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ARTICLE

Proposal of Single Sideband Modulation Scheme Using Frequency Domain Filtering

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ABSTRACT

With the rapid development of wireless systems, the demand for frequency resources has been increasing in recent years. Therefore, it is necessary to consider the high-quality communication method that efficiently utilizes finite frequency resources. In this paper, Single Sideband 16 Pulse Amplitude Modulation (SSB 16PAM) scheme for the uplink communication is proposed. It transmits data in only Lower Sideband (LSB) without extra Hilbert components. Under Additive White Gaussian Noise (AWGN) channel environment, Bit Error Rate (BER) performance of the proposed scheme is superior by 3 dB in terms of Carrier-to-Noise Ratio (CNR) to 256 Quadrature Amplitude Modulation (256QAM) scheme with the same frequency efficiency and the same Peak-to-Average Power Ratio (PAPR).

Our proposed scheme employs the original frequency domain filter on the transmitter side to form an ideal spectrum. The configuration of its process is almost similar to Single Carrier-Frequency Division Multiple Access (SC-FDMA), moreover, half of the input data on the frequency domain is removed. The proposed frequency domain filter produces the SSB-modulated spectrum with a roll-off rate of zero without degrading the BER performance.

1. Introduction

Recently, the frequency resources of wireless systems are being depleted, so the high-priority issue for the subsequent wireless systems is to make a revolutionary modulation scheme with higher frequency efficiency and higher quality. In order to address this issue, several researchers in recent years have shown an interest in SSB modulation scheme combined with the quadrature modulation scheme.

The SSB modulation scheme sends data at half the occupied bandwidth compared to that of the Double Sideband (DSB) system. The SSB signal can be produced by combining the Hilbert transform and quadrature multiplexing, and its modulation scheme is only effective in scalar modulation [1,2]. In contrast, quadrature multiplexing scheme, which is a typical DSB modulation method, employs two carrier waves of the same frequency that are out of phase with each other by 90°. If the SSB modulation can be incorporated with the quadrature modulation, the spectrum efficiency might be twice as high as the conventional scheme. Unfortunately, they are not independent of each other as both modulations use the same signal processing for quadrature multiplexing. The in-phase component comprises the I-data and Hilbert transform of Q-data, and the quadrature component

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comprises Q-data and Hilbert transform of I-data on the receiver side. Thus, lossless demodulation cannot be performed analytically because of extra Hilbert components.

In this paper, SC-FDMA SSB 16PAM using the frequency domain filter is proposed, which does not require consideration of the problem of extra Hilbert components. It employs scalar modulation, so it can be SSB-modulated without extra Hilbert components. In addition, our proposed scheme employs the frequency domain filter on the transmitter side to form an ideal spectrum. The configuration of its process is almost similar to SC-FDMA, moreover, half of the input data on the frequency domain is removed to form SSB-modulated spectrum. The advantages of the proposed frequency domain filter are that it can form the spectrum with a roll-off rate of zero without the penalty of the BER performance [1], and that PAPR is improved over the conventional SSB modulation scheme. The proposed SC-FDMA SSB 16PAM reduces the peak power generated by the Hilbert transform and has the same PAPR as the SC-FDMA 256QAM with the same frequency efficiency.

Currently, the SSB processing is used for the downlink communication and not for the uplink communication, so the proposed scheme is expected to improve the quality of the single carrier transmission for the uplink communication. Under AWGN channel environment, BER performance of the proposed scheme is superior by 3 dB in terms of CNR to 256QAM scheme with the same frequency efficiency and the same PAPR. In addition, the same beneficial effect can be obtained with SC-FDMA SSB M-PAM modulation scheme.

2. Related Works

Several researchers have shown research results on the Quadrature SSB (Q-SSB) modulation [3-6]. It is expected that the spectrum efficiency can be doubled by multiplying the SSB modulation and the quadrature modulation. The transmission signal of Q-SSB modulation scheme is represented as follows:

$$S_u(t) = \left\{I_u(t) + \widehat{Q_u}(t)\right\} cos\, 2\pi f_c t + \left\{-\widehat{I_u}(t) + Q_u(t)\right\} sin\, 2\pi f_c t. \quad (1)$$

where $\widehat{Q_u}(t)$ and $\widehat{I_u}(t)$ are Hilbert components.

However, each of the in-phase component and the quadrature component contains the extra Hilbert component on the receiver side, and no method has been considered to analytically remove its component completely. Therefore, the Q-SSB signal cannot be properly demodulated by the normal method. The demodulation method with the strong equalizer such as the turbo equalizer can suppress the extra Hilbert component and the demodulation can be performed without significantly degrading the BER performance, but it becomes difficult to form an ideal spectrum with a roll-off rate of zero [7]. In addition, the Q-SSB modulation scheme is vulnerable to phase-error due to the effect of extra Hilbert components. The BER performance deteriorates under the fading channel environment. In order to avoid that problem, the 4ASK-based SSB modulation scheme in which only the quadrature component is used as the Hilbert component was proposed [1].

On the other hand, the SSB signal also has the drawback that the PAPR of the time domain signal is significantly deteriorated due to the Hilbert transform on the transmitter side. When a high peak signal is input to the non-linear device such as a transmitter power amplifier, the output signal is distorted. Its distortion causes the deterioration of the transmission quality and the increase of out-of-band radiation to adjacent systems. Although it is possible to reduce the distortion for large peak signals by using the device with excellent input/output characteristics, this method leads to the increased power consumption.

From the above, the main issues of the SSB modulation scheme are to solve the problem of the extra Hilbert components, to form the practical spectrum without deterioration of the BER performance, and to improve the PAPR that is degraded by the Hilbert transform.

3. SSB Modulation

This section describes how SSB modulation is applied. The details of the modulation method are explained in other articles [1,8,9]. In this paper, the detail of the spectrum transition of the SSB modulation on the transmitter side is described. Figure 1 depicts the spectrum transition of the SSB modulation [10,11]. The Hilbert transform depicted in Figure 1 is the method of delaying the phase by 90° at the positive frequency domain and advancing the phase by 90° at the negative frequency domain. As depicted in Figure 1, $S_{USB}(t)$ *and* $S_{LSB}(t)$ of the transmission signal is represented as follows:

$$S(f) = S_I(f) \pm S_Q(f) S(f) = S_I(f) \pm S_Q(f). \quad (2)$$

In (2) and Figure 1, when $S_I(f)$ and $S_Q(f)$ are added, SSB modulation by LSB is performed; on the contrary, when they are subtracted, SSB modulation by USB is performed.

Here, the spectrum transition as indicated in (2) is explained in detail. The SSB-modulated transmission signal $S(t)$ using Hilbert transform is represented as

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follows:

$$S(t) = S_I(t) \pm S_Q(t)$$
$$= m(t) \cdot cos\, 2\pi f_c t \pm \hat{m}(t) \cdot sin\, 2\pi f_c t, \quad (3)$$

where $m(t)$ is the modulated signal, f_c is the carrier frequency. The relationship between $m(t)$ and $M(f)$ is represented as follows:

$$m(t) \Leftrightarrow M(f) = \begin{Bmatrix} M_+(f), & f > 0 \\ M_-(f), & f < 0 \end{Bmatrix} \quad (4)$$

On the other hand, the relationship between $\hat{m}(t)$ and $\hat{M}(f)$ is represented as follows:

$$\hat{m}(t) \Leftrightarrow \hat{M}(f) = \begin{Bmatrix} -jM_+(f), & f > 0 \\ jM_-(f), & f < 0 \end{Bmatrix} \quad (5)$$

From (3), (4), and (5), $S_I(f)$ and $S_Q(f)$ are represented as follows:

$$S_I(f) = \frac{1}{2}\{M_+(f - f_c) + M_-(f - f_c)\} + \frac{1}{2}\{M_+(f + f_c) + M_-(f + f_c)\} \quad (6)$$

$$S_Q(f) = \frac{1}{2}\{-M_+(f - f_c) + M_-(f - f_c)\} + \frac{1}{2}\{M_+(f + f_c) - M_-(f + f_c)\} \quad (7)$$

Therefore, the LSB signal $S_{LSB}(f)$ and the USB signal $S_{USB}(f)$ depicted in Figure 1 are represented as follows:

$$S_{LSB}(f) = S_I(f) + S_Q(f) = M_-(f - f_c) + M_+(f + f_c) \quad (8)$$

$$S_{USB}(f) = S_I(f) - S_Q(f) = M_-(f + f_c) + M_+(f - f_c) \quad (9)$$

From (6), (7), (8) and (9), it can be seen that the SSB modulation scheme can transmit data at half the occupied bandwidth as compared with the DSB modulation scheme.

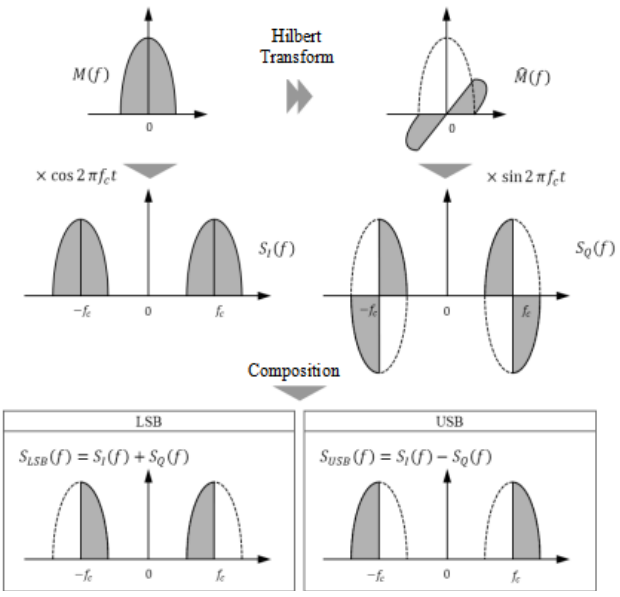

Figure 1. Generating SSB modulation signal

4. SC-FDMA

Since our proposed method assumes single-carrier transmission in uplink communication, SC-FDMA, which is important for explaining the proposed method, is described in this section. Figure 2 depicts the configuration around the Discrete Fourier Transform-Spread-Orthogonal Frequency Division Multiplexing (DFT-s-OFDM) used in SC-FDMA.

As depicted in Figure 2, the transmission signal is first mapped as a single carrier with low PAPR and the data is transformed into frequency components by Fast Fourier Transform (FFT). The signal on the frequency domain by the FFT is represented as follows:

$$F(k) = \frac{1}{\sqrt{N}} \sum_{t=0}^{N-1} f(t) \cdot e^{-\frac{j2\pi kt}{N}}, \quad (10)$$

where $k = 0,1, \ldots, N-1, F(k)$ is the frequency-domain signal, and $f(t)$ is the time-domain signal. The 4-times oversample is performed to form a spectrum with a roll-off rate of zero [1]. The reason why the position to insert nulls is in the middle of the symbol is that the data position in the frequency domain of the FFT block in MATLAB/Simulink is as depicted in Figure 3. The FFT size in Figure 3 is 256.

As depicted in Figure 3, the nulls in the 4-times oversample are arranged outside the original data, so that the side lobe of the transmission spectrum can be suppressed. In this paper, the processing to insert nulls on the frequency domain is called the frequency domain filtering.

The frequency domain filter is equivalent to multiplying a rectangular wave in the frequency domain, as depicted in Figure 4. The signal on the frequency domain after 4-times oversample is represented as follows:

$$F_R(f) = F(f) \cdot rect(f), \quad (11)$$

where

$$rect(f) = \begin{cases} 0, & N < |f| \le 4N \\ 1, & |f| \le N \end{cases}. \quad (12)$$

The inverse Fourier transform of (12) is represented as

$$\int_{-N}^{N} rect(f) \cdot e^{j2\pi ft} dt = \frac{1}{\pi t} sin\left(\frac{\pi t}{T}\right), \quad (13)$$

where N=$1/2T$, and T is the sampling interval. If $f_R(t)$ is the inverse Fourier transform of $F_R(f)$, then

$$f_R(t) = f(t) * \frac{1}{\pi t} sin\left(\frac{\pi t}{T}\right) = \int_{-\infty}^{\infty} f(t') \frac{1}{\pi(t - t')} sin\left\{\frac{\pi(t - t')}{T}\right\} dt'. \quad (14)$$

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As $f_R(t)$ is the discrete signal, $t' = nT$. Then,

$$f_R(t) = \sum_{n=-\infty}^{\infty} f(nT)\frac{1}{\pi(t-nT)} sin\left\{\frac{\pi(t-nT)}{T}\right\}, \quad (15)$$

where $t = kT/4$; IFFT size increases to 4N, which is the size of 4-times the oversample. As indicated in (15), the time-domain signal after the frequency domain filter is represented by the sum of the *sinc* function. As shown in the red waveform in Figure 5, the output signal is formed so as to smoothly complement between the original signals.

Figure 6 depicts the actual time-domain signal after the frequency domain filtering. The red waveform is the signal before the frequency domain filtering, and the blue waveform is the signal after the frequency domain filtering. As indicated in Figure 6, the oversampling interpolates the part of the transient response between the original signal points, i.e., the 4-times oversample interpolates three points between two signal points.

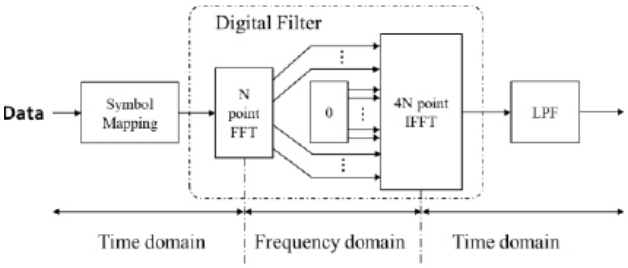

Figure 2. Configuration of DFT-s-OFDM

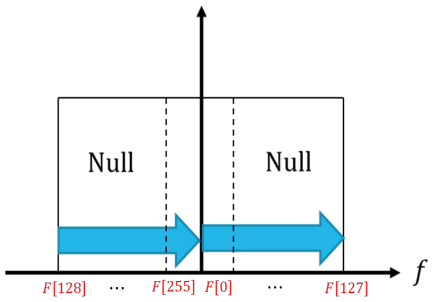

Figure 3. The data position of the FFT block

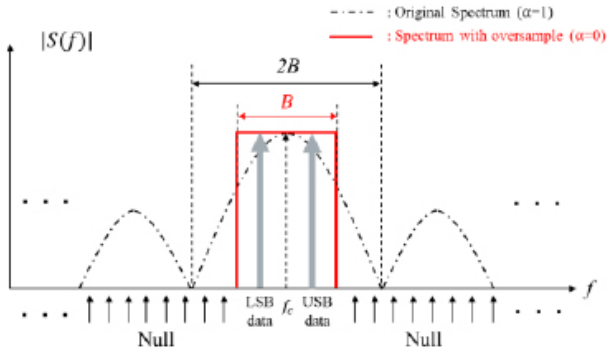

Figure 4. Spectrum with the digital filter

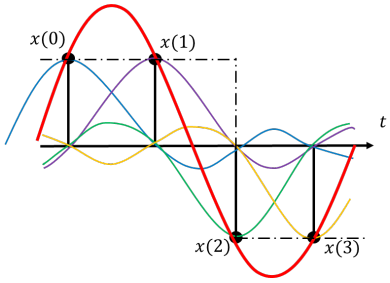

Figure 5. Time domain signal $f_R(t)$, as in (15)

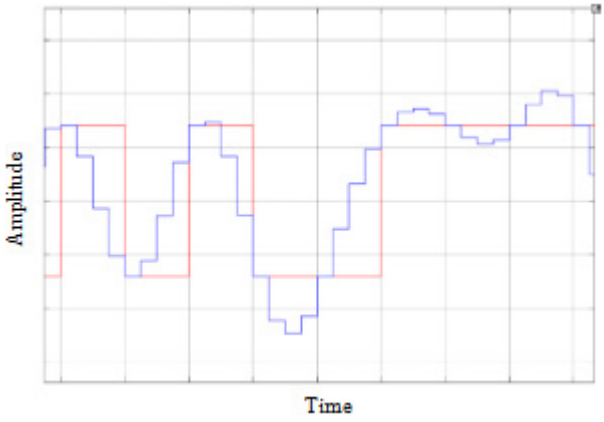

Figure 6. the signal after the frequency domain filtering

5. Proposed Method

In this section, our proposed SSB modulation scheme using the frequency domain filtering is described. In the proposed method, the SSB modulation processing is incorporated into the frequency domain filtering, by utilizing the complex conjugate centered on Direct Current (DC) component of the spectrum of the Amplitude-Shift Keying (ASK).

On the transmitter side, the signal $F(k)$ when FFT is applied is represented as (10). Here, assuming that the ASK signal is $f(t)$, the frequency component $F(k)$ is the complex conjugate centered on N/2 as depicted in Figure 7 since $f(t)$ does not have the quadrature component. In order to perform SSB modulation processing on the frequency domain filtering, only the data $S(k)$ with $k = 0 \sim N/2$ is used from $F(k)$ as depicted in Figure 8, and the other data are removed by multiplying by zero. Then, when $S(k)$ is converted into the time domain signal by IFFT, the original signal $f(t)$ is generated on the in-phase component and the Hilbert component $\hat{f}(t)$ is generated on the quadrature component.

In this way, the spectrum of the SSB modulation signal is formed after the quadrature multiplexing. On the other hand, in the case of the symbol of QAM containing the quadrature component, the signal on the frequency

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domain is asymmetric centered on DC so that the SSB modulation processing cannot be performed. In this paper, the frequency domain filtering is contained both the above SSB processing and 4-times oversample. The part of the proposed transmitter configuration is depicted in Figure 9.

On the receiver side, the spectrum copy is introduced as a demodulation method of Received signal $R(k)$. As can be seen from Figure 10, the spectrum copy complements the complex conjugate data of the original received signal $R(k)$ with the part removed by the transmitter side. Since the in-phase component is the original signal $f(t)$ on the receiver side, the demodulation is possible without implementing the spectrum copy that uses the quadrature component. However, the spectrum copy is needed to improve the CNR by 3dB on the receiving side. The reason for the improvement in CNR is that the proposed demodulation method is almost the same as the DB-SSB modulation scheme [12]. The variance σ^2 is doubled and the average amplitude E_b of the signal is also doubled in AWGN channel environment by implementing the spectrum copy that mixes the in-phase component and the quadrature component. Therefore, $|E_b|^2/|\sigma|^2$ is improved by 3dB.

In addition, The SSB processing by the proposed frequency domain filtering improves PAPR over the conventional SSB modulation scheme. Figure 11 compares the 4-times oversampled baseband time domain signals of the conventional SSB modulation method and the proposed SSB modulation method. Since the SSB processing by the frequency domain filtering can be performed before D/A conversion, the peak power generated by SSB modulation processing is reduced.

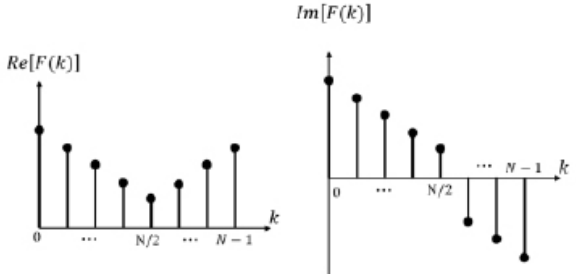

Figure 7. Spectrum of ASK signal with complex conjugate

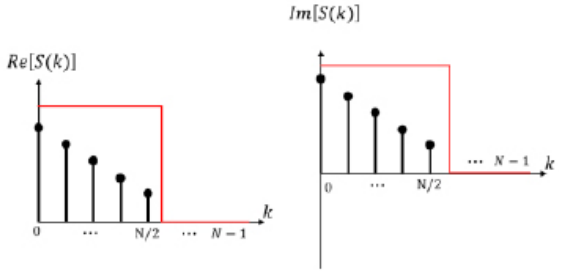

Figure 8. Spectrum of SSB-modulated ASK signal

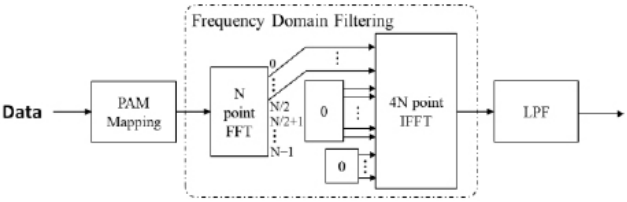

Figure 9. Transmitter configuration using the frequency domain filtering

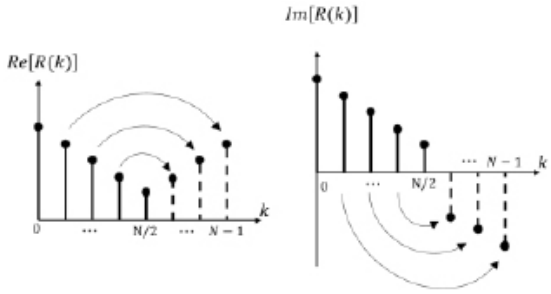

Figure 10. Spectrum copy

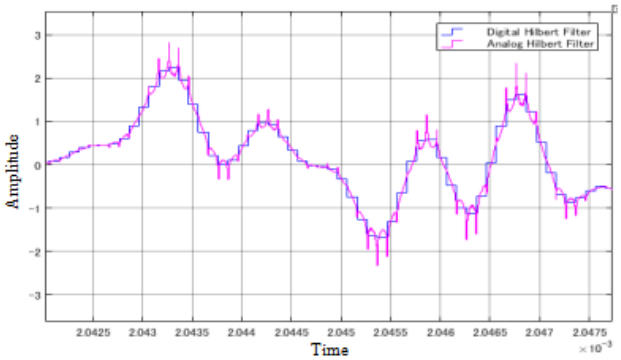

Figure 11. Comparison of the conventional SSB processing and the proposed SSB processing

6. Results

In this paper, the spectrum, PAPR, BER performance of the SSB 16PAM signal using the frequency domain filtering are confirmed. Under AWGN channel environment, BER performance of the proposed scheme is superior by 3 dB in terms of CNR to DSB 256QAM scheme with the same frequency efficiency and same PAPR.

6.1 Simulation Specification

A simulation using MATLAB/Simulink was performed for our proposed method. Table 1 illustrates the simulation specifications used in this research. As illustrated in Table 1, the SSB 16PAM is compared with the DSB 256QAM. Similarly, other PAM-based SSB modulation methods are compared.

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Figure 12 depicts a block diagram of the proposed system on the transmitter side. When evaluating the spectrum and BER performance, the proposed SC-FDMA SSB 16PAM depicted in (a) and SC-FDMA DSB 256QAM depicted in (b) are compared. On the other hand, when evaluating PAPR, the proposed SSB 16PAM depicted in (a) and the conventional SSB 16PAM depicted in (c) are compared. In this paper, Complementary Cumulative Distribution Function (CCDF) is used to evaluate PAPR. CCDF is represented as follows:

$$Pr(PAPR > PAPR_0) = 1 - \int_{-\infty}^{PAPR_0} \varphi_p(x)dx, \quad (16)$$

where $\varphi_p(x)$ is the probability density function of PAPR. CCDF indicates the probability that PAPR above a certain value $PAPR_0$ is present in the symbol.

Figure 13 depicts a block diagram of the proposed system on the receiver side. The interleave function in Figure 13 regenerates complex conjugate components which were removed in the transmitter side. Here, the regeneration is performed by permutation of the first half of N point FFT components as shown in Figure 10.

Table 1. Simulation Specification

Parameter	Proposed system	Comparison system
Primary Modulation	(4PAM,8PAM), 16PAM	(16QAM,64QAM), 256QAM
Secondary Modulation	SC-FDMA with SSB Modulation	SC-FDMA
FFT Size	64	64
IFFT Size	256	256
Data Rate	4Mbps	4Mbps
Carrier Frequency	40.25 MHz	40 MHz
Data Size	Single Carrier	Single Carrier
Channel Model	AWGN	AWGN

6.2 Simulation Results

Figure 14 depicts the spectrum of the SC-FDMA SSB 16PAM and SC-FDMA DSB 256QAM. Figure15 depicts CCDF of the proposed method. Figure 16 depicts the BER performance in the CNR of the proposed method. Figure 17 depicts the BER performance in the case of other modulation orders.

