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Dynamic Pricing Research for Container Terminal Handling Charges based on Demand Forecast

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ABSTRACT

A dynamic pricing model was established based on forecasting the demand for container handling of a specific shipping company to maximize terminal profits to solve terminal handling charges under the changing market environment. It assumes that container handling demand depends on the price and the unknown parameters in the demand model. The maximum quasi-likelihood estimation (MQLE) method is used to estimate the unknown parameters. Then an adaptive dynamic pricing policy algorithm is proposed. At the beginning of each period, through dynamic pricing, determining the optimal price relative to the estimation value of the current parameter and attach a constraint of differential price decision. Meanwhile, the accuracy of demand estimation and the optimality of price decisions are balanced. Finally, a case study is given based on the real data of Shanghai port. The results show that this pricing policy can make the handling price converge to the stable price and significantly increase this shipping company's handling profit compared with the original "contractual pricing" mechanism.

1. Introduction

In recent years, due to the unstable global economic development, the shipping market fluctuates wildly. Consequently, the terminal, as an essential logistics point that provides handling services to shipping companies, will bear the impact of market fluctuations. However, according to the terminal's operational data, container handling capacity has gradually become saturated, and merely attracting handling volume cannot significantly increase terminal revenue. Therefore, in the changing market environment, to develop reasonable and practical terminal handling charges is a crucial way to maximize revenue.

At present, most terminals charge for container terminal handling charges through a contractual pricing mechanism^[1]. Under this mechanism, the terminal and the shipping company negotiated and signed a contract about terminal handling charges for the next year. There may be two potential drawbacks. On the one hand, the terminal handling charges critically depend on the manager's own experience, which is difficult to reflect the labor value of terminal. On the other hand, the terminal handling charge is fixed during the contract period, which is challenging to accommodate the market changes. Therefore, the terminal needs to improve and optimize container handling charges, with the changing market environment dynamically adjusting price trends.

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The core idea of dynamic pricing is to establish the relationship between price and variable factors, such as changing periods, changing inventory, and changing demand. For terminals, the changing handling demand is an essential factor affecting the pricing of terminal handling charges. Generally, terminal handling operations are based on unit containers. On the one hand, different handling demand levels affect the composition of terminal handling costs. According to the theory of Time-driven Activity-based costing (TDABC), different types of containers are operated with different degrees of difficulty, resulting in different time consumption and thus different costs^[2]. On the other hand, the handling demand directly affects the terminal's revenue. To date, most academic studies implicitly assumed the handling demand obeys a specific distribution or a specific functional relationship, and they can fit it according to historical data^[3]. However, this method is generally applicable to a stable market environment, where the handling demand of shipping companies has little variation. The demand model can well reflect the future demands to set reasonable handling charges and improve terminal revenue. In fact, the operators of the terminal have insufficient data, and the fluctuating factors in the market environment, such as the pricing strategies of competitors, strategic behaviors of shipping companies, and market economic fluctuations, directly affect handling demand. In this case, the model of handling demand based on historical data cannot accurately reflect the shipping company's actual future demand.

This paper proposes a dynamic pricing mechanism for container terminal handling charges. This pricing mechanism is developed from the following two aspects: one is to describe the uncertainty demand of the handling volume; the other is to design a pricing policy under the uncertainty demand of the handling volume. Then, according to the physical properties and handling requirements of containers, we classify them into different types. This study's main objective is to explore the impact of different container handling charges on terminal revenue to maximize the terminal's profits, considering the terminal handling costs.

The rest of this paper is organized as follows. Section 2 is a literature review. Then, the framework of the dynamic pricing model is described in Section 3. Section 4 introduces the approach to solving the dynamic pricing problem of container terminal handling charge. Section 5 provides a case study about implementing the dynamic pricing model of the container terminal handling charge at the Shanghai Terminal. Finally, conclusions and future work are discussed in Section 6.

