the estimation of parameters rather than their identification. This implies that the adaptive parameters may not converge to their true values. Only the boundedness of the parameters' estimation is guaranteed in this paper.

5. Simulation study

In this section, a simulation is conducted to demonstrate the obtained results. For the simulation purpose, the following generalized Gaussian distribution depicted in **Figure 2** from the works [22,23] is chosen as the desired speed signal.

$$y_d(t) = \frac{6}{\Gamma(\frac{1}{6})} \cdot e^{-(\frac{|t-10|}{5})^6}, \;\; t \geq 0 \quad (38)$$

where $\Gamma(\cdot)$ denotes the Gamma function. This signal is consistent with the practical train system running between two stations.

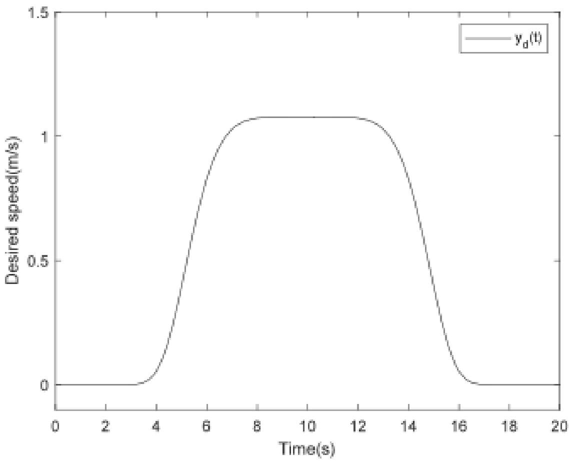

Figure 2. Time responses of the desired signal.

The nominal parameters of all bodies are set as:

$M_1 = M_2 = M_3 = M_4 = 126000\ kg,$
$d_1 = d_2 = d_3 = 80 \times 10^4\ Ns/m,$
$k_1 = k_2 = k_3 = 100 \times 10^6\ N/m,$
$b_{1o} = b_{2o} = b_{3o} = b_{4o} = 6.362 \times 10^{-3}\ N/kg,$
$b_{1v} = b_{2v} = b_{3v} = b_{4v} = 1.08 \times 10^{-4}\ Ns/(mkg),$
$b_{1a} = 2.06 \times 10^{-5}\ Ns^2/(m^2kg).$

The initial condition is set as $[x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32}, x_{41}, x_{42}]^\top = [0\ 0.8\ 0\ 0.5\ 0\ 0.2\ 0\ 0]^\top$. The controller gains are determined as $k_1 = 0.7$, $k_2 = 5$, $k_3 = 2$ and $k_4 = 1$.

With the distributed sliding mode control (DSMC) proposed in this paper, the speed of each body asymptotically tracks the desired signal $y_d(t)$, as illustrated by the blue line in **Figure 3**. For comparison, a robust distributed controller (DC) in the work [16] is also considered as shown by the red line in **Figure 3**. It can be observed that the controller using DSMC achieved rapid convergence within the first 5 seconds, while the controller using DC exhibited poorer tracking performance. Concurrently, the displacement errors $\delta_i(t)$ for $i = 1, 2, 3, 4$ remain stable, as shown in **Figure 4**. The adaptive parameters $\hat{\alpha}_1(t)$, $\hat{\beta}1(t)$, $\hat{\alpha}_{i1}(t)$, $\hat{\alpha}_{i2}(t)$, $\hat{\beta}i1(t)$, $\hat{\beta}i2(t)$, $(i = 2, 3)$, $\hat{\alpha}_4(t)$ and $\hat{\beta}4(t)$ are bounded, as demonstrated in **Figure 5**. The simulation results align with the theoretical findings, validating the proposed approach.

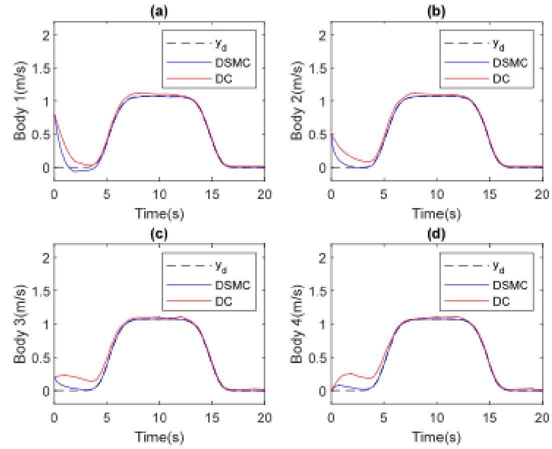

Figure 3. Tracking the performance of the system. (a). A comparative tracking results of the body 1. (b). A comparative tracking results of the body 2. (c). A comparative tracking results of the body 3. (d). A comparative tracking results of the body 4.