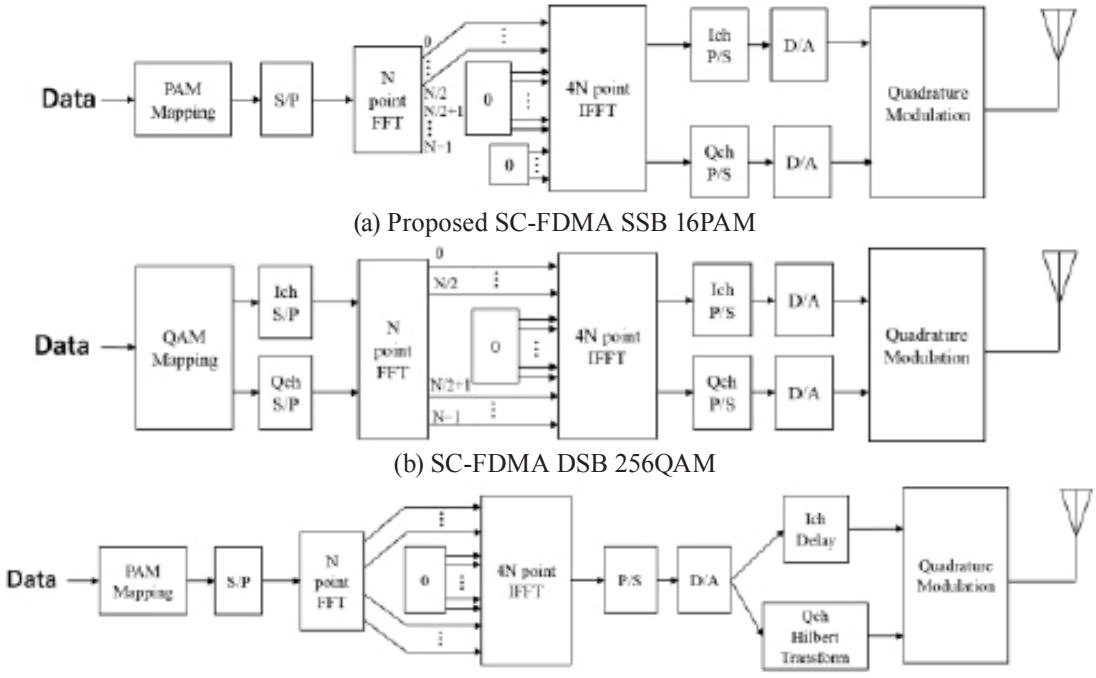

Figure 12. Block Diagram on the Transmitter Side

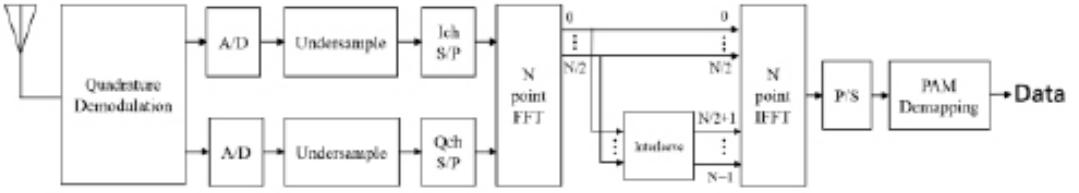

Figure 13. Block Diagram of the Proposed System on the Receiver Side

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As observed from Figure 14, it can be seen that the proposed method forms the spectrum with roll-off rate of zero. In the spectrum of the proposed method, the left side of the main lobe suppresses the side lobe by the 4-times oversample, and the right side suppresses the side lobe by SSB modulation processing. Since the radiation level of the side robe is lower than 40dBm, the spectrum of the proposed method is considered to satisfy the spectrum mask that defines the upper limit of the allowed radiation level. In addition, since the spectrum bandwidths of SC-FDMA SSB 16PAM and SC-FDMA DSB 256QAM are the same and have the same data rates from Table I, it can be confirmed that those spectrum efficiencies are the same.

As observed from Figure 15, focusing on $PAPR_0$ with respect to 0.01 of CCDF, it can be seen that the proposed SSB modulation method is 2dB superior to the conventional SSB modulation method that performs Hilbert transform after D/A conversion. The proposed method has PAPR equivalent to SC-FDMA DSB 256QAM by suppressing the peak signal generated by Hilbert transform, which is a drawback of SSB modulation scheme.

As observed from Figure 16, focusing on 0.001 of BER, it can be seen that the SC-FDMA SSB 16PAM has 3dB better BER performance under the AWGN environment in CNR than the SC-FDMA DSB 256QAM. The reason why the BER performance of the proposed method is superior to that of the comparison method is that the proposed method implements the spectrum copy on the receiver side. When the spectrum copy is performed on the received signal, the data is supplemented to the part on the frequency domain that was removed on the transmitting side, so the signal amplitude E_b after the spectrum copy is doubled. In contrast, the noise applied to the receiving side is different between the in-phase component and the quadrature component, and the variance σ^2 is doubled at the time of spectrum copy. Therefore, CNR of the proposed method can be improved by 3dB.

As shown in Figure 17, it is confirmed that the BER performance of the proposed SSB M-PAM methods is 3dB superior to that of the corresponding DSB M-QAM methods even if the number of the modulation order changes.

Finally, the proposed method and the Q-SSB modulation method described in Section Ⅱ are compared. Since the Q-SSB modulation method using the turbo equalization contains Hilbert components in each of the in-phase component and the quadrature component, the eye pattern is significantly deteriorated and the BER performance is also deteriorated when the roll-off rate is low [7]. However, in the proposed method, the original signal and its Hilbert component are separated into the in-phase component and the quadrature component. Therefore, it can form the practical spectrum with a roll-off rate of zero without deteriorating the BER performance.

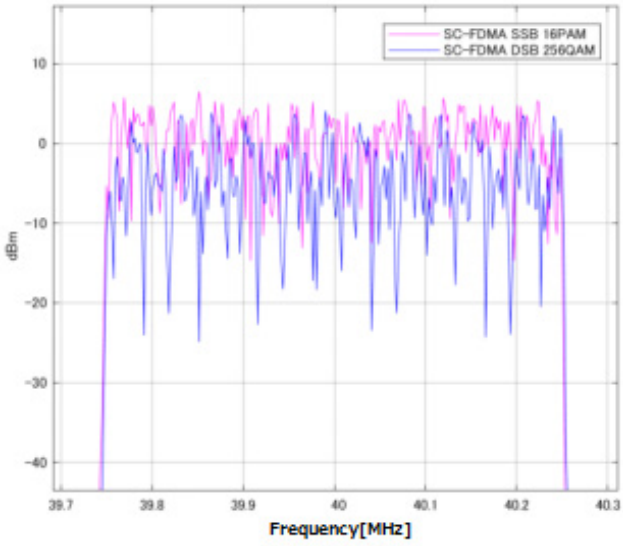

Figure 14. Spectrum of the proposed method

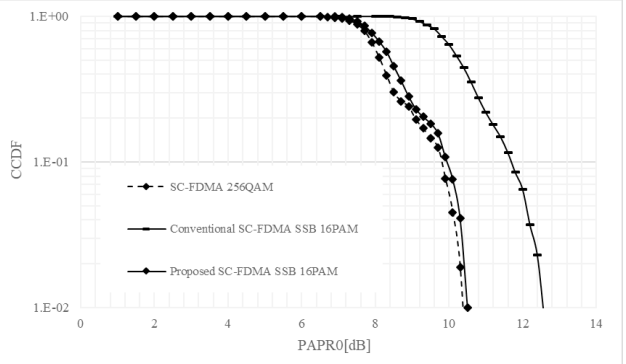

Figure 15. CCDF of the proposed method

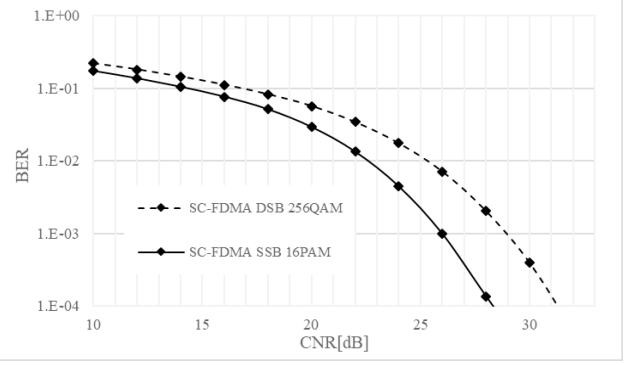

Figure 16. BER performance of SSB 16PAM and DSB 256QAM

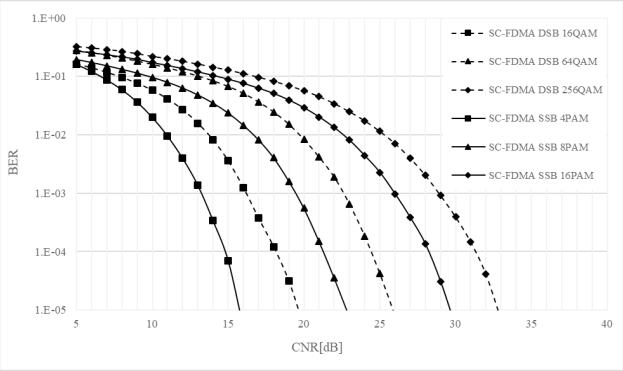

Figure 17. BER performance in the case of other modulation orders

7. Conclusions

In this Paper, 16PAM-based SSB modulation scheme using the frequency domain filtering is proposed. The proposed method prevents the deterioration of the quality due to the extra Hilbert component, and suppresses the peak power generated by the Hilbert transform. It has been confirmed that the BER performance of the SC-FDMA SSB 16PAM is superior by 3dB to the SC-FDMA DSB 256QAM for the same spectral efficiency and PAPR. Its characteristic of the proposed method is the same even if the modulation order changes. Therefore, in the case of a single-carrier transmission, it is considered that the PAM-based SSB modulation scheme is superior in terms of the quality to the QAM-based DSB modulation scheme.

References

[1] H. Waraya and M. Muraguchi, "Proposal of Single Sideband Modulation Scheme with Ideal Low Pass Filter for Wireless Communication Systems [J], "International Journal on Advances in Telecommunications, VOL. 13, NO. 3&4, pp. 33-42.

[2] H. Waraya and M. Muraguchi, "Proposal of a Quadrature SSB modulation Scheme for Wireless Communication Systems [D]," The Nineteenth International Conference on Networks (ICN2020), pp. 1-6, 2020.

[3] B. Pitakdumrongkija, H. Suzuki, S. Suyama, and K. Fukawa, "Single Sideband QPSK with Turbo Equalization for Mobile Communications [J]," 2005 IEEE 61st Vehicular Technology Conference, pp. 538-542, May 2005.

[4] Y. Jiang, Z. Zhou, M. Nanri, G. Ohta, and T. Sato, "Performance Evaluation of Four Orthogonal Single Sideband Elements Modulation Scheme in Multi-Carrier Transmission Systems [J]," 2011 IEEE 74th Vehicular Technology Conference, pp. 1-6, September 2011.

[5] Y. Jiang, Z. Zhou, M. Nanri, G. Ohta, and T. Sato, "Inter-Signal Interference Cancellation Filter for Four-Element Single Sideband Modulation [J]," 2012 75th Vehicular Technology Conference, pp. 1-5, 2012.

[6] A. M. Mustafa, Q. N. Nguyen, T. Sato, and G. Ohta, "Four Single-Sideband M-QAM Modulation using Soft Input Soft Output Equalizer over OFDM [J]," The 28th International Telecommunication Networks and Applications Conference (ITANAC 2018), pp. 1-6, November 2018.

[7] B. Pitakdumrongkija, H. Suzuki, S. Suyama, and K. Fukawa, "Coded Single-Sideband QPSK and Its Turbo Detection for Mobile Communication Systems [J]," IEEE Transactions on Vehicular Technology, VOL. 57, NO. 1, pp. 311-323, January 2008.

[8] X. Wang, M. Hanawa, and K. Nakamura, "Sideband Suppression Characteristics of Optical SSB Generation Filter with Sampled FBG Based 4-taps Optical Hilbert Transformer [J]," 15th APCC, pp. 622-625, 2009.

[9] C. C. Tseng and S. C. Pei, "Design of discrete-time fractional Hilbert transformer [J]," IEEE International Symposium on Circuits and Systems, pp. 525-528, May 2000.

[10] K. Takao, N. Hanzawa, S. Tanji, and K. Nakagawa, "Experimental Demonstration of Optically Phase-Shifted SSB Modulation with Fiber-Based Optical Hilbert Transformers [J]," OFC/NEOEC, 2007.

[11] J. G. R. C. Gomes and A. Petraglia, "A switched-capacitor DSB to SSB converter using a recursive Hilbert transformer with sampling rate reduction [J]," ISCAS 2000, pp. 315-318, 2000.

[12] S. A. Mujtaba, "A Novel Scheme for Transmitting QPSK as a Single-Sideband Signal [J]," IEEE GLOBECOM 1998, pp. 592-597, 1998.

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ARTICLE

User and Systems Requirement Framework for Electronic Land Registration Systems Decentralisation in Ghana

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ABSTRACT

Although land registration systems are constantly changing based on entrenched institutional frameworks, they cannot fulfill their technical objectives. Technical designs based on loopholes in the existing land registration system are the common steps in advanced countries. However, the missing link between the implementation of technical designs and their uses and users, affect the prospects of land registration decentralisation in developing countries. User and System requirements are used to integrate and enhance land registration services delivery across land information systems. This study assesses and develops framework for decentralised electronic land registration systems in Ghana. Semi-structured interviews were used to collect user and system requirement data from clients and technical staff of the Lands Commission in Accra. The framework pointed out five main policy outline strategies. These include land registration process modeling (process reduction and turnaround time monitoring), technical function (file tracking), the effect of changes in the registration loupe and absorption of land registration report, land registration workflow improvement, and the use of aerial images. The study recommends a systemic monitoring and evaluation of staff job roles.

1. Introduction

Land administration represents government concern to provide security of tenure and information about land tenure for property markets and public activities. Land administration systems offer structures for implementation of land management policies and strategies. These include processes and frameworks, adherence to standards, legal frameworks, land information systems, and users required to contribute, support and control the use of these structures [11]. A significant increase in land transaction and land development projects are the driving force of these structures [15].

Based on that governments across the globe have taken advantage of electronic platforms to facilitate land registration for economic transformation. However, most land information systems have limitedly achieved their intended aims due to lack of adherence to user

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and systems requirements. Governments are interested in harmonising policies and reforms as system design strategies than recognising the need for consultation and feedback from users and stakeholders in the design process [5]. These lead to staff resistance across most developing land administration systems. The concept of user and system requirements is usually applicable in the manufacturing and computer industries. This is because, it is fairly recognized that about 80% of production in the manufacturing industry is determined by the collaborative decision of stakeholders in the early stages of the design cycle, therefore the ability of stakeholders to understand what they need and expect from the production plays an important role in the industry's development [2]. User and system requirements are major components for enterprises to improve their product design. So a precise thought of customer requirement information is vital to provide dependable design decisions. In today's competitive global marketplace, competitive advantage lies with those companies that understand and respond quickly to dynamic user and systems requirements that guarantee quality, reliability, and performance [32]. However, most land registration practices can be said to be monopolistic. That is, land registration systems are managed and controlled by centralised state institutions [20,4,18,27]. However, one of the key visions of every land administration institution is to satisfy user needs through the application of soft land registration systems and practices thereby ensuring tenure security. This can be achieved by an increase in public responsiveness regarding benefits of the formalisation of property rights, receive feedback on public acceptance, and evaluation of the land registration services provided [23]. Feedback from the land registration system requires a spatial definition of registered administrative sectors in varied land registers, coordinated exclusively to capture the spatial extent of tenure, land institutional efficiency, and improved data integration [10]. Others argue that efficient management of geographic data and data interoperability among different institutions and stakeholders helps to improve the land cadastre [1,4,11]. In Ghana, the land registration system is partly electronic and complex. The procedures are long due to interwoven classes of land ownership and registration requirements [5]. Often, there are institutional challenges such as overlapping claims of ownership by users, unclear land registration processes, presence of unknown staff (ex-national service workers), poor conditions of service, lack of collaboration among the land registration institutions and their stakeholders [3,8]. Few studies on land registration have been carried out in Ghana to identify user needs with the view to satisfying them[3,5,8,33]. Until now these studies have focused on analysing land administrative practices, and they proposed numerous theoretical approaches for proper land registration system at the centralised level in Ghana. However, these studies have ignored the technical gap (system requirements) and user requirements needed for the implementation of a comprehensive land registration system at the decentralised level in Ghana. Therefore, this study assesses the prospects of a decentralised electronic land registration system from the perspective of user and system requirements. Users of land registration systems have a gatekeeping role in ensuring an efficient and transparent land registration system through collaboration with land registration institutions. In Ghana, users who are unable to register their lands are often disadvantaged. This contradicts the view in customary land law in Ghana that land rights are birthrights. This study contributes towards the facilitation and improvement of the electronic land registration system in Ghana. Aside from that, there are benefits in this study for the drive to scale up the integrated land information system to other regions in Ghana. Theoretically, the study contributes to the land administration discourse by fusing user and system requirements approaches from computing and manufacturing domain to land registration. Practically, identifying systems and user needs will avoid staff resistance, improve land registration turn-around time, and improve productivity across the value chain. The paper is structured in six sections. Section two reviews literature on Ghana's land registration systems, and user and systems requirements. Section three presents the methodology of the study. The results are presented in section four and discussed in section five. The conclusion is presented in section six

2. Ghana's Drive towards Electronic System of Land Registration

The drive to transform land registration systems from manual to electronic systems started in 1999. The 1999 national land policy heralded the commencement of land registration reforms [5]. The policy provided a guideline that saw the implementation of the Land Administration Project (LAP) I and II. LAP I aimed to 'fashion a well-functioning and sustainable land administration system that ensures efficient, decentralised, fair, and cost-effective decentralised land administration (World Bank, 2013, p. 4). The First Phase of the project aimed to (i) improve valuation and information, and land titling registration (ii) harmonising land regulatory and policy framework for sustainable land administration and (iii) institutional development and reform. In 2012, LAP II introduced

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new departments within the Lands Commission to serve as an entry point for electronic transactions [5]. These include the Clients Services Access Unit (CSAU) and the Ghana Enterprise Land Information System (GELIS). Clients Services Access Unit (CSAU) aimed to improve service delivery through the provision of customer-focused and quality services to the general public. The CSAU serves as case management and a single point of contact that safeguards between the client information and the complex processes of land registration. In 2016, the government aimed to re-engineer the business processes of land registration with the introduction of the Ghana Enterprise Land Information Systems (GELIS). The GELIS facilitates full automation of the land registration processes and provides a holistic Land Information System that permits all users including public land institutions and other government institutions to digitally carry out their business activities effectively and efficiently using common databases.

2.1 User Requirements in Land Administration

Sustainable land registration systems are required to meet the needs and benefits of the people concerning their relationship to the land [9]. However, these require that well-functioning land registration processes are built around the organization's mandate driven by stakeholders and user needs [14]. A study in Turkey showed that land registration projects in the 1990s failed because stakeholders did not meet data infrastructure standards due to the failure to integrate user needs [1]. Recently, a study on the institutional design of the land administration system in Ghana confirms this claim [5]. Studies on user requirements in land administration and management are limited. User requirements serve as means of data gathering in the form of service quality, time, dissemination, and storage for a well-functioning land registration system [7]. A study showed that user requirements answer the following questions: What do the users or clients need and want from a new system? Who needs the information and what information is necessary? For what purpose will the information be used by the client? When, and in what time interval does the client need the information? In what quality, standard, and quantity of information is needed as a minimum or maximum? In what way is information transferred to related persons or institutions? [25]. Another study revealed that user requirements are enabled by interviewing users and officials through collecting data in the form of rules, regulations, reports, complaints, and recommendations of land registration [28]. A study in Macedonia shows the use of questionnaires as a means to gather user requirements from both external sources (NGO's), citizens and internal sources (Government officials) to improve the business services of its Cadastral Organisation [29]. The integration of user requirements in the system design of organisations leads to the performance of its business services and the opportunity for further improvement in the existing system [30]. User requirements also enable one to know the gap in the existing architecture and then help to translate this requirement into the system design. Technically, model building in support of the land administration domain depends on user requirements that support land administration system implementation [16].

2.2 System Design and Implementation

Land administration systems in some parts of the world are confronted with a lack of spatial and descriptive land information, parcel mapping and outdated maps, metadata, common standards for descriptive data, duplication, and storage of the same information into individual systems [25]. Others are noncontinuous data update, technology systems in some agencies that might complicate the interoperability amongst systems, tracking of information changed and accessed, insufficient control of internal user access, and security of the information accessed [25]. System requirements form the foundation for the original assessment and ideas for developing and validating any product [12]. Aside from that system requirements are established to adhere to the wishes of the client and communicate with the client, thereby improving the ability of the designer to react to changes in the client's wishes, and to explain design choices to the client [21]. Technically, systems requirements control the logical process of an electronic system which cannot be distracted by implementation details [12]. This strategy enhances communication accuracy between the designer and the client while being easier to comprehend [12]. Systems requirements are mostly characterised by the ability of the system to access data. Data accessibility includes the ability to visualise, query, generate tables [31]. An electronic system review in China showed the need to impute the latest data of clients, correcting the existing data, and deleting outdated data, to ensure data integrity and recognition [17]. A similar study proved that database systems are the supporting tool for manipulating data [31]. The database should be able to support a diversity of data types and formats, including but not limited to: spatial data imagery, raster, vector, tabular data, documents, photographs, static and time-series, and URL links [31]. System requirements in land administration in some jurisdictions are based on the ability of the system to improve transparency and system integrity through the

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ability of the system to generate land reports, digitize all interest in land, identification, and elimination of problems of double plotting and double referencing for titles, carry out instant land searches, and creation of digital base maps [13].

3. Materials and Methods

The case study approach was used to study the land registration system in Accra. The study assessed the challenges of user and systems requirements of the electronic land registration system in Accra. Accra was selected because the electronic land registration system is being piloted in Accra. Primary data was collected from the technical staff of the different divisions of the Lands Commission. The divisions include the Public and Vested Land Management Division (PVLMD), Client Service and Access Unit (CSAU), Land Title Registry (LTR), Land Valuation Division (LVD), and the Survey and Mapping Division (SMD).

3.1 User and Systems Requirement Definition

system users are defined as clients who register their lands at the Lands Commission and the staff working with the land registration system. Users (clients) were selected using the simple random sampling technique. A list of 100 selected clients was obtained from the Land Title Registration Division of the Lands Commission. These form 5% of clients who received their land title documents in 2021. Key technical users (staff) were purposively selected from all the divisions. Semi-structured interviews and key informant interviews were used for clients and technical staff respectively. User requirements focused on land information verification, data processing, and services quality. System requirements on the other hand are electronic land registration system capabilities required for efficient system operation and services delivery. System requirements focused on parcel search systems ability, document tracking, systems monitoring and evaluation, and database availability

3.2 Data Analysis

The data were grouped into themes and analysed with Atlas Ti software. Quotations and descriptive statistics were used to present a summary of user and systems requirements.

4. Results

4.1 Brief Description of the Land Registration Processes in Ghana

The study found three distinct levels of land registration processes in Ghana depending on the land ownership type. These processes include the private (family lands) land registration processes, stool lands registration processes, Public and Vested Lands registration processes. However, the processes are not straightforward but follow an up-down approach of data flow within every division of the Lands Commission. The main institutions responsible for the registration of these land ownership types include the Survey and Mapping, Client Service Unit and Access Unit, Lands Valuation Division, Public and Vested Lands Management Division, and the Land Title Registration Division. Within these divisions, some units perform specific tasks to ensure the movement of land documents. Aside from these institutions, other stakeholders include the customary land secretariat and the Land Use and Spatial Planning Authority.