2. Literature Review

In port pricing studies, few studies address the pricing policy of container terminal handling charges, and mainly from qualitative research. Khalld^[4] proposed different pricing targets from the perspective of different pricing entities. From the perspective of port managers, the goal of pricing is to maximize the terminal's profits. Jia^[5] studied the price management mechanism of bulk cargo handling operation in Tianjin Port, and concluded that the existing pricing methods for port handling operation mainly include marginal cost pricing, cost-plus pricing, and game theory pricing. Wang^[6] used the analytic hierarchy process to analyze the influencing factors of container handling pricing in Port L, and the results showed that handling cost was the first important factor influencing pricing, and competition between terminals was the second key factor. Ding and Chen^[2] used price discrimination method to study the problem of container terminal handling pricing, established a quantity discount pricing model based on the assumption that the handling demand of shipping companies obeys uniform distribution, and solved the problem through particle swarm optimization. Meersman et al.^[3] discussed the principle, structure and model of port pricing and put forward the viewpoint of applying revenue management theory to port pricing. Yu and Ding^[7] introduced revenue management theory, established dynamic pricing model based on changing handling time of shipping company, and solved it by Q-Learning algorithm.

Dynamic pricing has received much research attention. Arnoud^[8] proposed the literature on dynamic pricing can roughly be classified as follows: one is models where the demand function is dynamically changing over time; the other is models where the demand function is static, but where pricing dynamics are caused by the inventory level. Further, according to whether the form of demand function is known, it can be divided into model-determined dynamic pricing which the functional form of random model is known and determined, and the dynamic pricing with uncertain models which the functional form of random model is unknown or partially known^[9]. However, Kalyanam^[10] proposed that the model-determined dynamic pricing has the following shortcomings in practical applications: compared with reality, the demand model has a certain degree of error; and the demand model reflects the historical situation, while the decision makers care about the customer's future reactions. Therefore, most researchers pay more attention to dynamic pricing research with uncertain models. Bertsimas^[11] studied the dynamic pricing in which the number of customers arriving in the

sales period was uncertain. Assuming that the demand function was linear but the parameters were unknown, the least square method was used to estimate the unknown parameters in the demand function. Levina^[12] constructed the sales process as a bernoulli process, assumed that the function form was known but the parameters were unknown, and used Bayesian theory to update the unknown parameters. Boer^[13] studied the single product dynamic pricing problem, assumed that the function form was known but the parameters were unknown, estimated the parameters by using the maximum quasi-likelihood estimation method, and proposed the control variance pricing strategy. Later, Boer^[14] extended the single-product pricing problem to multiple product dynamic pricing and proposed an adaptive pricing strategy.

In conclusion, the research on dynamic pricing has been quite completely, but on terminal handling charge is not enough. Therefore, our contributions are as follows: we apply dynamic pricing to container terminal handling

charges, introduce a model of dynamic pricing in the changing market environment, using parametric demand model describe the market process, and the value of these unknown parameters can be estimated by maximum quasi-likelihood estimation (MQLE); then propose an adaptive pricing policy to solve this problem.

3. Model Description

This paper studies by specific shipping company. We consider the handling operations of container terminal require the cooperation of m types of equipment. Classifying containers into n types, in each time period $k \in K$, the manager of terminal decides on the handling charge of each type container $p_i(k)$. After selecting the price, the manager of terminal observes the handling demand for each type of container s_{ik} . We assume that all demand can be met, in other words, terminal capacity-outs do not occur. Table 1 presents all notations used in this article.

Table 1. Notations given in the proposed model

I	Set of all the containers, $i \in I = \{1, 2, \dots, i, \dots, n\}$
J	Set of all the equipment, $j \in J = \{1, 2, \dots, j, \dots, m\}$
K	Set of all the period, $k \in K = \{0, 1, 2, \dots, k, \dots, z\}$
Θ	Set of admissible prices, $\Theta = \{1\} \times \prod_{i=1}^n [p_{li}, p_{hi}]$; where p_{li}, p_{hi} denotes the lowest and highest price for container i
$p_i(k)$	The terminal handling price of container i in period k
$P(k)$	The terminal handling price vector, $P(k) = (p_0(k), p_1(k), \dots, p_n(k))$
P^T	Transpose vector of terminal handling price
ρ	The matrix of price decisions; $\rho(k) = (P(0)^T, P(1)^T, \dots, P(k)^T)$
λ_{\min}	The smallest eigenvalue of price matrix ρ
$tr(\rho)$	The trace of price matrix ρ
P_{opt}	The correct optimal price
P_{cepq}	The certainty equivalent price
$S_{ik}(P(k))$	The handling demand for container i in period k , given price vector $P(k)$.
S_{ik}	The realized handling volume for container i in period k
r	The total expected handling revenue the specific shipping company
c_j	The unit resource-time cost of equipment j
Q_j	The total resource-time cost of equipment j
α_j	The resource-time of equipment j operating container i
C	The total expected terminal handling cost of the specific shipping company
R	The total expected terminal handling profit
R'	The total realized terminal handling profit
β	The unknown parameters in the demand model