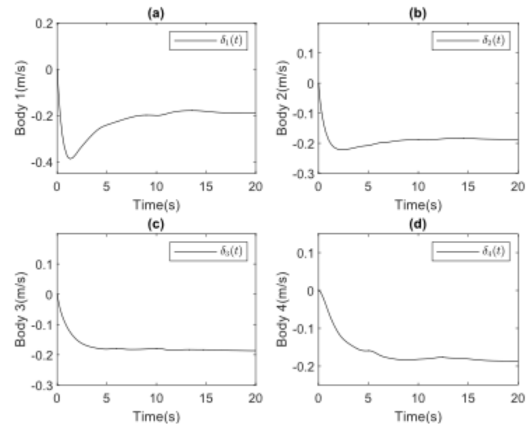

Figure 4. Displacement errors between the desired displacement S_d & the displacement of each body with DSMC. (a). Displacement error of the body 1. (b). Displacement error of the body 2. (c). Displacement error of the body 3. (d). Displacement error of the body 4.

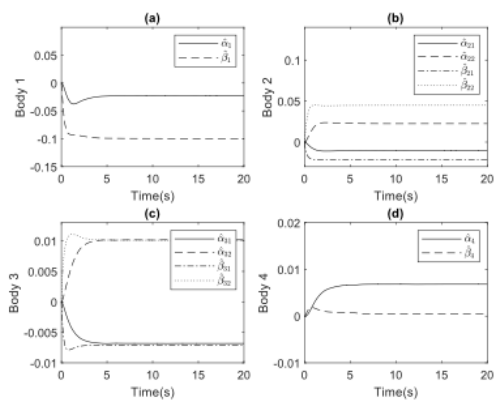

Figure 5. The time response of the estimated parameters. (a). Estimated parameters of the body 1. (b). Estimated parameters of the body 2. (c). Estimated parameters of the body 3. (d). Estimated parameters of the body 4.

6. Conclusions

This paper introduces a distributed tracking control method for a four-body train system with unknown uncertainties in its interconnections, leveraging sliding mode techniques. Unlike previous methods, our approach accommodates time-varying desired signals. The proposed distributed sliding mode control scheme based on the *Barbashin-Krasovskii* theorem has been proposed to fulfil the reachability condition, ensuring the reachability condition is met. Additionally, the unknown interconnections are approximated by using adaptive techniques. Simulation results for the four-body system validate the effectiveness and practicality of the proposed approach.

In the process of train control, the use of distributed control may lead to the entire system coming to a halt. For this, one of the advantages of decentralised control is that the controller of each subsystem only collects local information, meaning that even if other subsystems experience malfunctions, decentralised control can still ensure the normal operation of the entire system. Therefore, combining decentralised control with the tracking control of trains is a promising research direction.

Author Contributions

This paper has been researched and written by Yueheng Ding and Xinggang Yan.

Conflict of Interest

We declare that there is no conflict of interest regarding the publication of this paper.

References

[1] Zhao, L., Cai, B., Xu, J., et al., 2013. Study of the track-train continuous information transmission process in a high-speed railway. IEEE Transactions on Intelligent Transportation Systems. 15(1), 112–121. DOI: https://doi.org/10.1109/TITS.2013.2274617

[2] Lin, P., Huang, Y., Zhang, Q., et al., 2020. Distributed velocity and input constrained tracking control of high-speed train systems. IEEE Transactions on Systems, Man, and Cybernetics: Systems. 51(12), 7882–7888. DOI: https://doi.org/10.1109/TSMC.2020.2987339

[3] He, Y., Wu, J., Zheng, Y., et al., 2022. Track defect detection for high-speed maglev trains via deep learning. IEEE Transactions on Instrumentation and Measurement. 71, 1–8. DOI: https://doi.org/10.1109/TIM.2022.3151165

[4] Yan, X.G., Wang, J.J., Lu, X.Y., et al., 1998. Decentralized output feedback robust stabilization for a class of nonlinear interconnected systems with similarity. IEEE Transactions on Automatic Control. 43(2), 294–299. DOI: https://doi.org/10.1109/9.661085

[5] Yan, X.G., Xie, L., 2003. Reduced-order control for a class of nonlinear similar interconnected systems with mismatched uncertainty. Automatica. 39(1), 91–99. DOI: https://doi.org/10.1016/S0005-1098(02)00194-2

[6] Tan, L.N., 2018. Distributed H∞ optimal tracking control for strict-feedback nonlinear large-scale systems with disturbances and saturating actuators. IEEE Transactions on Systems, Man, and Cybernetics: Systems. 50(11), 4719–4731. DOI: https://doi.org/10.1109/TSMC.2018.2861470