The Customary Lands Secretariat is the institution that is responsible for recording rights in the land on behalf of customary landowning groups such as the family and stool in Accra. The Land Use and Spatial Planning Authority is responsible for providing and allocating land use plans to clients. The key land registration documents required include an indenture, Site plan, Allocation note, and consent letter. Among these documents required, the survey showed that 46% and 43% of respondents used site plan and the indenture as an entry point in the registration system. The purpose of these two documents according to respondents is to give them the legal backing of parcel ownership even before the registration process, but it cannot confer title to the land. The other 11% of respondents admitted the use of a consent letter as an entry point. The reasons for consent letters are respondents who have ever registered state or vested lands. For consistency in land registration requirements, an officer at the CSAU confirmed that:

> [...] the said two documents must conform to legal, survey, and mapping standards of the Lands Commission. This includes the signature of a licensed surveyor and a solicitor from the high or supreme court of Ghana.

The Survey and Mapping Division investigates and prepares cadastral plans on which plotting of parcel coordinates is done to assist the Land Title Registry to issue land title certificates. The Survey and Mapping Division interacts with the Land Title Registry and Public and Vested Lands Division in delivering survey search and mapping request. The complex nature of land ownership and registration processes requires detailed background checks before the purchase and registration of lands to avoid double sales and land disputes in Accra. The

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study discovered that lands are registered for sale, lease, mortgage, and use (see Figure 1).

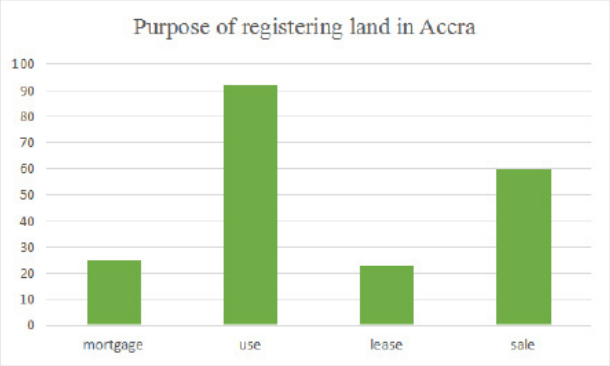

Figure 1. Purpose of registering land in Accra

4.2 Cross-Cutting Issues of Electronic Land Registration Processes

The issues are related to the verification and document processing in the land registration system.

4.2.1 Verification of documents

The emerging issues regarding verification are division-specific. These include; parcel search, capabilities of assessment report templates, and inconsistency in manual, digital verification of site plan, and electronic deficiencies. The Customer service and Access Unit is the first point of information flow in the land registration system. Concerning parcel search, an officer at the CSAU explained that:

> [...] Verification of documents interface does not permit the Customer Services and Access Unit attendant to absorb and write a report on the status of a client's application. The Customer Service and Access Unit verification process does not track the progress of a client's application"

According to the CSAU, the system supports the search of documents. However, due to the high volumes of applications received in a day, it makes it difficult for the system to provide quick search reports. In other instances, because of a lack of sensitization regarding the processes of registration, some clients are requested to return when their documents do not meet their requirements during the verification process. According to a senior officer at the CSAU:

> [...] Customer Service and Access Unit receives on average 200 applications per day regarding first registration, which takes about a day to review all these applications when an applicant's documents meet all requirements.

Also, the design of the document's assessment template creates some difficulties for the assessors as they are not able to check the details of the documents, as such the registration interface remains secondary. According to one officer at the Lands Title Registry:

> [...] The template was designed not to include all relevant information that concerns statutory ones. According to the officer, this should appear on the same page to enable the assessment of documents without any difficulties. This delays the workflow because most registration processes get stuck somewhere in the flow where statutory or vested lands are registered.

Moreover, verification of stool lands at the Land Use and Spatial Planning Authority slows the process. Clients who wish to register stool lands can verify whether their lands conform with the master plan of Accra. When the request is made, the officers within the LUSPA also undertake another search on the field. These processes alone take a month to get a report. One side effect of this process is that the client is made to make double payment for site inspection.

There are conflicting interests in the visit and preparation of site plans from the Survey and Mapping Division and the Land Use and Spatial Planning Department. By law, the Land Use and Spatial Planning Authority visit the site when a client wants to put up a landed property. However, the registration of stool lands only gives them the mandate to verify an applicant's site in office but not on the field. This process needs to be wiped off as suggested by the Lands Commission.

Finally, verification of documents at all divisions was affected with internet problems.

> [...] According to an officer at the GELIS unit, the internet system goes off during the verification process. This affects the payment of fees and slows the whole process.

4.2.2 Data Processing

Processing in the study involves capturing, plotting, and back coding of search parcels at the Commission. The study found that there was non-capturing of relevant information including statutory information during the verification process. This affects the processing of search documents at the SMD. According to an officer at the SMD:

> [...] Some data which are required on the checklist are not provided especially the statutory items such as consideration/rent, solicitor, etc. by the front officers of the CSAU. Such documents must be returned at the point of submission to the Public and Vested Land Management Division for a check. This results in the back-and-forth movement of documents within these two divisions.

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Also, the back coded site plan processes introduced by the Lands Commission in the registration process was to ensure that site plans conform to survey standard and are assigned a secret number. However, this process delays because a certified surveyor must re-visit the site to determine the veracity of the site plan submitted by the client. According to an officer:

> [...] The yellow card we use before makes the registration process easy because you don't have to visit the field for any inspection because it takes two days for survey plans to be ready"

However, there are issues regarding the plotting of site plans for both stool lands and family lands between the two divisions. According to officers:

> [...] clients can plot their parcel at both survey division and Pubic and Vested Land Management Division. Therefore, there is the likelihood of double plotting hence stacking the process at the Pubic and Vested Land Management Division during the registration process. This results in disputes and the return of applications. It is also possible for a client to plot family lands at the Land Title Registry and the Survey and Mapping Division.

Sometimes parcel area or dimensions are not captured properly; hectare is sometimes used for acreage creating problems with assessment, hence delay in the release of a file to the next stage of the registration process. Also, inadequate survey and cartographic instruments slow the business processes of the Survey and Mapping Division. Most survey processes are laborious because the equipment (plotters) used are outmoded and not enough to quickly complete the survey and cartographic work of the Survey and Mapping Division. An officer at SMD indicated that:

> [...] Out of 211 survey plans received from the Land Title Registry, the Survey Division can process only 10 percent a day and forward to the cartographic department for plotting.

These make survey and plotting of site plans take one month to complete. There is a lack of a database in place for the Survey and Mapping Division; Pubic and Vested Land Management Division to ensure they keep up-to-date files. The Land Title Registry indicated that they do not have a database, and so when a file gets missing they go back to the cartographic department for missing pottered files.

4.3 User Requirements

The clients that use the land registration system include individuals, banks, estate companies, banks, estate companies, and firms. The informal sector workers in the registration process include traders. Collection of client requirements helps to get different views from clients on the land registration process and how their responses would change their present situation. User requirements are a relevant component of information technology because it allows for improvement of the existing design [32]. This implies that land registration should be built to meet the needs and benefits of the people concerning their relationship to the land.

According to most clients of the Lands Commission, their documents are not scanned at the entry point of the registration i.e., the Customer Service and Access Unit, as such they have to carry the documents from one office to the other which sometimes results in some documents or files getting missing. From the viewpoint of the Customer Service and Access Unit, all documents are scanned at the last stage of the process to get records of applicants. As a result, files get missing before they get to the last stage, especially when the number of applicants is many. The Customer Service and Access Unit suggests that it would be easy when applicants' documents are scanned at the entry point to allow digital movement of files. On the contrary, few clients support the fact that the scanning would be helpful because they have the feeling that their documents can be edited and their rights transferred to another party. Thus the general idea is that clients prefer the land registration process is digitized to avoid missing and spoiled documents, but are also concerned that they may lose the land rights altogether.

Moreover, Clients indicated that traveling a long distance to the Lands Commission is still something that needs to be addressed. Generally, the Client survey suggests that the processes should be reduced firstly and decentralised to improve the land registration process. This is because the Lands Commission in Accra serves the large population of Accra. Some officials indicated that the Metropolitan, Municipal, and District Assemblies (MMDAs) be given the right to register lands just as the Land Use and Spatial Planning Department. Clients indicated that there is a need to decentralise the processes to the Metropolitan, Municipality, and District levels so that the regional Lands Commission would have a seamless database to share data and information among them. This implies the need to interoperate data within local government offices (see Figure 2).

In addition, clients are of the view that sending text messages to them delays information sharing. According to clients, sharing information is affected by a poor mobile network. Furthermore, clients noted that the text messages which are sent out to applicants are not the best in terms of content and timing. Clients complain of receiving several

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text messages in respect of documents they know nothing about. These text messages are sent to them many times in a day. Some individuals complained of receiving more than 20 messages in a day. Thus, the clients suggested the use of mobile phone calls, emails, or letter posts. This is necessary because it would reduce the stress clients go through since some of the divisions are not located near each other. Therefore, application forms should make provision for other mobile contacts so that clients can be reached on time.

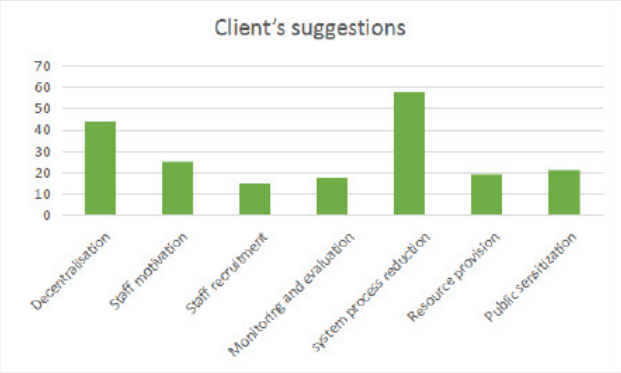

Figure 2. Client's suggestions

Survey site plans are necessary and should be checked by the Commission, but clients suggested that, going to the field for site inspection be discontinued. Rather, there should be a desktop system for verifying and inspecting land documents. This way, land valuation and ground rent assessment can be facilitated easily at the office.

Finally, Clients indicated that the plotting and signing stage of certification takes more time. According to clients, it takes sometimes, more than a month for the documents to get there. It was also found that there is no interface available to all the divisions to keep track of each other, and the movement of documents. The information shows that certifications are done every three months, as such, clients indicated that there should be a system in the workflow where an officer can be tasked to monitor and check the processes of registration to reduce the delays. As a result of these flaws in the system, the user perception score on registration time was average (table 1).

User perception of the land registration system

Table 1. Client perception index on the registration system

User perception	Level of perception	Percentage % of respondent	User description
Service quality • Delivery of information • Conformance with registration requirements • Reliability of service officials	Average	48	Messages are not delivered on time. Clients receive several text messages. Lack of awareness of land registration requirements delays the registration process because the client must come to the registration centre to inquire about the necessary documents. It is also not reliable to contact any of the officials without identification for registration. Some officials sometimes receive monies from clients that are not accounted for. This leads to double payments, most especially when a promised client's registration fail.
Data quality • Quality and content of the registration documents	Satisfactory	59	There is a lack of uniformity in the site plan template. Clients who contract 'quack' surveyors present site plans that lack all the requirements of a plan. Sometimes grid lines are missing. This makes it difficult for some plans to be approved at the Commission. The clients are of the view that when the Commission provides a uniform template to surveyors it will ease that.
Registration timeline	Average	32	Lands Commission is envisaged to reduce the land registration period to three months. But, the processes currently take about five months depending on the type of registration. where clients have queries of their files, the registration period sometimes exceeds six months. Even to years.
Registration cost	Average	20	Every stage of the registration process requires the payment of money. It is not known how much a client may legally pay for land registration services because the client pays unapproved fees to officials that are not accounted for. Therefore, the client is not able to provide the exact amount that is required to register their documents. Clients are part of this development because they influence these staff themselves to enable them to get quick registration. The poor client's document would be in the registration chain for years because they cannot pay

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4.4 System Requirements

These are system needs that enable the system to perform adequately. These requirements ensure efficiency, flexibility, interoperability, integrity, and testing of the new system [24]. To gather system requirement data, interviews were conducted at all the divisions of the Accra Lands Commission. Survey and Mapping Division indicates that there is a need to reduce the double search of a parcel on site. There should be an interface where all private surveyors would directly upload coordinates picked from the site to the Survey and Mapping Division to enable surveyors to directly prepare site plans on a uniform template. This will avoid the issues of indeterminate boundary, asymmetric and scaling problems of site plans, thereby improving parcel searches.

The use and access to the internal and external database should be the starting point within each division of the Lands Commission to enable them to track documents of applicants. Interviews from all the divisions show that officers need a database to store both spatial and non-spatial survey plans and historic files. This should support a multi-user database within and outside the Lands Commission..

Other system requirements include system monitoring and evaluation. According to the staff, there is no laid down system workflow indicating monitoring and evaluation of actors within each division. The study found that there is the need for well assembled and networked computers, well customized and functional intranet and internet systems that can ensure faster execution of data on the interface of the system within the divisions Lands Commission to ensure monitoring.

Provision should be made for copies (extra copies) presented by the client to be easily identified. The interface indicating the forms of every division should be created for the number of copies. This would reduce the complaints of submitting four documents, yet officers of a division are giving one or two depending on the type of registration. For example, the interface template should include all relevant information including statutory information.

The verification of documents interface does not permit the CSAU attendant to absorb and write a report on the status of a client's application. The CSAU verification process does not track the progress of a client's application. However, it supports the search of documents. Results show that the CSAU receives on average 200 applications per day regarding first registration and other registration inquiries which takes about a day to review all these applications when an applicant's documents meet the requirements of all requirements. This slows the preparation of reports on all inquiries made by applicants.

Technically, the study found that the workflow model should be able to support the survey maps and other attribute data, communication within other connected programs, and provide the level of access permission of officers within each division of the Lands Commission such as an ID or a password.

Finally, ensuring quick access to registration information was a challenge. To ensure quick access to registration information, it was suggested that there should be an online system where clients can upload and find information regarding the registration processes. These according to some staff, would reduce the mistakes clients make regarding what should be submitted. Table 2 shows system requirements from the client's perspective.

Table 2. Land registration system requirements from Lands commission staff perspective

System requirements	Institution concerned
System ability to reduce double parcel search	Survey and Mapping Division
Data interoperability	All divisions
System ability to monitor workflow performance	Administration
Transaction reporting and absorption of report	All divisions of the Lands Commission
Online registration of parcels	All divisions of the Lands Commission

5. Discussions

The purpose of user and systems requirements in this study is to link client and systems needs to draw a policy framework for an improved land registration system in Ghana. User requirements serve as means of data gathering in the form of service quality, time, dissemination, and storage for a well-functioning land registration [7]. The results have shown that user and system requirements can be analysed into process modeling and technical function. i.e., what clients need and how the clients want the entire land registration system to operate. In this study, it was found that the requirements that the user and system need can be analysed into three main themes; digitalisation and decentralisation of the system, reduction of the length of registration, and security of document (non-compliance with scanning). These would help to improve the business performance of the Lands Commission and open opportunities for further improvement in the existing system [29].

Concerning decentralization, clients indicated a need to decentralise the system in the MMDA's to enable

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easy registration. Although this finding has economic importance, it is not possible because Accra lacks a complete cadastral system where all parcels are mapped despite the declaration of every part of Accra as Title Registration District. This is likely to face challenges similar to a study in Kenya where data standardisation and data interoperability was an issue facing the implementation of its Land Information System (LIS) [19]. The ripple effect is that there could be chaos regarding determining parcel size and parcel boundary when the land registration system is decentralised. The best alternative to tackle the decentralisation of the system would be to reduce the length of registration process. Based on this we can infer that reducing the duration of the registration process can be the first stage. The second will be to decentralise it later after there is a complete cadastre in Accra. This can be achieved by a drive change in aggregating the processes through complete digitalisation and prototyping but not to reform local land institutions as suggested in some studies [21-23]. Even though, the digitalisation drive has its consequences regarding security. The study discovered that users require the digitalisation of land documents. That notwithstanding, it is necessary to consider the security of clients' documents during the registration process. Figure 3 presents a theoretical approach that can facilitate the technical perspective of digitalisation and decentralisation drive to complete land information systems design and implementation in Ghana.

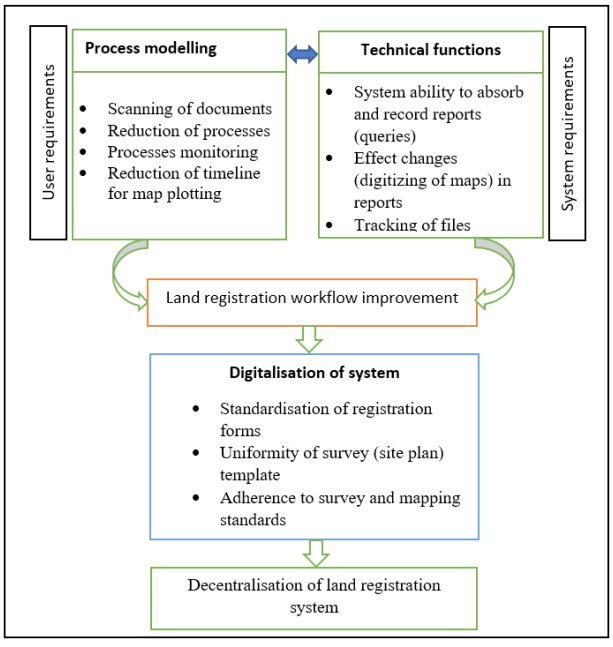

Figure 3. Electronic decentralised land registration systems framework

6. Conclusions

The theoretical background of this study identified the gaps concerning the application of user and system requirements in the land registration domain. The study revealed the emerging issues that necessitate the integration of user and system requirements in land registration systems. Based on semi-structured interviews, the paper developed a theoretical framework that defines overall user requirements and system design towards an efficient land registration system in Ghana. System requirements are the land registration systems capabilities required for an efficient land services delivery. The study found that the land registration process is confronted with poor coordination of functions, bureaucracy, inadequate logistics, inadequate staff, and monitoring equipment. Users require a streamline of the land registration processes, a decentralized system, scanning of documents at the entry point, desktop inspection of survey plans, and assessment of stamp duty. The study suggests that decentralisation of the system as an ultimate objective may be achieved through the use of aerial images as a starting point to facilitate desktop parcel inspection, survey, titling, and valuation. However, the drivers of data interoperability malfunctioning need to be checked. Technically, system design requires, absorbing processed reports, quick access to information, display of survey maps, and the ability to connect with other software programs. These have a significant implication on the transition of the manual system to a fully digitalised land information system. Finally, land registration system designers must be able to respond, predict and address clients' needs effectively during the registration process.

References

[1] Aydinoglu, A. C., & Bovkir, R. (2017). Generic land registry and cadastre data model supporting interoperability based on international standards for Turkey. *Land Use Policy*, *68*(March 2016), 59-71. https://doi.org/10.1016/j.landusepol.2017.07.029.

[2] Azadegan, A., Papamichail, K. N., & Sampaio, P. (2013). Computers in Industry Applying collaborative process design to user requirements elicitation : A case study. *Computers in Industry*, *64*(7), 798-812. https://doi.org/10.1016/j.compind.2013.05.001.

[3] Barry, M., & Danso, E. K. (2014). Tenure security, land registration and customary tenure in a peri-urban Accra community. *Land Use Policy*, *39*, 358-365. https://doi.org/10.1016/j.landusepol.2014.01.017.

[4] Bennett, R., Tambuwala, N., Rajabifard, A., Wallace, J., & Williamson, I. (2013). On recognizing land

 DOI: https://doi.org/10.30564/jeisr.v3i2.3692

administration as critical, public good infrastructure. *Land Use Policy*, *30*(1), 84-93. https://doi.org/10.1016/j.landusepol.2012.02.004.

[5] Biitir B. S., Miller, A.W., Musah, I, C, (2020). Land Administration Reforms: Institutional Design for Land Registration System in Ghana. *Journal of Land and Rural Studies*. SAGE. 1-25. DOI: 10.1177/2321024920968326.

[6] Dennis, A., Wixom, B. H., & Roth, R. M. (2012). *System Analysis and Design*. (B. L. Golub & E. Mills, Eds.) (5th ed.). United States of America: Don Fowley.

[7] Didigwu Augustus, O. O. M. (2016). The importance of cadastral survey information for effective land administration in Nigeria. *International Journal of Environment and Pollution Research*, *4*(1), 26-32.

[8] Ehwi, R. J., & Asante, L. A. (2016). Ex-Post Analysis of Land Title Registration in Ghana Since 2008 Merger: Accra Lands Commission in Perspective. *SAGE Open*, *6*(2). https://doi.org/10.1177/2158244016643351.

[9] Enemark, S., Bell, K., Lemmen, C., & Mclaren, R. (2014). *Building Fit-for-Purpose Land Administration Systems* (Engaging the Challenges, Enhancing the Relevance). Kuala Lumpur, Malaysia.

[10] Flego, V., & Roi, M. (2018). Land tenure registration on the marine areas in Croatia, (July 2017). https://doi.org/10.1016/j.ocecoaman.2018.03.008.

[11] Furuholt, B., & Oystein, S. (2015). Land Information System for Development (LIS4D): A Stakeholder Perspective. In J. . Steyn, J., Van Belle (Ed.), *9th IDIA conference: Beyond development. Time for a new ICT4D paradigm* (pp. 286-298). Nungwi, Zanzibar: IDIA.

[12] Hussain, A., Mkpojiog, E. O. C., & Kamal, F. M. (2016). The Role of Requirements in the Success or Failure of Software Projects. *International Review of Management and Marketing*, (2146-4405).

[13] Jacobs, G., & Orlova, N. (2014). *Development and Implementation of Land Information Systems : Building an Effective Partnership to Reform Uganda' s Land Administration and Management System* (FIG Congress 2014, Engaging the Challenges - Enhancing the Relevance Kuala Lumpur, Malaysia). Kuala Lumpur, Malaysia.

[14] Kalantari, M., Dinsmore, K., Urban-karr, J., & Rajabifard, A. (2015). Land Use Policy A roadmap to adopt the Land Administration Domain Model in cadastral information systems. *Land Use Policy*, *49*, 552-564. https://doi.org/10.1016/j.landusepol.2014.12.019.

[15] Kurwakumire, E. (2014). Digital Cadastres Facilitating Land Information Management. *South African Journal of Geomatics*, *3*(1), 64-77.

[16] Lemmen C., Oosterom P. V., Uitermark Harry, Z. K. de. (2013). Land Administration Domain Model is an ISO Standard Now. In *Annual World Bank Conference on Land and Poverty* (p. 21). World Bank, Washington DC: World Bank.

[17] Liang, Y., Sun, W., Diao, H., & Li, Y. (2011). The Design and Development of the Land Management System in Dingzhuang Town Based on Spatial Data, 57-65.

[18] Lorenzo Cotula, Camilla Toulmin, C. H. (2004). *Land Tenure and Administration in Africa: Lessons of Experience and Emerging Issues*. London: SMI distribution services.

[19] Mburu, P. N. (2017). *Strategies to modernize the land registration system in Kenya*. University of Groningen.

[20] Mitchell, D., Mwasumbi, A., Plessis, J. Du, Sait, S., Barnes, G., & Todorovski, D. (2017). Towards a Curriculum on Responsible Land Administration. In B. World (Ed.), *2017 World Bank Conference on Land and Poverty* (pp. 1-11). Washington DC.

[21] Mogk, N. W. (2014). A Requirements Management System based on an Optimization Model of the Design Process. *Procedia - Procedia Computer Science*, *28*(Cser), 221-227. https://doi.org/10.1016/j.procs.2014.03.028.

[22] Mwangi, E., Patrick, E., Kagwanja, J., Adams, M., Turner, S., Mcauslan, P., & Kameri-mbote, P. (2006). *Land Rights for African Development From Knowledge to Action*. Washington DC ; World Bank.

[23] Oput, R., Milledrogues, A., & Burke, C. (2018). *Implementation of the national land information system in Uganda: Strengthining land governance* (world bank conference on land and poverty). Washington DC ; World Bank.