3.1 Estimation of Container Terminal Handling Demand

In this paper, we assume that there is a functional relationship between the demand for container handling and the handling price of container terminal. Considering that estimation of demand have certain deviations in the changing market environment, therefore, this article uses parametric demand model with unknown parameter β to describe the changing market process. At the same time, the handling volume of container terminals does not obey any specific distribution, and the assumptions about this in the previous studies are ideal and simple. Therefore, assuming that the variance of the handling demand for container i is a function of expectation. The parametric model is defined as follows:

$$E(S_i(P)) = h_i(P^T \beta_i); \quad P \in \rho \tag{1}$$

$$Var[S_i(P)] = \sigma_i^2 v_i(E[S_i(P)]); \quad P \in \rho \tag{2}$$

Here, the functions h_i express the relationship between the expectation of handling demand and handling price. The functions v_i express the relationship between the variance of handling demand and expectation. The form of functions h_i, v_i are known, and parameter β are unknown.

The unknown parameters β can be estimated with maximum quasi-likelihood estimation. This is a natural extension of ordinary maximum-likelihood estimation to settings where only the function relationship between expectation and variance are known [15]. Therefore, given the price vectors $\{P(k) : k = 1, \dots, k-1\}$ and realized handling volume $\{s_{ik} : k = 1, \dots, n; i = 1, \dots, k-1\}$, by solving equation (3) we can obtain the parameters β in period k , and denote as $\hat{\beta}_i(k)$.

$$l_{ik}(\beta_i) = \sum_{k=1}^{k-1} \frac{\partial h_i(P^T(k)\beta_k)}{\partial \beta \cdot v_i(h_i(P^T(k)\beta_i))} P(k)(s_{ik} - h_i(P^T(i)\beta_i)) = 0 \tag{3}$$

3.2 Accounting of the Terminal's Handling Profit

Considering that terminal's handling cost is an important factor influencing the pricing of terminal handling, the goal of this paper is to maximize the terminal's handling profits. In the k period, given price vector $P(k)$, the handling demand is $S(P(k))$, and the total expected terminal's handling revenue of the specific shipping company is defined as follows:

$$r(P) = \sum_i^n E[p_i \cdot S_i(P)] = \sum_i^n p_i \cdot h_i(P^T \beta_i^{(0)}) \tag{4}$$

The terminal's handling cost of the specific shipping company is calculated by TDABC method. TDABC is

a method that uses time as a driving factor to analyze costs [2]. Usually, operating each unit of container will generates corresponding operating resource-time of each equipment. However, due to the different types of container stacks in different blocks of the terminal and they required different operating standards, this causes the operating resource-time of each type of containers is also different. Therefore, it is applicable to the cost analysis of container terminal's handling operations. Based on TDABC method, the handling cost of the specific shipping company is the sum of the total resource-time of each equipment multiplied by the corresponding unit cost. Assuming that α_{ij} , the resource-time of equipment j operating container i is constant. So, the total expected handling cost of the specific ship is defined as :

$$C = \sum_j^m \sum_i^n S_i(P) \cdot \alpha_{ij} \cdot c_j \tag{5}$$

Therefore, in k period, given price vector $P(k)$, the handling demand is $S(P(k))$, and the total expected terminal's handling profits of the specific shipping company is defined as :

$$R(P) = \sum_i^n E[p_i \cdot S_i(P)] - \sum_j^m \sum_i^n S_i(P) \cdot \alpha_{ij} \cdot c_j \tag{6}$$