[7] Zhang, D., Nguang, S.K., Yu, L., 2017. Distributed control of large-scale networked control systems with communication constraints and topology switching. IEEE Transactions on Systems, Man, and Cybernetics: Systems. 47(7), 1746–1757. DOI: https://doi.org/10.1109/TSMC.2017.2681702

[8] Cai, H., Hu, G., 2017. Distributed tracking control of an interconnected leader—follower multiagent system. IEEE Transactions on Automatic Control. 62(7), 3494–3501. DOI: https://doi.org/10.1109/TAC.2017.2660298

[9] Liu, C., Zhang, H., Xiao, G., et al., 2019. Integral reinforcement learning based decentralized optimal tracking control of unknown nonlinear

large-scale interconnected systems with constrained-input. Neurocomputing. 323, 1–11. DOI: https://doi.org/10.1016/j.neucom.2018.09.011

[10] Ren, X.X., Yang, G.H., Li, X.J., 2019. Global adaptive fuzzy distributed tracking control for interconnected nonlinear systems with communication constraints. IEEE Transactions on Fuzzy Systems. 28(2), 333–345. DOI: https://doi.org/10.1109/TFUZZ.2019.2905827

[11] Pagilla, P.R., Dwivedula, R.V., Siraskar, N.B., 2007. A decentralized model reference adaptive controller for large-scale systems. IEEE/ASME Transactions on Mechatronics. 12(2), 154–163. DOI: https://doi.org/10.1109/TMECH.2007.892823

[12] Yan, X.G., Spurgeon, S.K., Edwards, C., 2018. Variable structure control of complex systems analysis and design. Springer: Berlin.

[13] Zhu, G., Nie, L., Lv, Z., et al., 2020. Adaptive fuzzy dynamic surface sliding mode control of large-scale power systems with prescribe output tracking performance. ISA Transactions. 99, 305–321. DOI: https://doi.org/10.1016/j.isatra.2019.08.063

[14] Ding, Y.H, Yan, X.G., Mao, Z.H., et al. (editors), 2022. Sliding mode based decentralized tracking control of underactuated four-body systems. 2022 13th Asian Control Conference (ASCC); 2022 May 4–7; Jeju, Korea. New York: IEEE. p. 1765–1770. DOI: https://doi.org/10.23919/ASCC56756.2022.9828154

[15] Song, Q., Song, Y.D., Tang, T., et al., 2011. Computationally inexpensive tracking control of high-speed trains with traction/braking saturation. IEEE Transactions on Intelligent Transportation Systems. 12(4), 1116–1125. DOI: https://doi.org/10.1109/TITS.2011.2143409

[16] Wang, X., Zhu, L., Wang, H., et al., 2019. Robust distributed cruise control of multiple high-speed trains based on disturbance observer. IEEE Transactions on Intelligent Transportation Systems. 22(1), 267–279. DOI: https://doi.org/10.1109/TITS.2019.2956162

[17] Mao, Z., Tao, G., Jiang, B., et al., 2021. Zero dynamics analysis and adaptive tracking control of underactuated multibody systems with flexible links. International Journal of Control. 94(7), 1931–1943. DOI: https://doi.org/10.1080/00207179.2019.1686179

[18] Mao, Z., Tao, G., Jiang, B., et al., 2020. Adaptive control design and evaluation for multibody high-speed train dynamic models. IEEE Transactions on Control Systems Technology. 29(3), 1061–1074. DOI: https://doi.org/10.1109/TCST.2020.2991119

[19] Tang, H., Wang, Q., Feng, X., 2017. Robust stochastic control for high-speed trains with nonlinearity, parametric uncertainty, and multiple time-varying delays. IEEE Transactions on Intelligent Transportation Systems. 19(4), 1027–1037. DOI: https://doi.org/10.1109/TITS.2017.2710138

[20] Rochard, B.P., Schmid, F., 2000. A review of methods to measure and calculate train resistances. Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit. 214(4), 185–199. DOI: https://doi.org/10.1243/0954409001531306

[21] Hassan, K.K., 2001. Nonlinear systems. Prentice Hall: Hoboken.

[22] Cui, S., Datcu, M., 2012. Statistical wavelet subband modeling for multi-temporal SAR change detection. IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing. 5(4), 1095–1109. DOI: https://doi.org/10.1109/JSTARS.2012.2200655

[23] Mallat, S.G., 1989. A theory for multiresolution signal decomposition: The wavelet representation. IEEE Transactions on Pattern Analysis and Machine Intelligence. 11(7), 674–693. DOI: https://doi.org/10.1109/34.192463