[24] Oshana, R. (2015). System Requirements. In K. Fowler (Ed.), *Developing and Managing Embedded Systems and Products. Methods, Techniques, Tools, Processes, and Teamwork* (pp. 159-188). The Boulevard, Langford Lane, Kidlington, Oxford UK: Elsevier Inc. https://doi.org/10.1016/B978-0-12-405879-8.00006-4.

[25] Silva, S. K. (2010). *Design of a conceptual land information management model for the rural cadastre in brazil*. University of New Brunswick.

[26] Sittie, R. (2006). Land Title Registration . The Ghanaian Experience. In *XXIII FIG Congress, Shaping the Change* (p. 11). Munich, Germany: FIG.

[27] Stig, E., Clifford, K. B., Christiaan, L., & Robin, M. (2014). *Fit-For-Purpose Land Administration*.

DOI: https://doi.org/10.30564/jeisr.v3i2.3692

International Federation of Surveyors (FIG). https://doi.org/https://www.fig.net/resources/publications/figpub/pub60/Figpub60.pdf.

[28] Thuy, L. E. P., Zevenbergen, J., & Lemmen, C. (2012). *Investigating the Conformity between the Land Administration Domain Model and the Vietnamese Land Administration System* (Knowing to manage the territory, protect the environment, evaluate the cultural heritage No. 5545). Rome, Italy.

[29] Todorovski, D. (2006). *Developing an ICT Strategy for the State Authority for Geodetic Works in the Republic of Macedonia.* Univeristy of Twente, ITC.

[30] Todorovski, D., & Lemmen, C. (2007). *Analyses of User Requirements - The First Step towards Strategic Integration of Surveying and Cadastral Services Analyses of User* (Integration Approaches in Land Administration). Hong Kong SAR, China.

[31] USDA. (2018). System Requirements Specification for stewards. Conservation Effects Assessment Project.

[32] Wang, Y., Yu, S., & Xu, T. (2017). Advanced Engineering Informatics. A user requirement driven framework for collaborative design knowledge management. *Advanced Engineering Informatics*, *33*, 16-28. https://doi.org/10.1016/j.aei.2017.04.002.

[33] Zevenbergen, J. Systems of land registration aspects and effects. PhD thesis, editors Prof.dr.ir. M.J.M. Bogaerts,Prof.dr.mr. J. de Jong.Delft University. NCG, Nederlandse Commissie voor Geodesie, Netherlands Geodetic Commission, Delft (November 11, 2002). NCG Nederlandse Commissie voor Geodesie. Retrieved from http://www.narcis.nl/publication/RecordID/oai%3Atudelft.nl%3Auuid%3A44e404e9-c1e9-4c20-b1e1-977ee9c11570.

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ARTICLE

The Adoption of E-learning by Students in Zimbabwean Universities in the Wake of COVID-19

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ABSTRACT

COVID-19 effects have been felt in the education sector worldwide where schools, colleges, and universities were closed as a way to reduce the spread of the deadly pandemic and loss of lives. The Ministry of Higher and Tertiary Education. advocate that no child should be left behind during the COVID-19 era, therefore gave a directive for universities to use other alternative means of teaching and learning to continuously provide teaching and learning to students during the series of lockdown. An efficient eLearning system in universities is very important as an alternative to face-to-face teaching and learning in this COVID-19 era to have continuity in teaching and learning during the induced lockdowns. Success in online learning can be achieved by understanding the level of readiness of online learning environments. The main objective of the study was to evaluate the adoption of online learning by students in Zimbabwean universities. A descriptive online survey employing questionnaires to collect data on the adoption of eLearning by Zimbabwean universities students was used. Results indicated various eLearning platforms have been introduced in Zimbabwean universities though there is a need for eLearning infrastructure to be availed, students to be trained or students to effectively adopt the eLearning.

1. Introduction

The Coronavirus-2019 (COVID-19) has disrupted various sectors including health, economy, education, business, religion among others world over [1]. Geographical differences of case severity were witnessed with African countries being reported to suffer the most due to poverty and low testing capacity [2]. Zimbabwe is one of the African countries which were affected by COVID-19, especially the education sector. The policy on "Education for all" [3,4] has paused a challenge to the policymakers in the education sector. In response to the increase in confirmed cases of COVID-19 in Zimbabwe, The President of Zimbabwe ordered colleges and universities to close in March 2020, with an unspecified date of resumption [5]. With the ongoing surge of COVID-19 cases in Zimbabwe, an alternative to continuing providing education services while minimizing potential infection spread was eLearning.

Literature shows that a lot of developed countries have adopted eLearning systems for university education, for example, Libya since 2005 has adopted eLearning for Higher education Institutions according to [6] though the rate of adoption was very low [7] due to various barriers

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such as management barriers; technological barriers; cultural barriers, and barriers due to other factors.

Several developing countries have tried to embrace the new eLearning space [8]; however, financial and acceptance factors remain to be a problem that limits its potential use. Drawn on the existing pandemic and potential shift to full eLearning, this study has focused on the assessment of the adoption of eLearning by Zimbabwean university students. This research aims to investigate how Zimbabwean universities can successfully adopt eLearning systems during the pandemics such as COVID-19.

2. Literature Review

The COVID-19 pandemic has accelerated the use of Information Communication Technologies (ICT) in every businesses sector world over, including education [9]. To offer continuous education during the pandemic, eLearning systems had to be adopted as an alternative teaching and learning method [10]. eLearning system is defined as an educational system that delivers information using information technology resources such as internet, intranet, satellite broadcast, and multimedia applications [11]. eLearning systems provide a platform where students can interact with their lecturers and learning materials online. The advantage of adopting eLearning systems is that students can learn from anywhere, at any time, and in a shortened time. [12] identified six elements about eLearning system benefits which include: (1) connectivity, where students will be able to access information anywhere in the world. (2) flexibility, students will be able to access learning materials at any time anywhere. (3) interactivity, is the "dialogue" between students and eLearning tools through which students become engaged and involved in the eLearning process, (4) collaboration or the use of discussion tools can support collaborative learning beyond the classroom, (5) extended opportunities in terms of e-content which can reinforce and extend classroom-based learning, and (6) motivation where multimedia resources can make learning fun.

With eLearning systems, teachers and students can share information virtually and be able to upload and download materials in different formats (Video, audio, presentations, podcasts, documents, etc) [13]. eLearning systems have certain features that facilitate and nurture the learning-teaching process [14]. eLearning is Internet-based and does not require the installation of additional tools to access the system, once the materials are uploaded on the platform, the content can be accessed by students at any time [13]. eLearning includes elements such as technological tools and design, eLearning platforms, content, users and participants [8]. There are a variety of platforms that have been developed such as Google Classroom, Moodle, Blackboard, Sakai, Microsoft Learning Content Development System (LCDS), etc [8]. Some of the platforms are open access and other subscriptions have to be paid. Before the COVID-19 pandemic, eLearning systems have been used by most universities to complement traditional teaching methods.

The difference between eLearning systems and traditional methods is that eLearning systems do not only focus on instruction but also on learning that can suit individuals or groups. In the traditional setup, the education is more engrossed on the teacher as opposed to the student which is the main thrust of eLearning [15]. Differences between traditional and online learning may also be acknowledged in terms of principal sources of information, assessment, or quality of education. While in traditional education, students are evaluated only by teachers, who also represent their main source of information, and the quality of education is dependent on teacher's knowledge and skills, in online learning, the evaluation may be done with the help of tools and systems, students can acquire information from various documents uploaded on the platform, and the quality of education is influenced by the level of training that teachers have in using technology and also their teaching style [15]. [16] identified and described eight principles that stand at the core of effective online teaching, task time, encouraging students to allocate more time for completing tasks, high expectations, the teacher should communicate their expectations to encourage and motivate students, diversified learning, and technology application.

Researchers about eLearning systems have tried to investigate the reasons for the success or failure of adopting online learning systems [17]. As a result, there exist many factors as being important in determining the success of eLearning. According to [18], the general factors that affect the successful adoption of eLearning are the audience, course structure, page design, content engagement, usability. In this case, the audience is students who are a critical factor in the process of developing eLearning courses. Course structure refers to how a course is intended for eLearning, page design refers to the eLearning page design (navigation, appearance, balance between text and graphics, consistency, and ease of scanning), content management refers to how students interact with the content of the course (content, explanations, graphics, exams) and usability refers to testing eLearning content and applications in the same environment that the student will complete the course. The eLearning success model [18] was adopted for this research.

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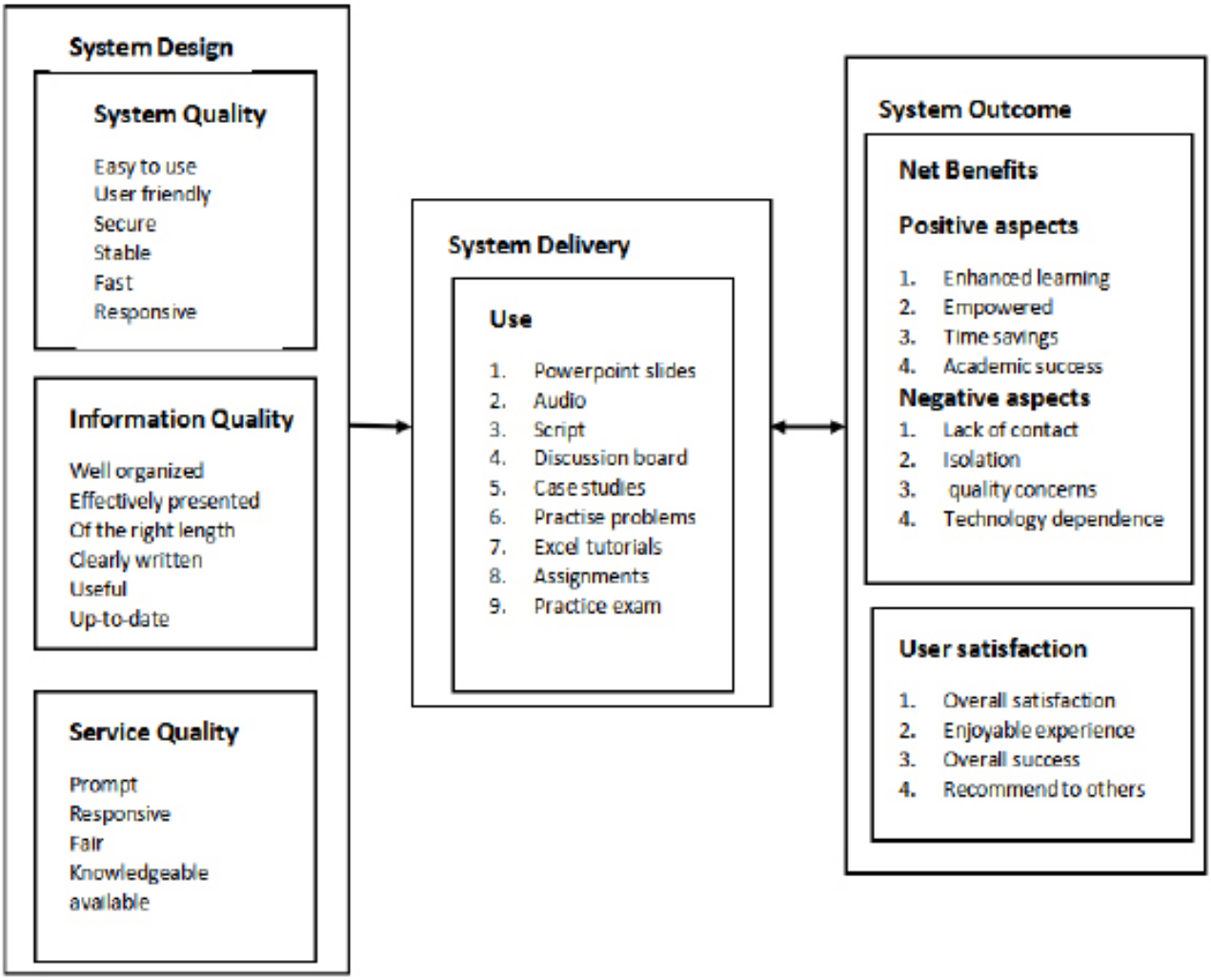

Figure 1. eLearning success model

As shown in Figure 1, the study is premised on the constructs of an eLearning success model based on the system design, service delivery, and system outcome. In the system design, the research focused on how user-friendly and accessible the eLearning system was to students. The students should also be provided with quality timely information which is well organized and useful. The quality of service for eLearning should be prompt and always available [18], With the advent of the COVID-19 pandemic, it meant that universities had to quickly adopt the use of eLearning systems at their workplace.

Higher education institutions are motivated to introduce innovative eLearning programs by expanding their educational boundaries during this COVID-19 era. Most of the universities in developing countries were not prepared for this new eventuality including Zimbabwe [2]. The development and implementation of successful eLearning systems are quite challenging to higher education institutions because of the investment required and the uncertain outcomes. This research aimed to identify the opportunities and challenges faced by university students in the adoption of eLearning systems in Zimbabwe. The study investigated how COVID-19 pandemic affected the adoption of eLearning and the challenges that were faced by students during online learning. Thus this research seeks to contribute to the successful adoption of eLearning systems in Zimbabwe or any other developing country during or post pandemics.

3. Methodology

This research was based on a case study of 20 Zimbabwean universities which include 13 public universities and 7 private universities. The mixed methodology was applied for the study where purposive sampling was used to collect data from students using an online platform due to COVID-19

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restrictions. 1158 students responded to the questionnaire and 30 interviews were conducted with final year students who had been exposed to the traditional face-to-face learning and eLearning methods. The interviews were further used to validate the collected data and fill in the gaps of missing information. The three constructs of the eLearning success model which are system design, system delivery, and system outcomes were used to guide the research. Data were systematically analysed and graphically presented.

4. Findings

The findings revealed that the higher and tertiary education system was disrupted due to the COVID-19 pandemic which forced institutions to switch to eLearning systems. This was important considering that the pandemic will be with us for a long time and there was a need to find continuous alternatives of providing teaching and learning services to students.

System Design

From Figure 2 above 84% of the students indicated that they own laptops, while 94% own smartphones that they use to access eLearning services. 10% of the students have access to desktops, 17% own tablets/ipads, and 0.04% of the students do not have access to any of the electronic gadgets. Most of the students possess the required devices which allow them to access eLearning services.

Figure 3 above shows that when off the campus, 59% of the students connect to the internet through smartphones mobile data, while 25% connect using home Wifi. 0.05% of students indicated that they use Wifi devices to connect to the internet and 10% of the students had no access to the internet at all when off the campus. The majority of the students have access to the Internet, hence easily be connected to the eLearning system.

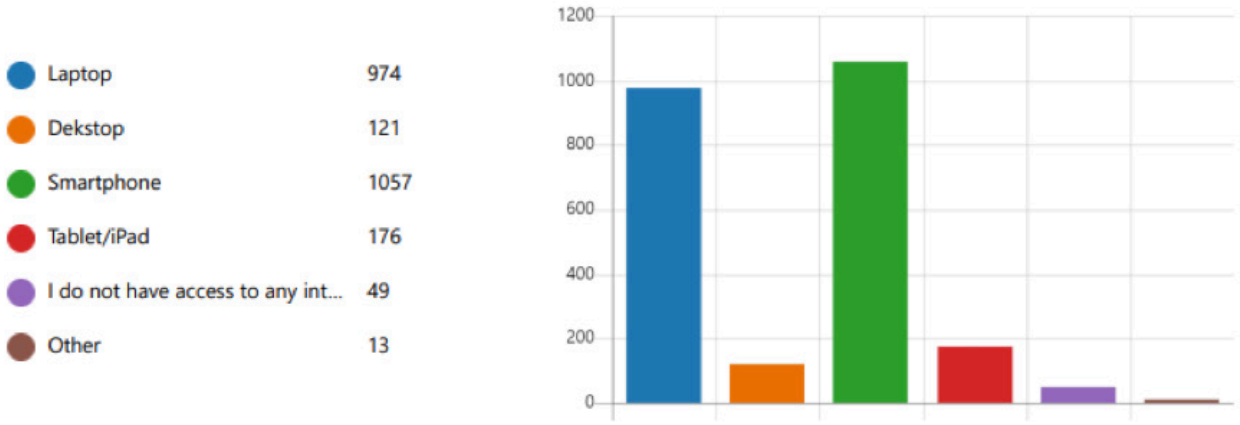

Figure 2. devices used by university students when accessing eLearning services

Figure 3. Internet connectivity when off the campus

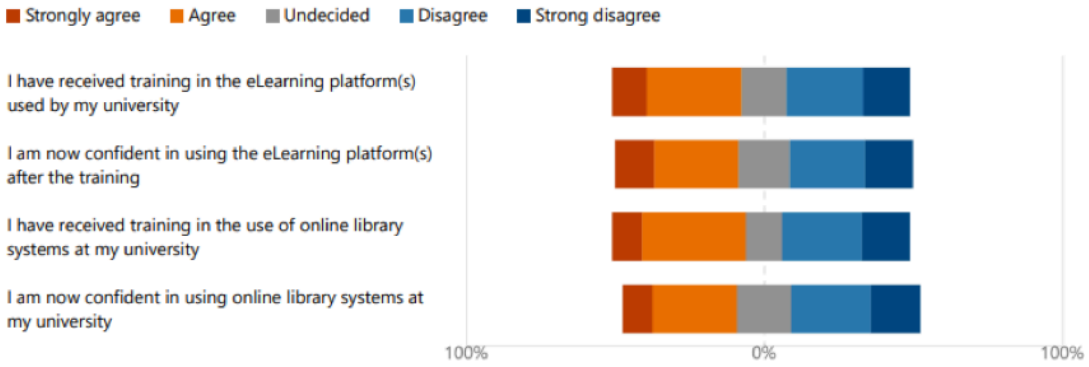

Figure 4. Highlighting eLearning training received by students

From Figure 4 above, the majority of students agreed that they received training on the use of eLearning systems and they indicated that they are now confident in using the platforms. Students also indicated that they received training on how to use online library systems. The training of the majority of students would help them to navigate through the eLearning system whilst away from their campuses.

Figure 5 above shows that the majority of students indicated that the online materials for the modules were inadequate though the materials were easily accessible. This can also reflect that the teachers were also facing challenges designing, developing, and uploading learning materials.

From the above results, the students revealed that they were able to use the eLearning systems since they had the necessary devices, and with minimum training provided they were able to navigate and interact with the system. Although the students were trained and with necessary gadgets but could get enough material from the eLearning system. The finding is important as it clearly shows that the adoption of eLearning is practical if all the necessary resources are provided. The finding would assist policy-makers in decision-making for the successful adoption of eLearning systems at universities.

System Delivery

From Figure 6 above, 84% of the students indicated that they use PowerPoint slides as learning materials from eLearning platforms. 51% of students use quizzes/

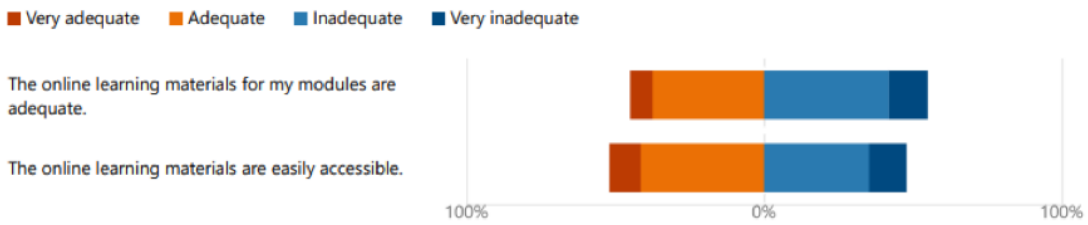

Figure 5. adequacy of eLearning Materials

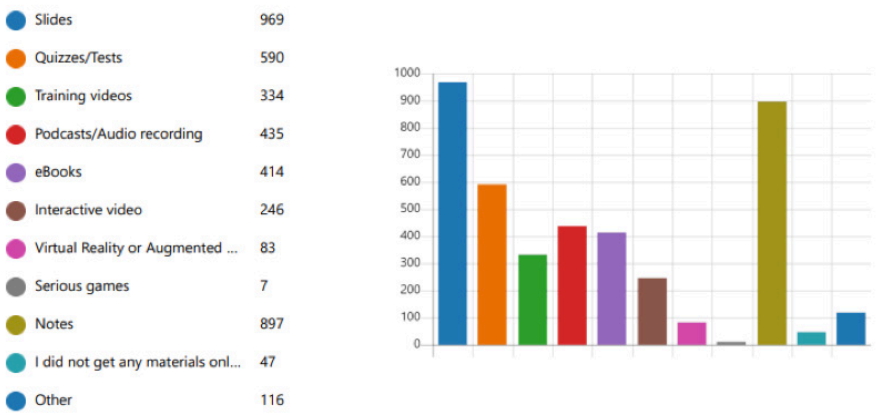

Figure 6. Highlights Learning Materials used by students

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tests. 29% showed that they receive training videos while 38% use podcasts/Audio recorded. 36% use eBooks, 21% use interactive video, 7% use virtual reality/Augmented, 77% use notes as learning materials. 4% indicated that they do not get any materials through eLearning systems. This finding shows that there is a wide variety of teaching methods that are being used across Zimbabwean universities.

From the above Figure 7, 74% of the students did not write online examinations while only 16% of the students were able to write the examination. This shows that some of the Zimbabwean universities had examinations written online while the majority could not. This was an advantage to those universities that can write examinations online as they were able to continue with their education system without disruptions.

System Outcome

Figure 8 highlights the eLearning support services when off campus which shows that the majority of students were strongly dissatisfied with the availability of data, and technical help when trying to access eLearning materials. Lack of data and support services hindered the students from fully adopting eLearning systems.

From Figure 9 above, the majority of the students were strongly dissatisfied with the support they received from the universities in terms of data bundles, securing of eLearning gadgets especially the disadvantaged students. Considering that students from the universities come from different backgrounds, it shows that the disadvantaged students were mostly affected.

The motivation for the use of eLearning was affected by lack of support through the provision of internet access and training on how to use the e-learning systems as well as lack of ICT devices. This finding will help decision-makers in developing strategies to motivate students in the adoption of eLearning systems.