3.3 Describing the Pricing Policy

At each k period, the pricing policy ϕ is a method to make a pricing decision of terminal handling based on the historical handling price of terminal $\{P(k) : k = 1, \dots, z\}$ and the realized handling volume of shipping company $\{s_{ik} : k = 1, \dots, n; i = 1, \dots, z\}$. The performance of a pricing policy ϕ is measured by the Regret, which is the loss of expected profits caused by not using the correct optimal price P_{opt} [14]. For a pricing policy ϕ , the Regret after z time periods is defined as:

$$Regret(z, \phi) = E \left[\sum_{k=1}^z R(P_{opt}, \beta^{(0)}) - R(P(k), \beta^{(0)}) \right] \tag{7}$$

The objective of the terminal is to find a pricing policy ϕ that attains the highest expected handling profits over z time periods. In other words, it's equivalent to minimizing Regret.

4. Adaptive Dynamic Pricing

Generally, the most simple and intuitive pricing policy based on the estimation of handling demand is: assuming that the estimation of unknown parameter in the model of handling demand is correct, based on which the optimal price can be calculated. This pricing policy is usually called certainty equivalent pricing, which is intuitive

and easy to understand but has poor performance. Den Boer ^[14] proposed an adaptive pricing policy to improve the certainty equivalent pricing. The key idea is given the estimation value of current parameter to choose the optimal price, with the additional constraint that $\lambda_{\min}(k)$, the smallest eigenvalue of the handling pricing matrix (8), grows with a certain rate(9), here is defined as the constraint function of differentiated price decision. This pricing rule balances the estimation of the parameter and the optimization of instant revenue at each period.

$$\rho(k) = \sum_{k=0}^z P(k)P^T(k) \tag{8}$$

$$\lambda_{\min}(k) \geq L(k) \tag{9}$$

Since, there is no simple explicit expression relating two consecutive smallest eigenvalues $\lambda_{\min}(k)$ and $\lambda_{\min}(k+1)$, we introduce the trace of the inverse design matrix $tr(\rho-1)$, for $\forall \rho$ conforming to the following relationship:

$$tr(\rho-1)^{-1} \leq \lambda_{\min}(\rho) \leq ntr(\rho-1)^{-1} \tag{10}$$

Therefore, the relationship between the two consecutive of the handling pricing matrix can be expressed as follows:

$$tr(\rho(k+1)^{-1} - tr(\rho(k)^{-1})) = -\frac{\|\rho(k)^{-1} \rho(k+1)\|^2}{1 + \rho^T(k+1)\rho(k)^{-1}\rho(k+1)} \tag{11}$$

A detailed pricing process with a constraint function of differentiated price decision is explained as follows:

Step 1: Initialization

Choose function $L \in \tau$; then choose $k+1$ linearly independent initial handling price vectors $P(1), P(2), \dots, P(k+1) \in \Theta$.

Step 2: Estimation

At the beginning of k period, for each type of container $i (i=1, \dots, n)$, calculate the estimated value of unknown parameter $\hat{\beta}_i(k)$ in the model of handling demand, using the equation (3).

Step 3: Pricing

(1) If for some types of container $i, \hat{\beta}_i(k)$ not exist, or the constraint $tr(\rho(k)-1)^{-1} \geq L_1(k)$ does not hold, then set $P(k+1) = P(1), P(k+2) = P(2), \dots, P(k+t) = P(t)$, where t is the smallest integer such that .

(2) If for all types of container $i, \hat{\beta}_i(k)$ exists, and the constraint $tr(\rho(k)-1)^{-1} \geq L_1(k)$ holds, then set

$P_{ceqp} = P(\hat{\beta}(k))$, and consider the following cases:

(2a) If the constraint $tr((\rho(k) + P_{ceqp}P_{ceqp}^T)^{-1})^{-1} \geq L_1(k+1)$ holds, then choose $P(k+1) = P_{ceqp}$.