During the interviews, the students indicated that they preferred blended learning which includes the traditional face-to-face method and eLearning systems. This was because some of the students did not have the necessary resources for eLearning services, although they would

Figure 7. Highlighting online examinations

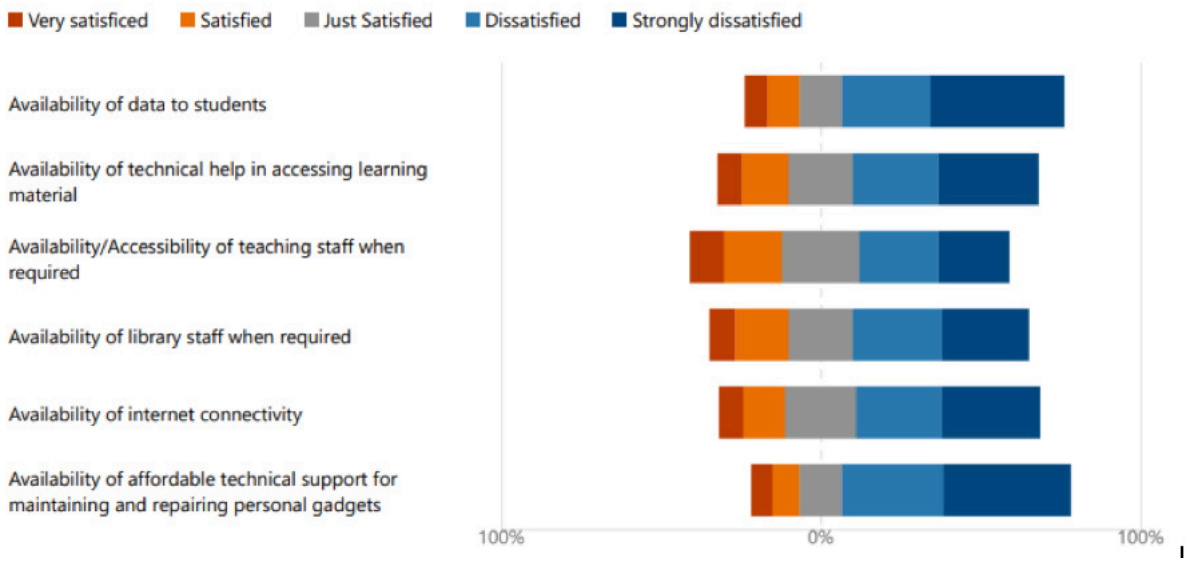

Figure 8. Indicating satisfaction with eLearning support services

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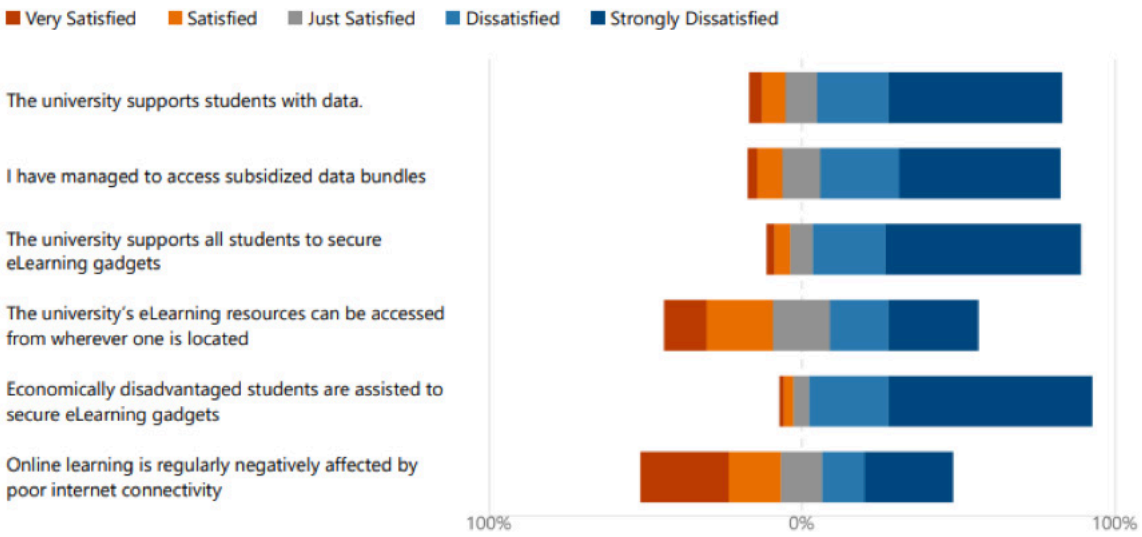

Figure 9. Universities eLearning support to students

want to continue learning. The challenges faced were internet connectivity especially for those disadvantaged and those living in the rural communities where there is lack of power and network.

5. Discussion

From the findings, it can be deduced that the adoption of e-learning is practical if all the necessary resources are provided. This is supported by [19] who states that successful implementation of eLearning systems require adequate resources such as ICT gadgets, internet access, and availability, and e-learning support services such as learning material and training.

Students indicated that they do not receive any support for purchasing data bundles and securing ICT devices for eLearning which makes it difficult for Higher learning Institutions to implement eLearning systems. This is highlighted in a study in Malaysia that showed that the adoption of eLearning systems for Higher Learning Institutions is greatly hindered by high data costs as a result students will not be able to fully participate in online learning [20]. Another study by [21] in private tertiary institutions amidst COVID-19 lockdown in Nigeria showed that implementation of eLearning was slowed down by the expensive eLearning resources such as data and materials.

Also from the findings of the interviews, students stated that blended learning was more feasible than complete eLearning, due to various challenges which range from poor connectivity, cost of eLearning resources, and lack of training. This is supported by [22] who studied students' perception of online learning during the COVID-19 pandemic discovered that most students preferred blended learning, while less than 5% opted for traditional face-to-face learning.

6. Conclusions

COVID-19 impacted negatively in the education sector and as a solution to provide continuous education, the Ministry of Higher and Tertiary Education switched to eLearning systems. Though Zimbabwean universities adopted eLearning systems as an alternative mode of learning, the students could not fully utilize the online learning system due to the following challenges, poor connectivity, high cost of eLearning resources, and lack of eLearning support services.

The Zimbabwean university students recommended the adoption of a blended learning system which includes eLearning systems complemented by face-to-face learning so that it can accommodate students from disadvantaged backgrounds who cannot afford eLearning resources such as data bundles and devices.

References

[1] A. Schleicher, "The impact of COVID-19 on education: Insights from education at a glance 2020," *OECD J. Econ. Stud.*, pp. 1-31, 2020.

[2] M. Alipio, "Education during COVID-19 era: Are learners in a less-economically developed country ready for e-learning?," *ZBW - Leibniz Inf. Cent. Econ. Kiel, Hamburg*, 2020.

[3] O. E. Maravanyika, *Implementing educational policies in Zimbabwe*, vol. 91. 1990.

[4] The Ministry of Primary and Secondary Education,

DOI: https://doi.org/10.30564/jeisr.v3i2.3722

"The national non-formal education for Zimbabwe: Promoting alternative pathways to increase access and quality education in Zimbabwe the national non-formal education policy for Zimbabwe," 2015.

[5] T. Mukeredzi, "Under pressure, the government closes universities and schools," *University World News*, 2020.

[6] T. A. A. KENAN, *E-LEARNING IMPLEMENTATION IN THE SCHOOL OF ENGINEERING AT TRIPOLI A thesis submitted to the University of Huddersfield in partial fulfillment of the requirements for the degree of Doctor of Philosophy The University of Huddersfield*. 2015.

[7] A. KENAN, T., PISLARU, C. OTHMAN, A. & ELZAWI, "'The social impact and cultural issues affecting the e-learning performance in Libyan Higher Education institutes'.," *Int. J. Inf. Technol. Comput. Sci. (IJITCS). ISSN I 2091 - 1610*|, vol. 12, no. 1, pp. 50-56.

[8] K. Heng and K. Sol, "Online learning during COVID-19: Key challenges and suggestions to enhance effectiveness," *Cambodian Educ. Forum*, no. December, pp. 1-15, 2020.

[9] Euro Group Consulting, "Leading positive transformation," 2021.

[10] A. Schleicher, "THE IMPACT OF COVID-19 ON EDUCATION INSIGHTS FROM EDUCATION AT A GLANCE 2020."

[11] J. A. D. Doculan, "E-L Earning R Eadiness a Ssessment T Ool F or Philippine Igher E Ducation I Institutions," *Int. J. Integr. Technol. Educ. Vol.5,* vol. 5, no. 2, pp. 33-43, 2016.

[12] A. A. Alqahtani, "The Effectiveness of Using E-learning, Blended Learning and Traditional Learning on Students ' Achievement and Attitudes in a Course on Islamic Culture : an Experimental Study," Durham University, 2010.

[13] B. Stauffer, "What's the difference between online learning and distance learning? The Applied Education System," 2020.

[14] U. Leif, E., Grové, C., Alfrey, L., Laletas, S., & Sharma, *5 tips on how unis can do more to design online learning that works for all students. The 13 Conversation.* 2020.

[15] V. Ilie, "Traditional Learning Versus E-Learning," pp. 1192-1201, 2019.

[16] J. Cable and C. Cheung, "Eight Principles of Effective Online Teaching: A Decade-Long Lessons Learned in Project Management Education," *PM World J. Eight Princ. Eff. Online Teach.*, vol. VI, no. Vii, pp. 1-16, 2017.

[17] T. FitzPatrick, "Key Success Factors of eLearning in Education: A Professional Development Model to Evaluate and Support eLearning.," *Online Submiss.*, vol. 9, pp. 789-795, 2012.

[18] A. Lee-Post, "e-Learning Success Model: an Information Systems Perspective," vol. 7, no. 1, pp. 61-70, 2009.

[19] C. Coman, L. G. Țîru, L. Meseșan-Schmitz, C. Stanciu, and M. C. Bularca, "Online teaching and learning in higher education during the coronavirus pandemic: Students' perspective," *Sustain.*, vol. 12, no. 24, pp. 1-22, 2020.

[20] M. F. Ramli, M. Majid, and B. Badyalina, "Impeding Factors Towards the Effectiveness of Online Learning During Covid-19 Pandemic among Social Sciences Students," *Int. J. Learn. Dev.*, vol. 10, no. 4, p. 37, 2020.

[21] W. O. Oyediran, A. M. Omoare, M. A. Owoyemi, A. O. Adejobi, and R. B. Fasasi, "Prospects and limitations of e-learning application in private tertiary institutions amidst COVID-19 lockdown in Nigeria," *Heliyon*, vol. 6, no. 11, 2020.

[22] M. Bączek, M. Zagańczyk-Bączek, M. Szpringer, A. Jaroszyński, and B. Wożakowska-Kapłon, "Students' perception of online learning during the COVID-19 pandemic: A survey study of Polish medical students," *Medicine (Baltimore).*, vol. 100, no. 7, p. e24821, 2021.

 DOI: https://doi.org/10.30564/jeisr.v3i2.3722

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ARTICLE

Factors Affecting Ion Thruster's Performance

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ABSTRACT

In this project, we investigated how ion thrusters produce propulsion and how the design of ion thrusters affects the performance of the thruster. In the experiment, we build a high voltage power supply (0- 50 kV) and foil rings to produce ion wind. When considering the design of the thruster, we focus on three variables: the volume of the space, where ions are produced and the electric field intensity. Thus, to investigate the first variable we made foil rings with different radius and change the distance between the ring and positive cathode. To determine the propulsion produced we use a speed sensor to determine the magnitude of the wind produced.

1. Introduction

An ion thruster, ion drive, or ion engine is a form of electric propulsion used for spacecraft propulsion. It creates thrust by accelerating ions using electricity. The thruster initially ionizes gas by depriving electrons of the gas molecules and therefore creating a group of positive ions [1]. Due to Coulomb force which states that "the magnitude of the electrostatic force of attraction or repulsion between two point charges is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distance between them" [2] the ions are repelled by each other and then accelerate in one direction.

Nowadays, there are already spacecraft such as "Deep Space 1" and "Dawn" [3] that use ion thruster as their propeller. The characteristic of ion thruster made it suitable for space transportation [4]. Firstly, ion thruster needs less fuel to achieve a certain speed than conventional chemical fuel. Secondly, even though, ion thruster is not widely used on earth, since its minuscule thrust can't overcome air resistance, this disadvantage can be minimized since there is no air resistance in the vacuum space.

To investigate more about the characteristics of ion thruster, our group conducted an experiment. There are two stages in the experiment. During the first stage, we will test that are we able to produce ion wind by just employing a high-voltage electric field. After confirming the result, in the second stage, we will design and make the test equipment. We will look at four variables: ring size, number of nails, distance, voltage, and how they account for the variations of the ion wind speed.

During the first stage of the experiment, we build a small piece of equipment that mimics the ion thruster. Firstly, we used aluminum-foil paper to build the middle part of the thruster. This part is connected to the cathode.

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And on top of this part, there is a round of copper wire which is connected to the anode of the power supply. We use these materials because they are good conductors of electricity. The layout of the equipment helps to create an electromagnetic field and therefore deprives electrons of air molecules. Therefore, produces ions and ion wind.

During the test period, we saw the thruster (equipment mentioned above) lift up from the ground. This shows the thruster is creating ion wind that causes uplift force. This proved what we thought previously. Additionally, we also smelled ozone during the experiment. This is the evidence that the oxygen molecules were ionized and then after being propelled from the thruster, the ions formed ozone.

$$3O_2====2O_3$$

After confirming the design, we built our test equipment. The equipment is composed of a power supply, test track, two sets of rings, and a speed sensor. This equipment will help us to find the relationship between the four variables (mentioned above) and the speed of the ion wind. To test the speed of the wind we used the wind speed sensor, which will tell us how fast the wind is by calculating the speed of its rotation. In different trials, we changed the variables and compared the speed.

2. Observation of Ion Wind

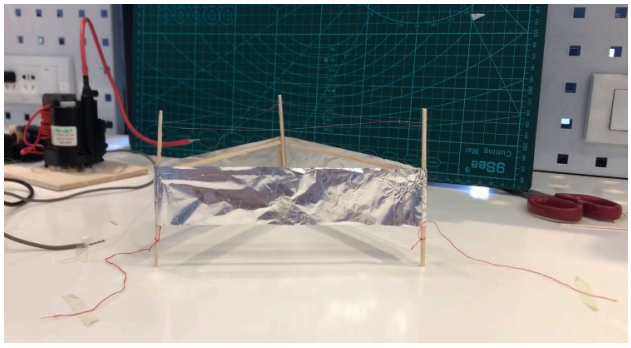

Figure 1. equipment before lifting up

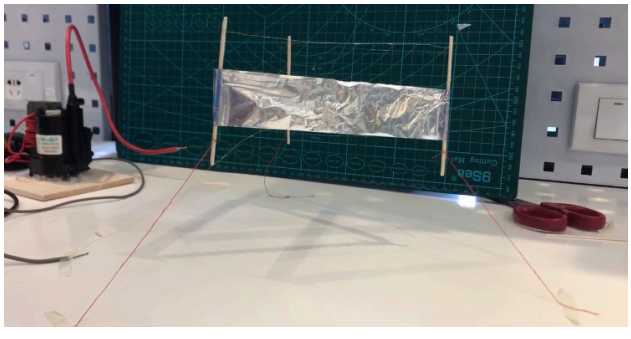

Figure 2. equipment after lifting up

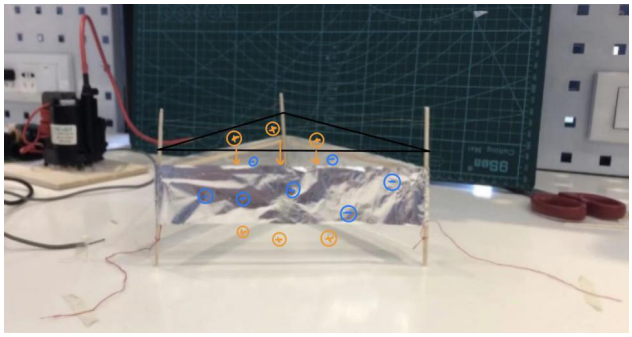

Figure 3. demonstration of the ions in the thruster(Orange color represents the positive ions, while blue color represents the negative charge on the foil paper).

During the first stage whether our electric power supply is sufficient to ionize molecules in the air. We used tin foil paper to wrap around wooden sticks and connect to the cathode of the power supply, and used the copper wire to wrap around the wooden sticks above the tin foil paper and connect to the anode of the power supply. This setup can provide an electrical field between the wire and tin foil paper. We also tied the edges of the wooden sticks to the threads that were glued on the table to prevent the device from flying away.

After applying electric power, we can see the device gained thrust and elevated. Under the space between the tin foil paper, we can detect a constant wide speed through the wind speed detector. During the test, we are also able to smell ozone. These phenomenons proved that our electric power supply is enough to provide an electric field need to ionize air molecules in this condition.

For this process, the ion wind is produced by the depletion of electrons of the molecules in the air. Because molecules are positively charged, the ionized molecules will propel each other and accelerate in one direction. Therefore, creating thrust for the equipment to lift.

3. Experiment

Question

How does the setting of ion engine equipment affect its performance?

Hypothesis

1) Number of nails will affect the performance of the ion engine: the more the nails are the greater impulse it could produce.

2) Distance affects the impulse available to run the rotor. The closer the engine to, the greater the impulse available.

3) The size of the ring affects the impulse generated,

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the greater the ring is the greater the impulse could be generated

4) Voltage will affect the impulse generated, the greater the voltage is the greater the impulse could be generated

Material

A roll of Tin foil paper	Glue gun	pencils
Thin wood plates	Three Copper nails	rulers
Copper wire	Speed sensor	Cotton thread
Potential transformer (220V to 50kV	Switcher	tapes
Small PE tube (1.5cm of diameter)	cutter	502 glues

Procedure

1) Set up the power supply which includes a switcher and a voltage transformer as Figure 4 shows.

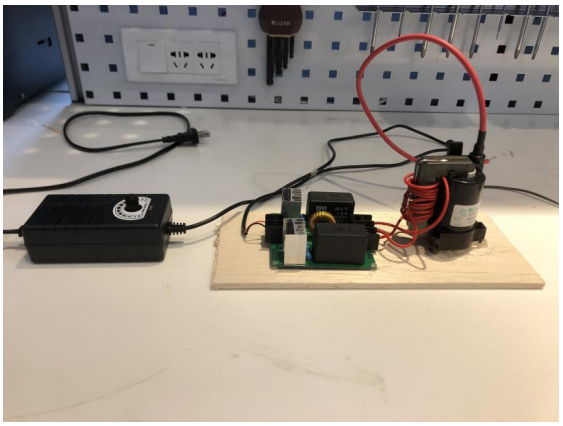

Figure 4. power supply

2) Set up the test track with foil paper, wood plates, copper nails and mark the scale on the plate using the pencil and ruler in order to clarify the distance. The fixation process is supported by a glue gun.

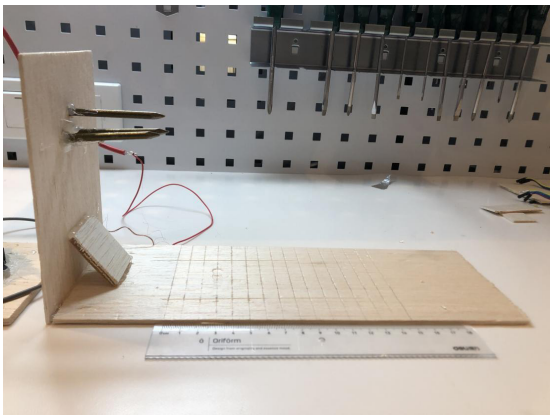

Figure 5. test track

3) Roll the foil paper around the PE tube and fix three of them in the shape of the pyramid by using glue gun.

4) Made another foil tube without an inner structure with a diameter of 6.5 cm.

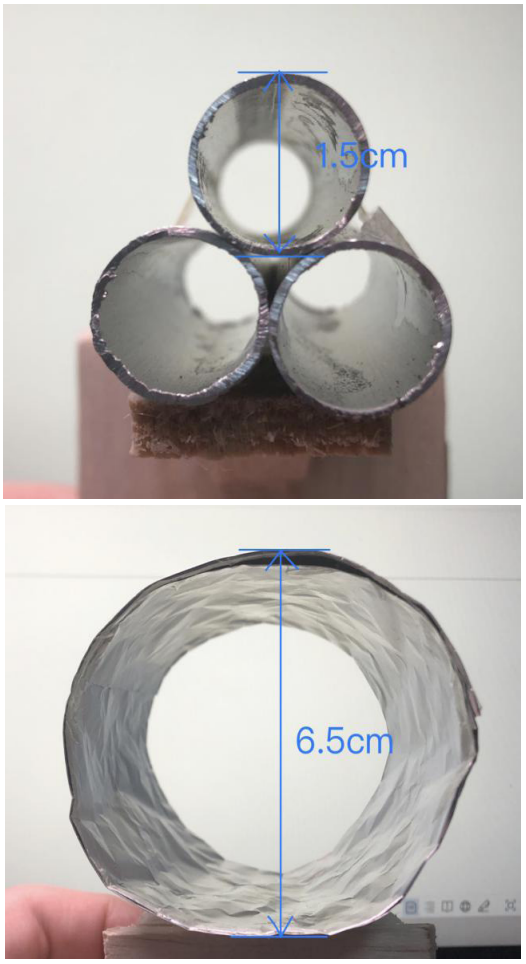

Figure 6. foil tubes

5) Connect all three nails to the transformer by copper wires, place the experiment tubes with three tubes based on the scale marked on the plate, place the wind speed sensor at the position that the blades of the sensor are just apart from the tube. Then open the power supply and take note of the wind speed data produced by the ion engine.

6) Repeat the experiment by adjusting the separation of nails and tubes with a distance of 1 cm, 2 cm, and 3 cm, record the data.

7) Disconnect one of the nails and repeat the procedure described in steps 5 and 6. Then disconnect two of them and repeat the experiment.

8) Lessen the voltage at the condition of 3 nails 2 cm distance and repeat the experiment.

9) Replace the three tubes system with the larger tube and repeat all of the procedures described in step 5, 6, and 7.

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10) In order to avoid the error as much as possible, all of the experiments described above will be repeated three times.

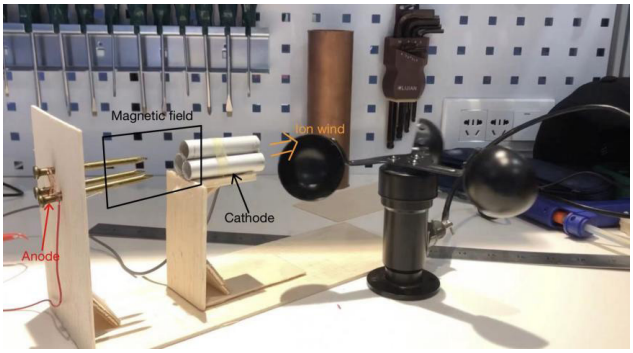

Figure 7. demonstration of the set up

4. Data and Results

Table 1. data collected from the experiment

Trial	Ring Size	Number of Nails	Distance	Voltage	Wind speed (round 1) (RPM)	Wind speed (round 2) (RPM)	Wind speed (round 3) (RPM)	AVG wind speed (RPM)
1	Small	3	1cm	50kV	N. A	N. A	N. A	N. A
2	Small	3	2cm	50kV	18	20	19	19
3	Small	3	3cm	50kV	0	0	0	0
4	Small	2	1cm	50kV	16	19	16.5	17.17
5	Small	2	2cm	50kV	0	0.333	0.5	0.28
6	Small	1	1cm	50kV	0	0	0	0
7	Small	3	2cm	25kV	0	0	0	0
8	Large	1	1cm	50kV	10	11.5	9.5	10.333
9	Large	1	2cm	50kV	0	0	0	0
10	Large	2	1cm	50kV	15	12	16	14.33
11	Large	3	1cm	50kV	17	17.5	16	16.83
12	Large	3	2cm	50kV	0	0	0	0

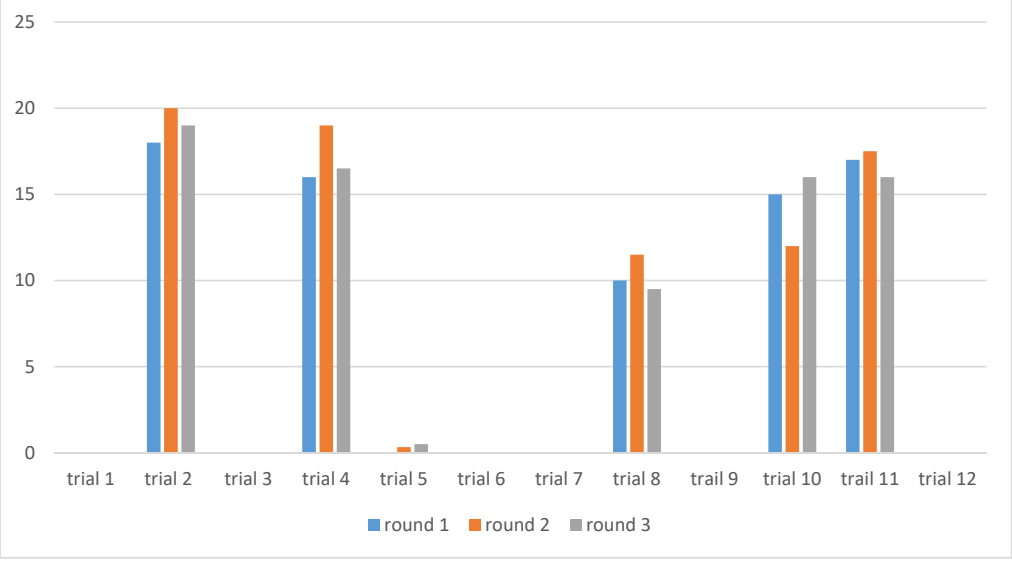

Figure 8. How different factors affect implus measured from ion engine

5. Discussion of Results

In the first three trials and last two trials of the experiment, we tested separately how the distance affects the impulse available with three nails. For the small rings group, the result is only available for a distance of 2 cm which is 18, 19, and 20 RPM respectively. While there is no wind speed detected when the distance increased to 3 cm, the experiment for a distance of 1 cm failed to give data because our tubes are too close to the nails, which lead to electric ace being generated between nails and tube so we have to stop the experiment to prevent any accident. This series of trials indicates that the closer the nails are to the tubes the greater the impulse is. However, it is not enough to conduct this barely based on one data available. Other trails with larger tubes further solidify the result that the wind speed could be only detected when the distance is 1cm. In the whole picture we could infer that when the distance is too far from the nail to the tubes, the ionized performance will be too weak that although there is a wind-generated, its speed is too slow to be detected. And with too close a distance, the ionized performance is too great that causes the electric ace.

In the trial 4 and 5, when the number of nails connected to the power supply decrease to 2, our ion engine generated the wind speed with the same pattern of decreasing with distance. It averages 17.17 when distanced 1cm and 0.28 when distanced 2 cm. comparing the data of 2cm distance with trial 2, we could find that with two nails the wind speed generated (avg 0.28 RPM)

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is much less than the test with 3 nails (avg 19 RPM), it is nearly not detectable for the test with two trails.

When we tested the one with only one nail available, even with the distance of 1cm there is nothing detected from the wind speed sensor, while the test with two nails at this distance generated a pretty fast wind speed avg 17.17 RPM.

The lower voltage is tested under the condition of trial 2 except the voltage is decreased to half of the original value. There is no wind detected.

The data collected from the large tube group though is out of our expectations. With three nails connected there is only wind available when the distance is 1cm which gives an avg 16.83 RPM, which is even then the result from two nails connected when we use the small tubes. And when we repeat the experiment with 2 and 1 nails connected there is no that significant decrease in wind speed happens as the test group with small tubes. At a distance of 1 cm, the numbers given by three different nails groups are avg 16.83 RPM, 14.33 RPM, and 10.33 RPM. Unlike the small tubes that generated their own ionized gas separately in one of the three tubes, a large tube is likely to provide a larger space for gas to defuse. So, when we remove one of the nails from the connection, we lose one-third of the ionized gas completely because the small tube only allows them to ionize part of the gas passing through them. The situation is different with large tube, since all three nails are within the range of a large tube, although one of them or two of them could not provide power, the rest of them could still ionized and make a great portion of gas passing through the tube due to the diffusion of a gas. Therefore, this result makes sense for us finally. Another thing to be noticed is that the large tube group all avg a lower wind speed when the remaining conditions are the same as the smaller tube group, which is opposite to our hypothesis. This could be explained by the efficiency of ionization performed by the foil paper. The effective ionized range may be too small for larger rings to ionize all of the gas inside the tube. Larger inside surface area helps smaller tube groups to generate more impulse inversely.

6. Conclusions

In this experiment, all of our hypothesis is expected except the one about tube side. In conclusion: the closer the distance between nails and tubes is the greater the impulse could be generated; the greater the number of nails is connected the greater the impulse could be generated; the higher the voltage the greater the impulse could be generated and lastly the three small tube system could generate more impulse compared to the large tube. The amount of ionized gas is the key to understanding all of those conclusions: that is the more the gas is ionized within the range of the tube, the more wind ion engine could blow which means more impulses are generated. Our result provides an intuitional demonstration of the electric rules and basic mechanism involved in ion thruster [5].

However, there are also limitations in our experiment. Firstly, we can't measure the wind velocity with great accuracy due to the minuscule thrust of the engine and the precision of our equipment. Secondly, this equipment in this experiment is a simplified model for ionizing gas to produce thrust. The actual ion thruster used on satellites is way complicated that involves the collision between electrons and propellant and the neutralization of the ions after the emission.

References

[1] Jahn, Robert G. (1968). Physics of Electric Propulsion (1st ed.). McGraw Hill Book Company. ISBN 978-0070322448. Reprint: Jahn, Robert G. (2006). Physics of Electric Propulsion. Dover Publications. ISBN 978-0486450407.

[2] Coulomb (1785b) "Second mémoire sur l'électricité et le magnétisme," Histoire de l'Académie Royale des Sciences, pages 578-611.

[3] Choueiri, Edgar Y. (2009). "New dawn of electric rocket". Scientific American. 300 (2): 58–65. Bibcode:2009SciAm.300b..58C.
DOI: 10.1038/scientificamerican0209-58. PMID 19186707.

[4] Choueiri, E. Y. "A Critical History of Electric Propulsion: The First 50 Years (1906–1956)". Retrieved 18 October 2016.

[5] Jahn, Robert G.; Choueiri, Edgar Y. (2003). "Electric Propulsion" (PDF). Encyclopedia of Physical Science and Technology. 5 (3rd ed.). Academic Press. pp. 125-141. ISBN 978-0122274107.

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ARTICLE

Generalization of Multiplication M-Sequences over Fp and Its Reciprocal Sequences

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ABSTRACT

M_p-Sequences or M-Sequence over F_p not used so much in current time as binary M-Sequences and it is pending with the difficult to construct there coders and decoders of Mp-Sequences further these reasons there is expensive values to construct them but the progress in the technical methods will be lead to fast using these sequences in different life's ways, and these sequences give more collection of information and distribution them on the input and output links of the communication channels, building new systems with more complexity, larger period, and security. In current article we will study the construction of the multiplication Mp-Sequence $\{z_n\}$and its linear equivalent, this sequences are as multiple two sequences, the first sequence$\{S_n\}$is an arbitrary Mp-Sequence and the second sequence $\{\zeta_n\}$ reciprocal sequence of the first sequence $\{S_n\}$, length of the sequence $\{z_n\}$, period, orthogonal and the relations between the coefficients and roots of the characteristic polynomial of $f(x)$ and it's reciprocal polynomial $g(x)$ and compare these properties with corresponding properties in M-Sequences.

1. Introduction

Sloane, N.J.A., studied the product or multiplication sequence $\{z_n\}$ on t degrees of the sequence $\{a_n\}$ which has the degree of complexity r and gave the answer that the degree of complexity of $\{z_n\}$ can't be exceeded [1,2].

Mokayes D. Al Cheikha A. H., studied the construction of the multiplication binary M-Sequences where the multiplication on one M-Sequence or on more than one sequence and gave an illustrated about the question "why the length of the linear equivalent shift register don't reached the maximum length?" [3-8].

Al Cheikha A. H., Omar E H.., studied the construction of the multiplication binary M-Sequences with it's reciprocal sequences, they studied the length of the result multiplication sequence$\{z_n\}$which is equals $[(\deg f(x))^2$-deg $(f(x))]$, linear equivalent, properties of the coefficients of the roots of the characteristic polynomial $f(x)$ and it's reciprocal sequence. Current article and showing the agreement and disagreement between these coefficients and corresponding roots, in the first side, in binary M-Sequences, and in the second side, Mp-Sequences [9-11].

2. Research Method and Materials

2.1 Mp-Sequences or M-Sequences over Fp

Mp- Sequences or M-Sequences over Fp $\{a_n\}$ is of the form;

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$a_{n+k} = \gamma_{k-1}a_{n+k-1} + \gamma_{k-2}a_{n+k-2} + ... + \gamma_0 a_n$

$+\gamma; \gamma \,\&\, \gamma_i \in F_p = \{0,1,2,..., p-1\}, i = 0,1,...,k-1$

Or;

$$a_{n+k} = \sum_{i=0}^{k-1} \gamma_i a_{n+i} + \gamma \tag{1}$$

where, $\gamma, \gamma_0, \gamma_1, ..., \gamma_{k-1}$ are in the field Fp and k is positive integer is called a linear recurring sequence of complexity or order k, *for* $\gamma = 0$ the sequence is called homogeneous sequence (H.L.R.S), in other case the sequence is called non-homogeneous, the vector $(a_0, a_1, ..., a_{k-1})$ is called the initial vector and the characteristic equation of the sequence is;

$$f(x) = x^k + \gamma_{k-1}x^{k-1} + ... + \gamma_1 x + \gamma_0 \tag{2}$$ [12-14]

We are limited in our article to $p > 2$.

2.2 Definitions and Theorems

Definition 1

The L.F.S.R is a linear feedback shift register if contains only addition circuits and the general term of the sequence $\{s_n\}$ generated through the shift register is the term of the output of the register [3].

Definition 2

The vector $\overline{X} = (\overline{a_1}, \overline{a_2}, ..., \overline{a_n})$ is the complement of the vector $X = (a_1, a_2, ..., a_n,)\, a_i \in F_p$ where

$$\overline{a_i} == p-1 \; if \; a_i \neq 0 \; and \; \overline{a_i} = 0 \; if \; a = 0 \tag{3}$$ [12-14]

Definition 3

The coefficient of correlation function of the two vectors $t = (t_0, t_1, ..., t_{n-1})$ and $l = (l_0, l_1, ..., l_{n-1})$ on F_p which denoted by $R_{t,l}$, is:

$$R_{t,l} = \left| \left(\sum_{i=0}^{n-1} (-1)^{t_i + l_i} \right) - \left(\left[\frac{n+1}{p} \right] - 1 \right) \right| \tag{4}$$

Where $\left[\frac{n+1}{p} \right]$ is the nearest integer of the number $\frac{n+1}{p}$, and we said the two vectors t and l are orthogonal if $\sum l = \sum_{i=0}^{n} l_i = 0, \sum t = \sum_{i=0}^{n} t_i = 0 \; mod \; p$, and $R_{t,l} \leq 1$ [8-10,13].

Definition 4

The periodic sequence $(a_i)_{i \in N}$ over F_p with the period $r = p^k - 1$ has the "Ideal Auto Correlation" property if and only its periodic auto Correlations $R_a(\tau)$ of the form:

$$a(\tau) = 0 \;\; and \;\; R_a(\tau) \leq 1 \tag{5}$$

When;

$$a(\tau) = \sum_{t=0}^{r-1} a_i (t+\tau) \; mod \; p, \tau = 0,1,...,r; \; R_a(\tau) =$$

$$\left| \sum_{t=0}^{r-1} (-1)^{a(t+\tau)+a(t)} - \left(p^{k-1} - 1 \right) \right|, \;\; for \; \tau \neq \frac{p^k - 1}{2} \tag{6}$$

Definition 5

The set $A = \left\{ t; t = (t_0, t_1, ..., t_{n-1}) \; t_i \in F_p, i = 0,1,...,n-1 \right\}$ is called an orthogonal set if and only if the set A satisfies the following two conditions;

$$1. \; \forall t \in A; \left| R_{t,0} \right| \leq 1. \tag{7}$$

$$2. \; \forall t, l \in A, \& \; t \neq l, \left| R_{t,l} \right| \leq 1. \tag{8}$$ [15,16]

Definition 6

If the function $f(x)$ has the degree n then the reciprocal function of $f(x)$ is the function;

$$g(x) = x^n f(1/x) \tag{9}$$

Theorem 7

If $\{s_n\}$ is a H.L.R.S sequence with the complexity k, period r and its characteristic polynomial is $f(x)$ then $r \,|\, ord\, f(x)$ and if the polynomial $f(x)$ is primitive and it's coefficients are in F_p then the period of the sequence $\{s_n\}$ is $p^k - 1$ and this sequence is called M_p-Sequence or M-Sequence over F_p [2,15-17].

Lemma 8. (Fermat's theorem).

If the finite field F of order q then each element x of F satisfies the equation;

$$x^q = x \tag{10}$$ [2,15-18]

Theorem 9

If $\{a_n\}$ is a H.L.R.S in F_p and $g(x)$ is its characteristic prime polynomial of degree k and α is a root of $g(x)$ in any splitting field of F_p then the general term of the sequence $\{a_n\}$ is;

$$a_n = \sum_{i=1}^{k} C_i \left(\alpha^{p^{i-1}} \right)^n \tag{11}$$

And the sequence $\{a_n\}$ has the period p^{k-1} [2,18].

Theorem 10

i. $(q^m - 1) \,|\, (q^n - 1) \Leftrightarrow m \,|\, n$ **(12)**

ii. any subfield of the field F_{p^n} is a field of order p^m where $m \,|\, n$ and if F_q is a field of order $q = p^n$ then any subfield of it is has the order p^m and $m \,|\, n$, and by inverse if $m \,|\, n$ then

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the field F_{p2^n} contains a subfield of order p^m [2,15-18].

*Our study is limited to the M_p-Sequence or M-Sequence over F_p and it's period is: $r = p^k - 1$.

3. Results and Discussion

Multiplication Two Reciprocal Mp-Sequences

If $f(x)$ is the characteristic prime polynomial of the M_p-Sequence $\{s_n\}$ of degree k in the field F_p, and $\alpha_1, \alpha_2, \ldots \alpha_k$ are it's independent different roots in the F_p^k, which is a splitting field of F_p then the general term of the sequence $\{s_n\}$ is

$$s_n = A_1\alpha_1^n + A_2\alpha_2^n + \ldots + A_k\alpha_k^n = \sum_{i=1}^{k} A_i\alpha_i^n \tag{13}$$

And if α is a root of $f(x)$ then we can find;

$$\alpha_1 = \alpha, \alpha_2 = \alpha^{p^{2-1}}, \ldots, \alpha_k = \alpha^{p^{k-1}}$$

And the general term of the sequence $\{s_n\}$ becomes;

$$s_n = A_1\alpha^n + A_2(\alpha^{p^{2-1}})^n + \ldots + A_k(\alpha^{p^{k-1}})^n = \sum_{i=1}^{k} A_i(\alpha^{p^{i-1}})^n \tag{14}$$

Suppose $g(x)$ is the reciprocal of $f(x)$, $\beta_1, \beta_2, \ldots, \beta_k$ are the different linear independent roots of $g(x)$, and $g(x)$ characteristic polynomial of the M_p-Sequence $\{\zeta_n\}$ reciprocal sequence of $\{s_n\}$ then the general term of the sequence $\{\zeta_n\}$ is

$$\zeta_n = B_1\beta_1^n + B_2\beta_2^n + \ldots + B_k\beta_k^n = \sum_{i=1}^{k} B_i\beta_i^n$$

Thus, if α_i is of the form $\alpha^{p^{i-1}}$ and β_i is reciprocal α_i then $\beta_i = \alpha^{p^k - p^{i-1} - 1}$ and ζ_n can be written in the form;

$$\begin{aligned}\zeta_n &= B_1\alpha^{n(p^k-2)} + B_2\alpha^{n(p^k-3)} + \ldots + B_k\alpha^{n(p^k-p^{k-1}-1)} \\ &= \sum_{i=1}^{k} B_i\alpha^{n(p^k-p^{i-1}-1)}\end{aligned} \tag{15}$$

Thus;

$$\begin{aligned}s_n\zeta_n &= \left(\sum_{i=1}^{k} A_i(\alpha^{p^{i-1}})^n\right)\left(\sum_{j=1}^{k} B_j(\alpha^{(p^k-p^{j-1}-1)})^n\right) \\ &= \left(\sum_{i,j=1,i\neq j}^{k} A_iB_j(\alpha^{p^{i-1}})^n(\alpha^{(p^k-p^{j-1}-1)})^n\right) + \left(\sum_{i=1}^{k} A_iB_i\right)\end{aligned} \tag{16}$$

Example 1

Suppose the polynomial $f(x) = x^2 + x + 2$ which is a prime characteristic polynomial the sequence $\{s_n\}$, $\forall n \in N; s_n \in F_3$ s_n satisfies;

$$s_{n+2} + s_{n+1} + 2s_n = 0; or\ s_{n+2} = 2s_{n+1} + s_n; s_0 = 0, s_1 = 1$$

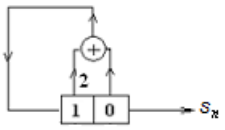

Figure 1. *Shift register generating the sequence $\{s_n\}$ on F_3*

The characteristic equation of the sequence $\{s_n\}$ is;

$$x^2 + x + 2 = 0 \tag{17}$$

If α is a root of the equation then;

$$\alpha^2 + \alpha + 2 = 0$$

And;

$$\begin{aligned}F_{3^2} &= \{0, \alpha^8 = 1, \alpha, \alpha^2 = 2\alpha + 1, \alpha^3 = 2\alpha + 2, \alpha^4 \\ &= 2, \alpha^5 = 2\alpha, \alpha^6 = \alpha + 2, \alpha^7 = \alpha + 1\end{aligned} \tag{18}$$

The general term of the sequence $\{s_n\}$ is of the form;

$$S_n = S_1\alpha^n + S_2(\alpha^3)^n$$

Solving the following equations;

$$\begin{cases} n = 0 \rightarrow S_1 + S_2 = 0 \\ n = 1 \rightarrow \alpha S_1 + \alpha^3 S_2 = 1 \end{cases}$$

We have;

$$S_1 = \alpha^6 = \alpha + 2, S_2 = \alpha^2 = 2\alpha + 1$$

And the general solution of the characteristic equation is;

$$S_n = \alpha^6(\alpha)^n + \alpha^2(\alpha^3)^n \tag{19}$$

Or;

$$S_n = (\alpha + 2)\alpha^n + (2\alpha + 1)(2\alpha + 2)^n$$

And the sequence is periodic with the period: 3^2-1 = 8 and the sequence is;

0 1 2 2 0 2 1 1 0 1 2 2 1 0 2 1 1 ….. (20)

We can see the important properties;

$$\left(\alpha^6(\alpha)\right)^3 = \left(\alpha^2(\alpha^3)\right) or\ (\alpha^6)^3 = \alpha^2\ \&\ (\alpha)^3 = \alpha^3 \tag{21}$$

By the same way;

$$\left(\alpha^2(\alpha^3)\right)^3 = \left(\alpha^6(\alpha)\right) or\ (\alpha^2)^3 = \alpha^6\ \&\ (\alpha^3)^3 = \alpha \tag{22}$$

The unitary reciprocal polynomial of the prime $f(x)$ is the prime polynomial $g(x) = x^2 + 2x^2 + 2$ and it is the characteristic polynomial of the recurring sequence $\{\zeta_n\}$ where $\zeta_{n+2} + 2\zeta_{n+1} + 2\zeta_n = 0$ or $\zeta_{n+2} = \zeta_{n+1} + \zeta_n$, the roots of $g(x)$ are;

$$\beta_1 = \frac{\alpha^8}{\alpha} = \alpha^7, \beta_2 = \frac{\alpha^8}{\alpha^3} = \alpha^5 \tag{23}$$

Is very easy looking that α^7, α^5, are roots of the characteristic polynomial $g(x)$ corresponding the roots

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α, α^3 of $f(x)$ and the general term ζ_n is;

$$\zeta_n = B_1(\alpha^7)^n + B_2(\alpha^5)^n$$

For the initial vector ($\zeta_0 = 1, \zeta_1 = 1$);

$$\begin{cases} n = 0 \to B_1 + B_2 = 1 \\ n = 1 \to \alpha^7 B_1 + \alpha^5 B_2 = 1 \end{cases}$$

We have $B_1 = \alpha^5, B_2 = \alpha^7$ and the general term of the sequence $\{\zeta_n\}$ is;

$$\zeta_n = \alpha^5(\alpha^7)^n + \alpha^7(\alpha^5)^n \quad (24)$$

We can see the important properties;

$$\left(\alpha^5(\alpha^7)\right)^3 = \left(\alpha^7(\alpha^5)\right) or\ (\alpha^5)^3 = \alpha^7\ \&\ (\alpha^7)^3 = \alpha^5 \quad (25)$$

By the same way;

$$\left(\alpha^7(\alpha^5)\right)^3 = \left(\alpha^5(\alpha^7)\right) or\ (\alpha^7)^3 = \alpha^5\ \&\ (\alpha^5)^3 = \alpha^7 \quad (26)$$

The sequence $\{\zeta_n\}$ is periodic with the period $2^3 - 1 = 8$ and it is the flowing sequence;

1 1 2 0 2 2 1 0 1 1 2 0 2 2 1 0 …. (27)

Figure 2 shows the shift register generating $\{\zeta_n\}$

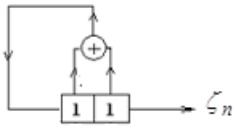

Figure 2. *shift register generating* $\{\zeta_n\}$

We can look that one period of the sequence $\{\zeta_n\}$ is one period of the sequence $\{s_n\}$but through reading it by inverse from the right to the left.

The multiplication sequence $\{z_n\}$, where $z_n = s_n.\zeta_n$ is;

$$\begin{aligned} z_n = s_n.\zeta_n &= \left[\alpha^6(\alpha)^n + \alpha^2(\alpha^3)^n\right]\left[\alpha^5(\alpha^7)^n + \alpha^7(\alpha^5)^n\right] \\ &= \alpha^{13}\alpha^{6n} + \alpha^7\alpha^{10n} + \alpha^9 + \alpha^{11} \\ &= \alpha^{13}\alpha^{6n} + \alpha^7\alpha^{10n} + 2 \end{aligned} \quad (28)$$

Or;

$$z_n = \alpha^5\alpha^{6n} + \alpha^7\alpha^{2n} + 2 \quad (29)$$

Thus, the sequence $\{z_n\}$ is a linear nonhomogeneous sequence with the length of its linear homogeneous equivalent is equals 2 that is equal to $(\deg f(x))^2 - \deg f(x) = 2$, The period of $\{z_n\}$is 4 and it's the half period of the sequence$\{s_n\}$and the sequence $\{z_n\}$ is: 0110 0110 0110 0110 …….

We can check that the set of the all periodic permutation of one period is not an orthogonal set for example , for one permutation of the period: 0110 is: 0011 and the sum of the two vectors is 0121 and is not belong to the set of permutations.

Figure 3 illustrates the feedback shift registers generating the sequence $\{z_n\}$;

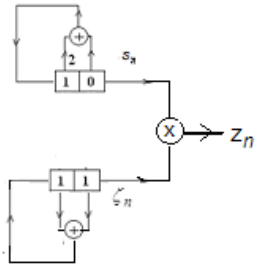

Figure 3. *Illustrated the multiplication sequence* $\{z_n\}$

xSuppose the linear homogeneous part of z_n is $LHP(z_n)$ $=\{z_n'\}$;

$$LHP(z_n) = z_n' = \alpha^5\alpha^{6n} + \alpha^7\alpha^{2n} \quad (30)$$

The sequence $\{z_n'\}$ is;

1 2 2 1 1 2 2 1 1 2 2 1 … (31)

The characteristic equation of the sequence $\{z_n'\}$is;

$$(x - \alpha^2)(x - \alpha^6) = 0$$

$$x^2 + 1 = 0 \quad (32)$$

And the recurring formula is $z_n' + z_n' = 0$ or $z_n' = 2z_n'$. Figure 4 illustrate the linear shift register generating sequence $\{z_n'\}$.

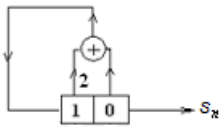

Figure 4. *shift register generating the sequence* $\{z_n'\}$

As the sequence $\{z_n\}$ the set of all periodic permutations of one period of the sequence $\{z_n'\}$ is not an orthogonal set.