(2b) If the constraint $tr((\rho(k) + P_{ceqp}P_{ceqp}^T)^{-1})^{-1} \geq L_1(k+1)$ does not hold, then choose $P(k+1)$ that maximizes (12) provided there is a feasible solution.

$$\begin{aligned} & \max_{\rho \in \Theta} R(\rho, \hat{\beta}(k)) \\ & s.t \frac{\|\rho(k+t)^{-1} \rho\|^2}{1 + \rho^T \rho(k)^{-1} \rho} \geq \frac{L_1(k)}{L_1(k)^2} \end{aligned} \tag{12}$$

(2c) If the constraint $tr((\rho(k) + P_{ceqp}P_{ceqp}^T)^{-1})^{-1} \geq L_1(k+1)$ does not hold, and (12) has no feasible solution, then put $P(k+1) = P(1), P(k+2) = P(2), \dots, P(k+t) = P(t)$, where t is the smallest integer such that.

5. Case Study

In this section, taking a terminal handling company in Shanghai port as an example, the dynamic pricing model is analyzed based on the estimation of handling demand of a specific shipping company. The data of this research comes from the operational data of the handling terminal company in 2017. Moreover, the proposed adaptive pricing algorithm is implemented in Matlab R2017a.

5.1 Basic Parameter Setting

According to the process of handling operation of the container terminal, the equipment on the terminal is divided into three types of resource groups ($j=1,2,3$): Yard crane, Truck and Quay crane. According to the physical characteristics and circulation direction, containers are divided into six types ($i=1, \dots, 6$): empty containers for export, full containers for export, empty containers for import, full containers for import, dangerous containers for export and special refrigerators containers for export. According to the data of the terminal's cost in 2017, c_j , the cost of unit resource-time of equipment j , and α_{ij} , the resource-time of equipment j operating container i are calculated as shown in Table 2. The data of lowest and highest handling price for container i are shown in Table 3.

Table 2. The cost of unit resource-time of equipment j (min/TEU) and The resource-time of equipment j operating container i (yuan/h)

α_j/min	Export-empty	Export-full	Import-empty	Import-full	Dangerous	Special	c_j
Yard crane	5	8	4	8	9	1	522
Tuck	21	13	16	18	14	26	157
Quay crane	3	2	2	2	1	3	1704

Table 3. The lowest and highest handling price for various containers (yuan/TEU)

$p(k)$	Export-empty	Export-full	Import-empty	Import-full	Dangerous	Special
pl	390	480	390	420	550	500
ph	1000	1000	1000	1000	1000	1000

5.2 Pricing Strategy Analysis

In this section, taking a specific shipping company as the research object, the interval between the shipping company's two arrivals at the terminal is a decision-making period. The demand for each container i is normally distributed with expectation and variance given by:

$$E[S_i(P)] = \beta_{i0} + \beta_{i1}p_1 + \dots + \beta_{in}p_n \quad (i = 1, \dots, 6)$$

$$Var[S_i(P)] = \sigma_i^2 \quad (i = 1, \dots, 6)$$

The set τ of constraint function of differentiated pricing decision in adaptive pricing is $L = \alpha * \sqrt{k \log(k)}$ ($0 \leq \alpha \leq 1$). First, take $\alpha=0.5$, according to the procedure of adaptive pricing in Section 4, the estimated handling volume of each type of container about the shipping company in the next period is $S=(10,415,117,36,333)^T$; the optimal handling pricing decision is $P_{opt} = (650,750,639,772,800,896)^T$; the expected handling profit is 476,300 yuan, which is an increase of 18,600 yuan compared with the actual profit of 457,400 yuan. Then, one year is set as a planning period to predict the demand of handling volume and determine the price about the handling operations of the shipping company at the terminal. The operating results are shown as (a)-(d) in Figure 1. Figure(a) shows a sample path of $tr(p(k)^{-1})^{-1}$ divided by $\sqrt{k \log k}$, it can be seen that the ratio of the two converges to 0.05, indicating that $L = 0.05 * \sqrt{k \log(k)}$, the constraint function of differential price decision of adaptive dynamic pricing policy is reasonable. Figure (b) shows the squared norm of the difference between

the parameter estimates and the true parameter, from that the adaptive dynamic pricing policy has a better convergence effect and the parameter estimation can converge to the real value. Figure (c) shows Cumulative Regret and Figure (d) shows the sample path of Regret divided by $\sqrt{k \log k}$, illustrating that the adaptive dynamic pricing policy has a good optimization effect and can obtain the correct optimal price.