Example 2

Suppose the polynomial $f(x) = x^3 + 2x + 1$ which is a prime characteristic polynomial the sequence $\{a_n\}$ as in figure5, $\forall n \in N; a_n \in F_3$ a_n satisfies;

$$a_{n+3} + 2a_{n+1} + a_n = 0; a_0\ or\ a_{n+3} = a_{n+1} + 2a_n; a_0 = 0,$$

$$a_1 = 1, a_2 = 1$$

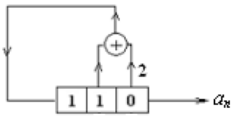

Figure 5. *Shift register generating the sequence* $\{a_n\}$ *on* F_3

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The characteristic equation of the sequence $\{a_n\}$ is;

$$x^3 + 2x + 1 = 0 \tag{33}$$

If α is a root of the equation then;

$$\alpha^3 + 2\alpha + 1 = 0 \tag{34}$$

And;

$$\begin{aligned} F_{3^3} = \{0, \alpha^{26} = 1, \alpha, \alpha^2, \alpha^3 = \alpha + 2, \alpha^4 = \alpha^2 + 2\alpha, \alpha^5 = 2\alpha^2 + \alpha + 2, \alpha^6 = \alpha^2 + \alpha + 1, \\ \alpha^7 = \alpha^2 + 2\alpha + 2, \alpha^8 = 2\alpha^2 + 2, \alpha^9 = \alpha + 1, \alpha^{10} = \alpha^2 + \alpha, \alpha^{11} = \alpha^2 + \alpha + 2 \\ \alpha^{12} = \alpha^2 + 2, \alpha^{13} = 2, \alpha^{14} = 2\alpha, \alpha^{15} = 2\alpha^2, \alpha^{16} = 2\alpha + 1, \alpha^{17} = 2\alpha^2 + \alpha, \\ \alpha^{18} = \alpha^2 + 2\alpha + 1, \alpha^{19} = 2\alpha^2 + 2\alpha + 2, \alpha^{20} = 2\alpha^2 + \alpha + 1, \alpha^{21} = \alpha^2 + 1, \\ \alpha^{22} = 2\alpha + 2, \alpha^{23} = 2\alpha^2 + 2\alpha, \alpha^{24} = 2\alpha^2 + 2\alpha + 1, \alpha^{25} = 2\alpha^2 + 1\} \end{aligned} \tag{35}$$

The general term of the sequence $\{a_n\}$ is of the form;

$$a_n = A_1\alpha^n + A_2(\alpha^3)^n + A_3(\alpha^{3^2})^n$$

Solving the following equations ;

$$\begin{cases} n = 0 \to A_1 + A_2 + A_3 = 0 \\ n = 1 \to \alpha A_1 + \alpha^3 A_2 + \alpha^9 A_3 = 1 \\ n = 2 \to \alpha^2 A_1 + \alpha^6 A_2 + \alpha^{18} A_3 = 1 \end{cases}$$

We have;

$$A_1 = \alpha^{22} = 2\alpha + 2, A_2 = \alpha^{14} = 2\alpha, A_3 = \alpha^{16} = 2\alpha + 1$$

We can see the properties;

$$(A_1)^3 = (\alpha^{22})^3 = \alpha^{14} = A_2$$
$$(A_2)^3 = (\alpha^{14})^3 = \alpha^{16} = A_3$$
$$(A_3)^3 = (\alpha^{16})^3 = \alpha^{22} = A_1$$

And the general solution of the characteristic equation is;

$$a_n = \alpha^{22}(\alpha)^n + \alpha^{14}(\alpha^3)^n + \alpha^{16}(\alpha^9)^n \tag{36}$$

Or;

$$a_n = (2\alpha + 2)\alpha^n + (2\alpha)(\alpha + 2)^n + (2\alpha + 1)(\alpha + 1)^n$$

And the sequence is periodic with the period: 3^3-1 = 26 and the sequence is;

0 1 1 1 0 0 2 0 2 1 2 2 1 0 2 2 2 0 0 1 0 1 2 1 1 2 0 1 1 1 0 …. (37)

The reciprocal polynomial of the prime $f(x)$ is the prime polynomial $g(x) = x^3 + 2x^2 + 1$ and it is the characteristic polynomial of the recurring sequence $b_{n+3} + 2b_{n+2} + b_n = 0$ or $b_{n+3} = b_{n+2} + 2b_n$, the roots of $g(x)$ are;

$$\beta_1 = \frac{\alpha^{26}}{\alpha} = \alpha^{25}, \beta_2 = \frac{\alpha^{26}}{\alpha^3} = \alpha^{23}, \beta_3 = \frac{\alpha^{26}}{\alpha^9} = \alpha^{17} \tag{38}$$

We can see the properties;

$$(\beta_1)^3 = (\alpha^{25})^3 = \alpha^{23} = \beta_2$$
$$(\beta_2)^3 = (\alpha^{23})^3 = \alpha^{17} = \beta_3$$
$$(\beta_3)^3 = (\alpha^{17})^3 = \alpha^{25} = \beta_1$$

Is very easy looking that $\alpha^{25}, \alpha^{23}, \alpha^{17}$ are roots of the characteristic polynomial $g(x)$ corresponding the roots $\alpha, \alpha^3, \alpha^{3^2} = \alpha^9$ of $f(x)$ and the general term b_n is;

$$b_n = B_1(\alpha^{25})^n + B_2(\alpha^{23})^n + B_3(\alpha^{17})^n$$

For the initial vector ($b_0 = 2$, $b_1 = 1$, $b_2 = 1$);

$$\begin{cases} n = 0 \to B_1 + B_2 + B_3 = 2 \\ n = 1 \to \alpha^{25}B_1 + \alpha^{23}B_2 + \alpha^{17}B_3 = 1 \\ n = 2 \to \alpha^{24}B_1 + \alpha^{20}B_2 + \alpha^8 B_3 = 1 \end{cases}$$

Thus, $B_1 = \alpha^{21}, B_2 = \alpha^{11}, B_3 = \alpha^7$

We can see the properties;

$$(B_1)^3 = (\alpha^{21})^3 = \alpha^{11} = B_2$$
$$(B_2)^3 = (\alpha^{11})^3 = \alpha^7 = B_3$$
$$(B_3)^3 = (\alpha^7)^3 = \alpha^{21} = B_1$$

And the general term of the sequence $\{b_n\}$ is;

$$b_n = \alpha^{21}(\alpha^{25})^n + \alpha^{11}(\alpha^{23})^n + \alpha^7(\alpha^{17})^n \tag{39}$$

The sequence $\{b_n\}$ is periodic with the period 3^3-1=26 and it is the following sequence;

2 1 1 2 1 0 1 0 0 2 2 2 0 1 2 2 1 2 0 2 0 0 1 1 1 0 2 1 1 2 1 0 1 0 0 …

Figure 6 shows register generating $\{b_n\}$;

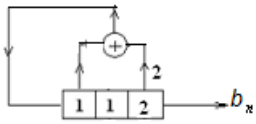

Figure 6. *shift register generating* $\{b_n\}$

We can look that one period of the sequence $\{b_n\}$ is one period of the sequence $\{a_n\}$ but through reading it by inverse from the right to the left.

Suppose the multiplication sequence $\{z_n\}$, where $z_n = a_n.b_n$, we have;

$$\begin{aligned} z_n = a_n.b_n = \left[\alpha^{22}(\alpha)^n + \alpha^{14}(\alpha^3)^n + \alpha^{16}(\alpha^9)^n\right] \\ \alpha^{21}(\alpha^{25})^n + \alpha^{11}(\alpha^{23})^n + \alpha^7(\alpha^{17})^n \\ z_n = \alpha^9(\alpha^2)^n + \alpha(\alpha^6)^n + \alpha^{11}(\alpha^8)^n + \\ \alpha^3(\alpha^{18})^n + \alpha^{21}(\alpha^{20})^n + \alpha^7(\alpha^{24})^n + 1 \end{aligned} \tag{40}$$

Thus, the sequence $\{z_n\}$ is a linear nonhomogeneous sequence with the length of its linear homogeneous equivalent is equals 6 that is equal to $(\deg f(x))^2 - \deg f(x) = 6$, The period of $\{z_n\}$ is 13, and the sequence $\{z_n\}$

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is: 0 1 1 2 0 0 2 0 0 2 1 1 0 0 1 1 2 0 0 2 0 0 2 1 1 0 …….

Figure 7 illustrates the linear feedback shift registers generating the sequence $\{z_n\}$;

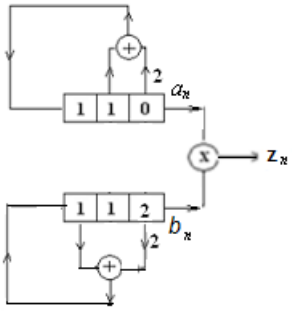

Figure 7. *Illustrated the multiplication sequence* $\{z_n\}$

We can check that the set of the all periodic permutation of one period is not an orthogonal set for example, for one permutation of the period: 0 1 1 2 0 0 2 0 0 2 1 1 0 is: 0 0 1 1 2 0 0 2 0 0 2 1 1and the sum of the two vectors is 0 1 2 0 2 0 2 2 0 2 0 2 1 and is don't belong to the set of all permutations of the first period.

The linear homogeneous part of z_n is $LHP(z_n) = \{z_n'\}$;

$$z_n' = \alpha^9(\alpha^2)^n + \alpha(\alpha^6)^n + \alpha^{11}(\alpha^8)^n + \alpha^3(\alpha^{18})^n + \alpha^{21}(\alpha^{20})^n + \alpha^7(\alpha^{24})^n \quad (41)$$

The characteristic equation of $\{z_n'\}$ is;

$$(x-\alpha^2)(x-\alpha^6)(x-\alpha^8)(x-\alpha^{18})(x-\alpha^{20})(x-\alpha^{24}) = 0$$

We can see that;

$$(-\alpha^2)(-\alpha^6)(-\alpha^8)(-\alpha^{18})(-\alpha^{20})(-\alpha^{24}) = 1$$

And the characteristic equation is of the form;

$$x^6 + \beta_1 x^5 + \beta_2 x^4 + \beta_3 x^3 + \beta_4 x^2 + \beta_5 x + 1 = 0$$

Calculated the coefficients we have;

$$\beta_1 = 0, \beta_2 = 1, \beta_3 = 2, \beta_4 = 0, \beta_5 = 1$$

Thus, the characteristic equation of $\{z_n'\}$ is;

$$x^6 + x^4 + 2x^3 + x + 1 = 0 \quad (42)$$

Or;

$$(x^3+1)(x^3+x+1) = 0$$

Thus, the recurring sequence $\{z_n'\}$is;

$$z_{n+6}' + z_{n+4}' + 2z_{n+3}' + z_{n+1}' + z_n' = 0 \quad (43)$$

Or;

$$z_{n+6}' = 2z_{n+4}' + z_{n+3}' + 2z_{n+1}' + 2z_n'$$

Figure 8 shows its feedback linear shift register of the sequence $\{z_n'\}$.

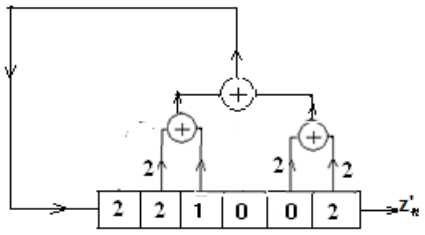

Figure 8. *Linear Equivalent of* $\{z_n'\}$ *with 6 complexity on* F_3

The sequence $\{z_n'\}$ is;

2 0 0 1 2 2 1 2 2 1 0 0 2 2 0 0 1 2 2 1 2 2 1 0 0 2 ….. (44)

As the sequence $\{z_n\}$ the set of all periodic permutations of one period of the sequence $\{z_n'\}$is not an orthogonal set.

Example 3

The recurring sequence $\{t_n\}$is giving by the shift register as in Figure 9;

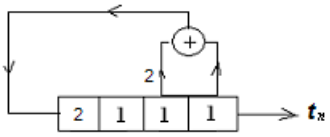

Figure 9. *Shift register generating sequence* $\{t_n\}$

The prime polynomial $f(x) = x^4 + x + 2$ is he characteristic polynomial of the sequence $\{t_n\}$in the field F_3, and it's characteristic equation is $x^4 + x + 2 = 0$, the recurring formula of the sequence is;

$$t_{n+4} + t_{n+1} + 2t_n = 0 \quad or \quad t_{n+4} = 2t_{n+1} + t_n \quad (44)$$

If β is a root of the characteristic equation then the general term of the sequence $\{t_n\}$ is of the form;

$t_n = \mu_1(\beta)^n + \mu_2(\beta^3)^n + \mu_3(\beta^9)^n + \mu_4(\beta^{27})^n$ or $t_n = \mu_1\beta^n + \mu_2\beta^{3n} + \mu_3\beta^{9n} + \mu_4\beta^{27n}$

Solving the following system (see appendix);

$$\begin{cases} n=0 \rightarrow \mu_1 + \mu_2 + \mu_3 + \mu_4 = 1 \\ n=1 \rightarrow \mu_1\beta + \mu_2\beta^3 + \mu_3\beta^9 + \mu_4\beta^{27} = 1 \\ n=2 \rightarrow \mu_1\beta^2 + \mu_2\beta^6 + \mu_3\beta^{18} + \mu_4\beta^{54} = 1 \\ n=3 \rightarrow \mu_1\beta^3 + \mu_2\beta^9 + \mu_3\beta^{27} + \mu_4\beta = 2 \end{cases}$$

Thus, $\mu_1 = \beta^{10}, \mu_2 = \beta^{30}, \mu_3 = \beta^{90}, \mu_4 = \beta^{270}$ or $\mu_1 = \beta^{10}, \mu_2 = \beta^{30}, \mu_3 = \beta^{10}, \mu_4 = \beta^{30}$

We can see that;

$$(\mu_1)^3 = (\beta^{10})^3 = \mu_2 \Rightarrow (\mu_3)^3 = \mu_4$$
$$(\mu_2)^3 = (\beta^{30})^3 = \mu_3, \Rightarrow (\mu_4)^3 = \mu_1$$

And t_n is;

$$t_n = \beta^{10}(\beta)^n + \beta^{30}(\beta^3)^n + \beta^{10}(\beta^9)^n + \beta^{30}(\beta^{27})^n \quad (45)$$

The sequence $\{t_n\}$ is periodic and its period is 3^4-1 =

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80, and it is;

1112002201 0221101012 1221201222 2000200120 2221001102 0112202021

2112102111 1000100210, 1112002201 0221101012 … (46)

The unitary polynomial $g(x) = x^4 + 2x^3 + 2$ is the reciprocal polynomial of $f(x)$ and the roots of $g(x)$ are;

$(\beta_1)^3 = (\beta^{79})^3, = \beta_2, (\beta_2)^3 = (\beta^{77})^3 \beta_3, , (\beta_3)^3 = (\beta^{71})^3 = \beta_4, (\beta_4)^3 = (\beta^{53})^3 = \beta_1$

Figure 10 shows the shift register generating sequence $\{\tau_n\}$ for the initial vector (0,1,2,0);

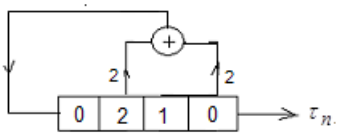

Figure 10. *Shift register generating sequence* $\{\tau_n\}$

The reciprocal sequence of $\{t_n\}$ is the sequence $\{\tau_n\}$ where;

$$\tau_n = \lambda_1(\beta^{79})^n + \lambda_2(\beta^{77})^n + \lambda_3(\beta^{71})^n + \lambda_4(\beta^{53})^n \quad (47)$$

For the initial vector ($\tau_0 = 0, \tau_1 = 1, \tau_2 = 2, \tau_3 = 0$);

$$\begin{cases} n = 0 \Rightarrow \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 0 \\ n = 1 \Rightarrow \beta^{79}\lambda_1 + \beta^{77}\lambda_2 + \beta^{71}\lambda_3 + \beta^{53}\lambda_4 = 1 \\ n = 2 \Rightarrow \beta^{78}\lambda_1 + \beta^{74}\lambda_2 + \beta^{62}\lambda_3 + \beta^{26}\lambda_4 = 2 \\ n = 3 \Rightarrow \beta^{77}\lambda_1 + \beta^{71}\lambda_2 + \beta^{53}\lambda_3 + \beta^{79}\lambda_4 = 0 \end{cases}$$

We have $\lambda_1 = \beta^9, \lambda_2 = \beta^{27}, \lambda_3 = \beta, \lambda_4 = \beta^3$, and τ_n is;

$$\tau_n = \beta^9(\beta^{79})^n + \beta^{27}(\beta^{77})^n + \beta(\beta^{71})^n + \beta^3(\beta^{53})^n \quad (48)$$

We can see that;

$(\lambda_1)^3 = (\beta^9)^3 = \lambda_2; (\lambda_2)^3 = (\beta^{\mathbf{?}}) = \lambda_3$

$(\lambda_3)^3 = (\beta^3)^3 = \lambda_4; \Rightarrow (\lambda_4)^3 = \lambda_1$

The period of $\{\tau_n\}$ is $3^4 - 1 = 80$ and the sequence is;

0120010001 1112012112 1202022110 2011001222 0210020002 2221021221

2101011220 1022002111, 0120010001 1112012112 …. (49)

We can see that one period of the sequence $\{\tau_n\}$ is one period of the sequence $\{t_n\}$ but by reading it by inverse from the right to the left.

Suppose the multiplication sequence $\{z_n\}$ where $z_n = t_n . \tau_n$ we have;

$$z_n = t_n.\tau_n = \left[\beta^{10}\beta^n + \beta^{30}\beta^{3n} + \beta^{10}\beta^{9n} + \beta^{30}\beta^{27n}\right] \left[\beta^9(\beta^{79})^n + \beta^{27}(\beta^{77})^n + \beta(\beta^{71})^n + \beta^3(\beta^{53})\right]$$

$$\begin{aligned} z_n = {} & \beta^{39}\beta^{2n} + \beta^{37}\beta^{6n} + \beta^{19}\beta^{8n} + \beta\ \beta^n \\ & + \beta^{31}\beta^{18n} + \beta^{57}\beta^{24n} + \beta^{39}\beta^{26n} + \beta^{13}\beta^{54n} \\ & + \beta^{33}\beta^{56n} + \beta^{13}\beta^{62n} + \beta^{11}\beta^{72n} + \beta^{31}\beta^{74n} \\ & + \beta^{37}(\beta^{78})^n + \beta^{11} + \beta^{19} + \beta^{57} + \beta^{33} \end{aligned} \quad (50)$$

Or;

$$\begin{aligned} z_n = {} & \beta^{39}\beta^{2n} + \beta^{37}\beta^{6n} + \beta^{19}\beta^{8n} + \beta\beta^n \\ & + \beta^{31}\beta^{18n} + \beta^{57}\beta^{24n} + \beta^{39}\beta^{26n} + \beta^{13}\beta^{54n} \\ & + \beta^{33}\beta^{56n} + \beta^{13}\beta^{62n} + \beta^{11}\beta^{72n} + \beta^{31}\beta^{74n} \\ & + \beta^{37}\beta^{78n} \end{aligned} \quad (51)$$

Thus, the sequence $\{z_n\}$ is a linear homogeneous sequence with the length 12 and equal to $(\deg f(x))^2 - \deg f(x) = 12$, periodic with the period 40 and this sequence is;

0120000001 0222002011 1102002220 1000000210, 0120000001 0222002011

1102002220 1000000210, ……. (52)

We can check that the set of the all periodic permutations of one period of $\{z_n\}$ is not orthogonal set.

Figure 11 illustrated the nonlinear feedback shift register generating $\{z_n\}$;

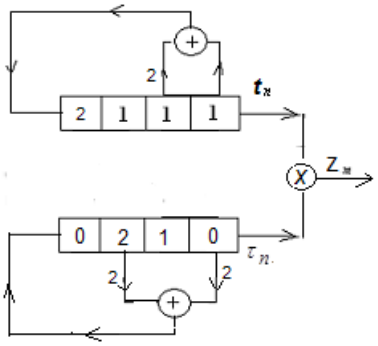

Figure 11. *Illustrated the multiplication sequence* $\{z_n\}$

We can see that $\beta^2.\beta^6.\beta^8.\beta.\beta^{18}.\beta^{24}.\beta^{26}.\beta^{54}.\beta^{56}.\beta^{62}.\beta^{72}.\beta^{74}.\beta^{78} = \beta^{480} = 1$.

Thus, $\{z_n\} = \{z_n'\}$ where $\{z_n'\}$ the linear part of $\{z_n\}$, the characteristic equation of $\{z_n\}$ is of the form;

$$x^{12} + \alpha_{11}x^{11} + \alpha_{10}x^{10} + \alpha_9x^9 + \alpha_8x^8 + \alpha_7x^7 + \alpha_6x^6 + \alpha_5x^5 + \alpha_4x^4 + \alpha_3x^3 + \alpha_2x^2 + \alpha_1x + 1 = 0$$

Or;

$$z_{n+12} + \alpha_{11}z_{n+11} + \alpha_{10}z_{n+10} + \alpha_9z_{n+9} + \alpha_8z_{n+8} + \alpha_7z_{n+7} + \alpha_6z_{n+6} + \alpha_5z_{n+5} + \alpha_4z_{n+4} + \alpha_3z_{n+3} + \alpha_2z_{n+2} + \alpha_1z_{n+1} + z_n = 0$$

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Thus, for n = 0,1,2,3,…,10 and adding for n = 11 if it is need we can find the coefficients of the characteristic equation of $\{s_n\}$ as following;

$$\begin{cases}2\alpha_{11}+\alpha_9+2\alpha_2+\alpha_1=1\\ 2\alpha_{11}+\alpha_{10}+\alpha_8+2\alpha_1=0\\ 2\alpha_{11}+2\alpha_{10}+2\alpha_9+\alpha_7=1\\ 2\alpha_{10}+2\alpha_9+2\alpha_8+\alpha_6=0\end{cases}$$

$$\begin{cases}2\alpha_9+2\alpha_8+2\alpha_7+\alpha_5=1\\ 2\alpha_{11}+2\alpha_8+2\alpha_7+2\alpha^6+\alpha_4=0\\ 2\alpha_{10}+2\alpha_7+2\alpha_6+2\alpha_5+\alpha_3=2\\ \alpha_{11}+2\alpha_9+2\alpha_6+2\alpha_5+2\alpha_4+\alpha_2=2\end{cases}$$

$$\begin{cases}\alpha_{11}+\alpha_{10}+2\alpha_8+2\alpha_5+2\alpha_4+2\alpha_3+\alpha_1=2\\ \alpha_{11}+\alpha_{10}+\alpha_9+2\alpha_7+2\alpha_4+2\alpha_3+2\alpha_2=1\\ \alpha_{11}+\alpha_{10}+\alpha_9+\alpha_8+2\alpha_6+2\alpha_3+2\alpha_2+2\alpha_1=0\end{cases}$$

Thus;

$\alpha_1=\alpha_3=0;\quad \alpha_2=\alpha_4=\alpha_5=\alpha_6=$

$\alpha_9=\alpha_{10}=\alpha_{11}=2;\quad \alpha_7=\alpha_8=1$

And the characteristic equation of the sequence $\{z_n\}$is;

$$x^{12}+2x^{11}+2x^{10}+2x^9+x^8+x^7+ 2x^6+2x^5+2x^4+2x^2+1=0 \tag{53}$$

And the recurring formula of the sequence $\{z_n\}$is;

$$z_{n+12}+2z_{n+11}+2z_{n+10}+2z_{n+9}+z_{n+8}+z_{n+7} +2z_{n+6}+2z_{n+5}+2z_{n+4}+2z_{n+2}+z_n=0 \tag{54}$$

Or;

$$z_{n+12}=z_{n+11}+z_{n+10}+z_{n+9}+2z_{n+8}+2z_{n+7} +z_{n+6}+z_{n+5}+z_{n+4}+z_{n+2}+2z_n \tag{55}$$

Figure 12 shows the linear equivalent of the $\{z_n\}$;

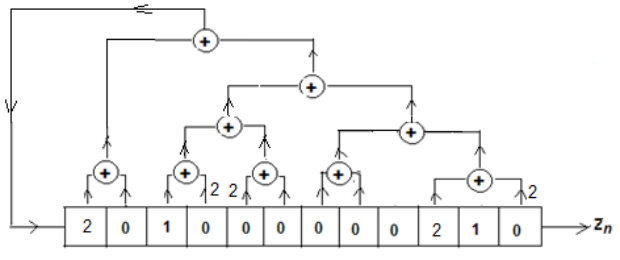

Figure 12. *Shift register generating sequence* $\{z_n\}$

Example 4

Given the recurring sequence $\{v_n\}$ over F_{5^2} with two degree where;

$$v_{n+2}+v_{n+1}+2v_n=0 \text{ or } v_{n+2}=4v_{n+1}+3v_n \tag{56}$$

Figure 13 shows the shift register generating it;