Further, changing the value of coefficient α in constraint function of differential price decision in the adaptive pricing policy, and take $\alpha \in \{0.1,0.3,0.5,0.7,0.9\}$. The coefficient α is larger, the difference of handling price between the shipping company arriving at the terminal for two adjacent periods is greater. Observe the change of the bound of cumulative Regret and the handling profit of specific shipping company. The calculation results are shown in Table 4. The results show that: for the upper bound of the cumulative Regret, it decreases first and then increases with the increase of coefficient α , when the coefficient α changes from 0.1 to 0.5, the upper bound of cumulative Regret drops to 1.3269 million yuan; changing from 0.5 to 0.9, the upper bound of cumulative Regret increased to 1.4835 million yuan. For handling profit of specific shipping company, it also increases first and then decreases with the increase of coefficient α , when the coefficient α changes from 0.1 to 0.5, the growth of handling profit of specific shipping company increases to 1.8268 million yuan; changing from 0.5 to 0.9, the growth of handling profit of specific shipping company decreased to 0.9219 million yuan.

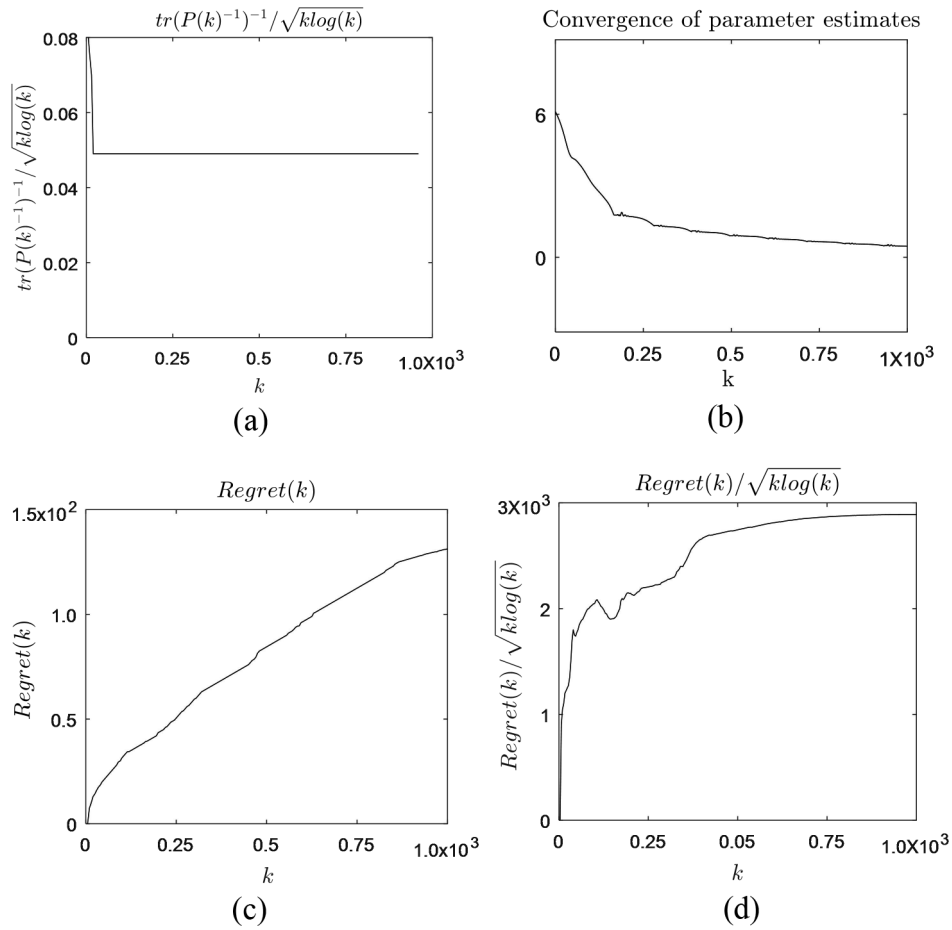


Figure 1. The solution results are shown in the diagram

Table 4. The upper bound of cumulative Regret and handling profit (wan yuan)