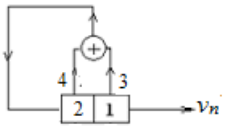

Figure 13. *Shift register generating the sequence* $\{v_n\}$

xAnd $\{v_n\}$ has the polynomial $f(x)=x^2+x+2$ as a prime characteristic polynomial, the roots $f(x)$ are; $\gamma, \gamma^5=4\gamma+4$ which are lie in the field F_{5^2} and;

$$\begin{aligned}F_{5^2}=\{&0,\gamma^{24}=1,\gamma,\gamma^2=4\gamma+3,\gamma^3=4\gamma+2,\gamma^4=\\ &3\gamma+2,\gamma^5=4\gamma+4,\gamma^6=2,\gamma^7=2\gamma,\gamma^8=5\gamma+1,\\ &\gamma^9=3\gamma+4,\gamma^{10}=\gamma+4,\gamma^{11}=3\gamma+3,\gamma^{12}=\\ &4,\gamma^{13}=4\gamma,\gamma^{14}=\gamma+2,\gamma^{15}=\gamma+3,\gamma^{16}=2\gamma+3,\\ &\gamma^{17}=\gamma+1,\gamma^{18}=3,\gamma^{19}=3\gamma,\gamma^{20}=2\gamma+4,\gamma^{21}=\\ &2\gamma+1,\gamma^{22}=4\gamma+1,\gamma^{23}=2\gamma+2\}\end{aligned} \tag{57}$$

And v_n is of the form;

$$v_n=A_1\gamma^n+A_2\gamma^{5n}$$

Or;

$$\begin{cases}n=0\Rightarrow A_1+A_2=1\\ n=1\Rightarrow A_1\gamma+A_2\gamma^5=2\end{cases}$$

Solving this system we have;

$$A_1=3, A_2=\gamma^{18}$$

But $(A_1)^5=(3)^5\neq A_2=\gamma^{18}$ and $(A_2)^5=(\gamma^{18})^5=\gamma^{18}\neq A_1$

Thus, v_n is equals;

$$v_n=3(\gamma)^n+\gamma^{18}(\gamma^5)^n \tag{58}$$

The sequence $\{v_n\}$ is period with the period 5^2-1 = 24, and the all cyclic permutations of one period is an orthogonal set;

1 2 1 0 3 2 2 4 2 0 1 4 4 3 4 0 2 3 3 1 3 0 4 1, 1 2 1 0 3 2 2 ……. (59)

The unitary reciprocal polynomial of $f(x)$ is the polynomial $g(x)=x^2+3x^3+3$ and the reciprocal sequence of $\{v_n\}$ is the sequence $\{w_n\}$ with the recurring formula;

$$w_{n+2}+3w_{n+1}+3w_n=0$$

Or;

$$w_{n+2}=2w_{n+1}+2w_n$$

The roots of $g(x)$ are;

$$\gamma_1=\frac{\gamma^{24}}{\gamma}=\gamma^{23},\gamma_2=\frac{\gamma^{24}}{\gamma^5}=\gamma^{19}$$

Is very easy looking that γ^{23},γ^{19} are roots of the

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characteristic polynomial $g(x), (\gamma^{23})^5 = \gamma^{19}$, $(\gamma^{19})^5 = \gamma^{23}$ and v_n is of the form;

$$w_n = B_1(\gamma^{23})^n + B_2(\gamma^{19})^n \tag{60}$$

For the initial vector ($w_0 = 1$, $w_1 = 4$ are the latest two values of one period of the sequence $\{w_n\}$ but by inverse we read them from the right to the left) and by solving the following system for $n = 0$, $n = 1$;

$$\begin{cases} B_1 + B_2 = 1 \\ \gamma^{23}B_1 + \gamma^{19}B_2 = 4 \end{cases}$$

Solving this system of equations we have:

$B_1 = \gamma^{17}, B_2 = \gamma^{13}$ and;

$(B_1)^5 = (\gamma^{17})^5 = \gamma^{13} = B_2$ and $(B_2)^5 = (\gamma^{13})^5 = \gamma^{17} = B_1$

Thus;

$$w_n = \gamma^{17}(\gamma^{23})^n + \gamma^{13}(\gamma^{19})^n \tag{61}$$

The sequence $\{w_n\}$ is periodic with the period $5^2 - 1 = 24$ and it is;

1 4 0 3 1 3 3 2 0 4 3 4 4 1 0 2 4 2 2 3 0 1 2 1, 1 4 0 3 1 3 3 2 0 … (62)

Figure14 illustrated shift register generating $\{w_n\}$;

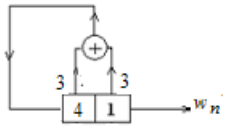

Figure 14. *Shift register generating sequence* $\{w_n\}$

One period of $\{v_n\}$ is an one period of the sequence $\{w_n\}$ but by inverse from the right to the left.

The multiplication sequence $\{z_n\}$ where $z_n = v_n.w_n$ is;

$$\begin{aligned} z_n &= v_n.w_n \\ &= \left[3(\gamma)^n + \gamma^{18}(\gamma^5)^n\right]\left[\gamma^{17}(\gamma^{23})^n + \gamma^{13}(\gamma^{19})^n\right] \\ z_n &= 3\gamma^{17} + 3\gamma^{13}\gamma^{20n} + \gamma^{11}\gamma^{4n} + \gamma^7 \\ z_n &= 3\gamma^{13}\gamma^{20n} + \gamma^{11}\gamma^{4n} + 3 \end{aligned} \tag{63}$$

Thus, the sequence $\{z_n\}$ is a linear nonhomogeneous sequence and periodic with the period 6 and it is;

1 3 0 0 3 1, 1 3 0 0 3 1 ……. (64)

The set of all cyclic permutations of one period is not orthogonal set.

Suppose the linear homogeneous part with the sequence is $\{z'_n\}$, which it's linear equivalent has the length $(\deg f(x))^2 - \deg f(x) = 2$, and the period of $\{z'_n\}$ is also 6 and it is;

3 0 0 0 0 3, 3 0 0 0 0 3, ….. (65)

And the set of the all cyclic permutations of one period of $\{z_n\}$ is not an orthogonal set.

Figure 15 Shows the nonlinear shift register generating sequence $\{z_n\}$;

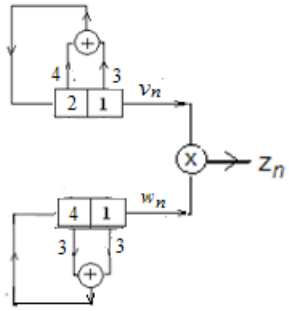

Figure 15. *Illustrated the multiplication sequence* $\{z_n\}$

According with the sequences $\{s_n\}$, its reciprocal sequence $\{\zeta_n\}$ and their multiplication sequence $\{z_n\}$ in the examples 1, 2, 3,4, we have;

$$\text{In example 1: } \begin{cases} S_n = \alpha^6(\alpha)^n + \alpha^2(\alpha^3)^n \\ \zeta_n = \alpha^5(\alpha^7)^n + \alpha^7(\alpha^5)^n \end{cases} \tag{66}$$

$$\text{In example 2: } \begin{cases} a_n = \alpha^{22}(\alpha)^n + \alpha^{14}(\alpha^3)^n + \alpha^{16}(\alpha^9)^n \\ b_n = \alpha^{21}(\alpha^{25})^n + \alpha^{11}(\alpha^{23})^n + \alpha^7(\alpha^{17})^n \end{cases} \tag{67}$$

In example 3:

$$\begin{cases} t_n = \beta^{10}\beta^n + \beta^{30}\beta^{3n} + \beta^{10}\beta^{9n} + \beta^{30}\beta^{27n} \\ \tau_n = \beta^9(\beta^{79})^n + \beta^{27}(\beta^{77})^n + \beta(\beta^{71})^n + \beta^3(\beta^{53})^n \end{cases} \tag{68}$$

$$\text{In example 4. } \begin{cases} v_n = 3(\gamma)^n + \gamma^{18}(\gamma^5)^n \\ w_n = \gamma^{17}(\gamma^{23})^n + \gamma^{13}(\gamma^{19})^n \end{cases} \tag{69}$$

Suppose, the basic sequence is $\{S_n\}$ and it's of the form;

$$\begin{aligned} s_n &= A_1\alpha^n + A_2(\alpha^{p^2-1})^n + \ldots + A_k(\alpha^{p^{k-1}})^n \\ &= \sum_{i=1}^{k} A_i(\alpha^{p^{i-1}})^n \end{aligned} \tag{70}$$

And it's reciprocal sequence is $\{\zeta_n\}$ and it's of the form;

$$\begin{aligned} \zeta_n &= B_1\alpha^{n(p^k-2)} + B_2\alpha^{n(p^k-3)} + \ldots + B_k\alpha^{n(p^k-p^{k-1}-1)} \\ &= \sum_{i=1}^{k} B_i\alpha^{n(p^k-p^{i-1}-1)} \end{aligned} \tag{71}$$

And the product sequence is $\{z_n\}$ where $z_n = S_n\zeta_n$ and it's of the form;

$$z_n = \left(\sum_{i=1}^{k} A_i(\alpha^{p^{i-1}})^n\right)\left(\sum_{j=1}^{k} B_j(\alpha^{(p^k-p^{j-1}-1)})^n\right) \tag{72}$$

Or;

$$z_n = \left(\sum_{i,j=1,i\neq j}^{k} A_iB_j(\alpha^{p^{i-1}})^n(\alpha^{(p^k-p^{j-1}-1)})^n\right) + \left(\sum_{i=1}^{k} A_iB_i\right) \tag{73}$$

We can see the following conclusions.

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4. Conclusions

*P*1. For one period of the sequence $\{S_n\}$ the corresponding period of its reciprocal sequence $\{\zeta_n\}$ is the same but by inverse from the right to the left.

*P*2. In the both form of the general term we have of each of them; $(A_i)^p = (A_{i+1})$ and $(B_i)^p = (B_{i+1})$ except example 4 that is, $(A_1)^p = (3)^5 = 3 \bmod 5 \neq \gamma^{18} = A_2$ also $(A_2)^5 = (\gamma^{18})^5 = \gamma^{18} \neq 3 = A_1$ but $(B_1)^5 = B_2$ & $(B_2)^5 = B_1$.

*P*3. In the both form of the general term we have of each of them; $(\alpha^{p^{i-1}})^p = (\alpha^{p^i})$ and $(\alpha^{(p^k - p^{j-1} - 1)})^p = (\alpha^{(p^k - p^j - 1)})$.

*P*4. The exponent of the coefficient of the first term in the general term in the sequence $\{S_n\}$ is larger than the corresponding coefficient in the sequence $\{\zeta_n\}$ by one except example 4.

*P*5. The length of the linear homogeneous part of the sequence $\{z_n\}$ is equal to $((\deg f(x))^2 - \deg f(x))$ Where the $f(x)$ is the characteristic polynomial of the sequence $\{S_n\}$.

P6. The period of the sequence $\{S_n\}$ equals the period of the sequence $\{\zeta_n\}$ and equals (p^k-1) but the period of the sequence $\{z_n\}$ in the examples1 to 3 is equal to half of this value but in example 4 is equals the quarter of this value.

P_7. The set of all cyclic permutations of one period of the multiplication sequence $\{z_n\}$ is not orthogonal set.

P_8. There is important difference between the properties of multiplication M-Sequences and M_p-Sequences.

P_9 – We did not come across one term in the basic M_p-Sequene $\{S_n\}$ as the sum of Exp.(A_i) and Exp. ($\alpha^{p^{i-1}}$) in formula of S_n are complement that is; (Exp.(A_i) + Exp. ($\alpha^{p^{i-1}}$) = p^k -1 consequently the same for the ζ_n and furthermore it the permutations between the "coefficients and roots which are in formula of S_n" will not be in the formula of ζ_n as in the binary M-Sequences.

References

[1] Sloane, N.J.A., (1976), "An Analysis Of The Stricture And Complexity of Nonlinear Binary Sequence Generators," *IEEE Trans. Information Theory* Vol. It 22 No 6, PP 732-736.

[2] Mac Wiliams, F. G & Sloane,N.G.A., (2006), "*The Theory of Error- Correcting Codes,"* North- Holland, Amsterdam.

[3] Mokayes D. Al Cheikha A. H., (2021- February) Study the Linear Equivalent of Nonlinear Sequences over *Fp* Where p is larger than two, *International Journal of Information and Communication* Sciences, ISSN: 2575-1700, Vol. 5, Issue 4, pp 53-75

[4] Al Cheikha A. H. (September,2014). Some Properties of M-Sequences Over Finite Field Fp. *International Journal of Computer Engineering & Technology*. IJCET. ISSN 0976-6367(Print),ISSN 0976 - 6375(Online),Vol.5, Issue 9. Pp. 61- 72.

[5] Al Cheikha A. H. (May 2014), " Matrix Representation of Groups in the finite Fields GF(p^n) "International Journal of Soft Computing and Engineering, Vol. 4, Issue 2, PP 118-125.

[6] Al Cheikha A. H. (2018).Generation New Binary Sequences using Quotient Ring Z/p^mZ. *Research Journal of Mathematics and Computer Science. RJMCS.* ISSN: 2576 -3989, Vol.2, Issue 11. Pp. 0001- 0013.

[7] Al Cheikha, A. H., (2019), Placement of M-Sequences over the Field *Fp* in the Space *Rn*, International Journal of Information and Communication Science, IJICS, ISSN: 2575-1700 (Print); ISSN: 2575-1719 (Online), Vol. 4, No.1, Pp. 24-34.

[8] Al Cheikha A. H. (May 5, 2014). Matrix Representation of Groups in the Finite Fields GF(p^n). *International Journal of Soft Computing and Engineering*, IJSCE, ISSN:2231- 2307, Vol. 4, Issue 2, pp. 1-6.

[9] Al Cheikha A. H., Omar Ebtisam. Haj., "Study the Multiplication M-sequences and its Reciprocal Sequences", *Journal of Electronic & Information Systems*. ISSN: 2661-3204, Vol. 03, Issue. 0, Pp. 13-22.

[10] Al Cheikha, A.H. (April 26, 2014). Matrix Representation of Groups in the Finite Fields GF(2^n). *International Journal of Soft Computing and Engineering*, IJSCE, ISSN: 2231-2307, Vol. 4, Issue 2. pp. 118-125.

[11] Al Cheikhaa A. H. A Theoretical Study for the Linear Homogenous Orthogonal Recurring Sequences. (5 May, 2004). In Almanara Journal, Alalbayt University, Jordan. No 2, 285/2004. (in Arabic), { In English: www.researchgate.net/profile/Ahmad_Al_Cheikha/ publications After select: Ahmad Al Cheikha | Ahlia University | Department of ... – ResearchGate After select: Research, and after select: Article, or Faulltexts, and the article between my items}.

[12] Golamb S. W. (1976), Shift Register Sequences, San Francisco – Holden Day.

[13] Lee J.S &Miller L.E, (1998), "*CDMA System Engineering Hand Book, "*Artech House. Boston, London.

[14] Yang S.C,"*CDMA RF* , (1998), *System Engineering,"*Artech House. Boston- London.

[15] Lidl, R.& Pilz, G., (1984), "*Applied Abstract Algebra*," Springer – Verlage New York, 1984.

[16] Lidl, R. & Niderreiter, H., (1994), "Introduction to Finite Fields and Their Application, " *Cambridge university* USA.

[17] Thomson W. Judson, (2013), "*Abstract Algebra: Theory and Applications*," Free Software Foundation.

[18] Fraleigh, J.B., (1971), "A First course In Abstract Algebra, *Fourth printing. Addison-Wesley publishing company* USA.

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Appendix

F_{3^4}		
0	$\beta^{26}=\beta^2+2\beta+1$	$\beta^{53}=\beta+1$
$\beta^{80}=1$	$\beta^{27}=\beta^3+2\beta^2+\beta$	$\beta^{54}=\beta^2+\beta$
β	$\beta^{28}=2\beta^3+\beta^2+2\beta+1$	$\beta^{55}=\beta^3+\beta^2$
β^2	$\beta^{29}=\beta^3+2\beta^2+2\beta+2$	$\beta^{56}=\beta^3+2\beta+1$
β^3	$\beta^{30}=2\beta^3+2\beta^2+\beta+1$	$\beta^{57}=2\beta^2+1$
$\beta^4=2\beta+1$	$\beta^{31}=2\beta^3+\beta^2+2\beta+2$	$\beta^{58}=2\beta^3+\beta$
$\beta^5=2\beta^2+\beta$	$\beta^{32}=\beta^3+2\beta^2+2$	$\beta^{59}=\beta^2+\beta+2$
$\beta^6=2\beta^3+\beta^2$	$\beta^{33}=2\beta^3+\beta\alpha+1$	$\beta^{60}=\beta^3+\beta^2+2\beta$
$\beta^7=\beta^3+\beta+2$	$\beta^{34}=\beta^2+2\beta+2$	$\beta^{61}=\beta^3+2\beta^2+2\beta+1$
$\beta^8=\beta^2+\beta+1$	$\beta^{35}=\beta^3+2\beta^2+2\beta$	$\beta^{62}=2\beta\alpha^3+2\beta^2+1$
$\beta^9=\beta^3+\beta^2+\beta$	$\beta^{36}=2\beta^3+2\beta^2+2\beta+1$	$\beta^{63}=2\beta^3+2\beta+2$
$\beta^{10}=\beta^3+\beta^2+2\beta+1$	$\beta^{37}=2\beta^3+2\beta^2+2\alpha+2$	$\beta^{64}=2\beta^2+2$
$\beta^{11}=\beta^3+2\beta^2+1$	$\beta^{38}=2\beta^3+2\beta^2+2$	$\beta^{65}=2\beta^3+2\beta$
$\beta^{12}=2\beta^3+1$	$\beta^{39}=2\beta^3+2$	$\beta^{66}=2\beta^2+\beta+2$
$\beta^{13}=2\beta+2$	$\beta^{40}=2$	$\beta^{67}=2\beta^3+\beta^2+2\beta$
$\beta^{14}=2\beta^2+2\beta$	$\beta^{41}=2\beta$	$\beta^{68}=\beta^3+2\beta^2+\beta+2$
$\beta^{15}=2\beta^3+2\beta^2$	$\beta^{42}=2\beta^2$	$\beta^{69}=2\beta^3+\beta^2+\beta+1$
$\beta^{16}=2\beta^3+\beta+2$	$\beta^{43}=2\beta^3$	$\beta^{70}=\beta^3+\beta^2+2\beta+2$
$\beta^{17}=\beta^2+2$	$\beta^{44}=\beta+2$	$\beta^{71}=\beta^3+2\beta^2+\beta+1$
$\beta^{18}=\beta^3+2\beta$	$\beta^{45}=\beta^2+2\beta$	$\beta^{72}=2\beta^3+\beta^2+1$
$\beta^{19}=2\beta^2+2\beta+1$	$\beta^{46}=\beta^3+2\beta^2$	$\beta^{73}=\beta^3+2\beta+2$
$\beta^{20}=2\beta^3+2\beta^2+\beta$	$\beta^{47}=2\beta^3+2\beta+1$	$\beta^{74}=2\beta^2+\beta+1$
$\beta^{21}=2\beta^3+\beta^2+\beta\alpha+2\beta$	$\beta^{48}=2\beta^3+2\beta+2$	$\beta^{75}=2\beta^3+\beta^2+\beta$
$\beta^{22}=\beta^3+\beta^2+2$	$\beta^{49}=2\beta^3+2\beta^2+2\beta$	$\beta^{76}=\beta^3+\beta^2+\beta+2$
$\beta^{23}=\beta^3+\beta+1$	$\beta^{50}=2\beta^3+2\beta^2+\beta+2$	$\beta^{77}=\beta^3+\beta^2+\beta+1$
$\beta^{24}=\beta^2+1$	$\beta^{51}=2\beta^3+\beta^2+2$	$\beta^{78}=\beta^3+\beta^2+1$
$\beta^{25}=\beta^3+\beta$	$\beta^{52}=\beta^3+2$	$\beta^{79}=\beta^3+1$

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Author Guidelines

This document provides some guidelines to authors for submission in order to work towards a seamless submission process. While complete adherence to the following guidelines is not enforced, authors should note that following through with the guidelines will be helpful in expediting the copyediting and proofreading processes, and allow for improved readability during the review process.

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- Program: Microsoft Word (preferred)
- Font: Times New Roman
- Size: 12
- Style: Normal
- Paragraph: Justified
- Required Documents

Ⅱ. Cover Letter

All articles should include a cover letter as a separate document.

The cover letter should include:

- Names and affiliation of author(s)

The corresponding author should be identified.

Eg. Department, University, Province/City/State, Postal Code, Country

- A brief description of the novelty and importance of the findings detailed in the paper

Declaration

v Conflict of Interest

Examples of conflicts of interest include (but are not limited to):

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- Honoria
- Employment or consultation
- Project sponsors
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- Other financial relationships/support
- Informed Consent

This section confirms that written consent was obtained from all participants prior to the study.

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- Trial Registration

Eg. Name of Trial Registry: Trial Registration Number

- Contributorship

The role(s) that each author undertook should be reflected in this section. This section affirms that each credited author has had a significant contribution to the article.

1. Main Manuscript
2. Reference List
3. Supplementary Data/Information

Supplementary figures, small tables, text etc.

As supplementary data/information is not copyedited/proofread, kindly ensure that the section is free from errors, and is presented clearly.

Ⅲ. Abstract

A general introduction to the research topic of the paper should be provided, along with a brief summary of its main results and implications. Kindly ensure the abstract is self-contained and remains readable to a wider audience. The abstract should also be kept to a maximum of 200 words.

Authors should also include 5-8 keywords after the abstract, separated by a semi-colon, avoiding the words already used in the title of the article.

Abstract and keywords should be reflected as font size 14.

Ⅳ. Title

The title should not exceed 50 words. Authors are encouraged to keep their titles succinct and relevant.

Titles should be reflected as font size 26, and in bold type.

Ⅳ. Section Headings

Section headings, sub-headings, and sub-subheadings should be differentiated by font size.

Section Headings: Font size 22, bold type

Sub-Headings: Font size 16, bold type

Sub-Subheadings: Font size 14, bold type

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Ⅴ. Introduction

The introduction should highlight the significance of the research conducted, in particular, in relation to current state of research in the field. A clear research objective should be conveyed within a single sentence.

Ⅵ. Methodology/Methods

In this section, the methods used to obtain the results in the paper should be clearly elucidated. This allows readers to be able to replicate the study in the future. Authors should ensure that any references made to other research or experiments should be clearly cited.

Ⅶ. Results

In this section, the results of experiments conducted should be detailed. The results should not be discussed at length in

this section. Alternatively, Results and Discussion can also be combined to a single section.

Ⅷ. Discussion

In this section, the results of the experiments conducted can be discussed in detail. Authors should discuss the direct and indirect implications of their findings, and also discuss if the results obtain reflect the current state of research in the field. Applications for the research should be discussed in this section. Suggestions for future research can also be discussed in this section.

Ⅸ. Conclusion

This section offers closure for the paper. An effective conclusion will need to sum up the principal findings of the papers, and its implications for further research.

Ⅹ. References

References should be included as a separate page from the main manuscript. For parts of the manuscript that have referenced a particular source, a superscript (ie. [x]) should be included next to the referenced text.

[x] refers to the allocated number of the source under the Reference List (eg. [1], [2], [3])

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J = Journal/Magazine

M = Monograph/Book

C = (Article) Collection

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S = Standards

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Kindly note that the order of appearance of the referenced source should follow its order of appearance in the main manuscript.

Graphs, Figures, Tables, and Equations

Graphs, figures and tables should be labelled closely below it and aligned to the center. Each data presentation type should be labelled as Graph, Figure, or Table, and its sequence should be in running order, separate from each other.

Equations should be aligned to the left, and numbered with in running order with its number in parenthesis (aligned right).

Ⅻ. Others

Conflicts of interest, acknowledgements, and publication ethics should also be declared in the final version of the manuscript. Instructions have been provided as its counterpart under Cover Letter.

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