α	0.1	0.3	0.5	0.7	0.9
Upper bound	149.74	144.87	132.69	139.11	148.35
R	6897.56	7098.99	7200.93	7162.22	7090.44

6. Conclusion

This paper proposes a dynamic pricing model for terminal handling charges based on handling demand estimation, using the maximum quasi-likelihood estimates of the unknown parameters in the demand model and adaptive pricing algorithm to solve the pricing process. The computational results suggested that: (1) The adaptive pricing policy can converge the terminal handling charges to the correct price, balance the accuracy of the estimation of handling demand and the optimality of pricing, and increase the terminal handling profits compared with the contractual pricing mechanism. (2) The range of

difference of handling price in adjacent periods will affect the variation of terminal handling profit and the upper bound of accumulated Regret. For example, the difference in handling price between the two adjacent periods changes slightly, and the handling profit decreases on the contrary. However, the difference in handling price is more tremendous; terminal profit growth is not necessarily more significant. Therefore, for the terminal, the constraint of the price difference should be determined according to the shipping company’s actual situation. In conclusion, the adaptive dynamic pricing policy with constraints of differentiated price decision is conducive to improving the terminal’s revenue and has specific reference value for pricing of terminal handling.

References

[1] Yi Ding, Kaimin Chen, Dongmin Xu. Dynamic pricing research for container terminal handling charge[J]. Maritime Policy & Management, 2020.
 [2] Yi Ding, Kaimin Chen, Guolong Lin. Cost apportioning and discount pricing for container terminal

- handling operation[J]. *Journal of Shanghai Maritime University*, 2017, 38(02): 22-28. (in Chinese)
- [3] Meersmn H, Strandenes S P, Eddy V D V. Port pricing: principles, structure and models[J]. Social Science Electronic Publishing, 2014.
- [4] Khalld Bichou. Port Operations, Planning and Logistics[M]. Information Law from Routledge, 2013, 121-134.
- [5] Aijun Jia. Study on the Price management mechanism of Bulk Cargo in Tianjin Port[D]. Jilin University, 2012:5-10. (in Chinese)
- [6] Zhaogang Wang. Study on container handling pricing and management in L port[D]. Yangzhou University, 2017: 3-7. (in Chinese)
- [7] Yu Jue, Ding Yi, Lin Guolong. The Strategy of Dynamic Pricing of Handling Operation for Port Container based on Q-Learning[J]. *Computer Applications and Software*, 2018, 35(12):123-130+221. (in Chinese)
- [8] Arnoud V. den Boer. Dynamic pricing and learning: Historical origins, current research, and new directions[J]. *Surveys in Operations Research and Management Science*, 2015, 20(1).
- [9] Lieberman W H. Pricing and Revenue Optimization: Maximizing Staff Effectiveness[M]. Revenue Management, 2011.
- [10] Kalyanam K. Pricing Decisions Under Demand Uncertainty: A Bayesian Mixture Model Approach[M]. 1996.
- [11] Bertsimas D, Perakis G. Dynamic Pricing: A Learning Approach[J]. 2001.
- [12] Levina, T, Levin, Y, McGill, J. Dynamic Pricing with Online Learning and Strategic Consumers: An Application of the Aggregating Algorithm[J]. 2010, 57(2): 327-341.
- [13] Boer A V D, Zwart B. Simultaneously Learning and Optimizing Using Controlled Variance Pricing[M]. *Inform*, 2014.
- [14] Den Boer, Arnoud V. Dynamic Pricing with Multiple Products and Partially Specified Demand Distribution[J]. *Mathematics of Operations Research*, 2014, 39(3).
- [15] Tian Xia, Xueren Wang. Strong consistency of maximum Quasi-Likelihood estimators in nonlinear quasi-likelihood models with adaptive design[J]. *Mathematics in Practice and Theory*, 2015, 45(21): 252-258. (in Chinese)