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Quasi Maximum Likelihood for MESS Varying Coefficient Panel Data Models with Fixed Effects

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ABSTRACT

The study of spatial econometrics has developed rapidly and has found wide applications in many different scientific fields, such as demography, epidemiology, regional economics, and psychology. With the deepening of research, some scholars find that there are some model specifications in spatial econometrics, such as spatial autoregressive (SAR) model and matrix exponential spatial specification (MESS), which cannot be nested within each other. Compared with the common SAR models, the MESS models have computational advantages because it eliminates the need for logarithmic determinant calculation in maximum likelihood estimation and Bayesian estimation. Meanwhile, MESS models have theoretical advantages. However, the theoretical research and application of MESS models have not been promoted vigorously. Therefore, the study of MESS model theory has practical significance. This paper studies the quasi maximum likelihood estimation for matrix exponential spatial specification (MESS) varying coefficient panel data models with fixed effects. It is shown that the estimators of model parameters and function coefficients satisfy the consistency and asymptotic normality to make a further supplement for the theoretical study of MESS model.

1. Introduction

The matrix exponential space specification (MESS), originally proposed by Lesage and Pace [1], has been used as an alternative to the spatial autoregressive (SAR) specification due to its significant advantages. There are few studies on MESS models. At present, Debarsy et al. [2] is the most representative one to study MESS cross-section data model. Relevant studies extending MESS to panel data include Figueiredo and Silva [3] and Zhang et al. [4].

Recently, the combination of spatial models and non-parametric or semi-parametric models have gradually become a research trend. Su and Jin [5] propose the profile quasi maximum likelihood (QML) estimation of partially linear spatial autoregressive models. On the basis of [5], Su [6] proposes a semi-parametric spatial autoregressive model with heteroscedasticity and spatial correlation of error terms, and obtains the semi-parametric GMM estimation through the two-step estimation method. For more literature, see Zhang [7], Sun [8], Koroglu and Sun [9], etc.

Variable coefficient model, as a special semi-parametric model, has been proposed by Hastie and Tibshirani [10]. Fan and Zhang [11] further propose a more general variable

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coefficient model. For more literature, please refer to Cai et al. [12] and Xia et al. [13]. By introducing variable coefficients to describe the nonlinear effects of the explanatory variables on the explained variables, not only can the problem of dimension curse in non-parameters be overcome, but also can avoid the risk of model error. Therefore, it is inevitable to introduce the variable coefficient model into the spatial econometric model. In terms of the cross-section data model, Sun [8] studied the variable coefficient spatial autoregressor model with non-parametric spatial weight matrix. With the increase of the availability of panel data, studies on variable coefficient panel data model include Cai and Li [14], Chen et al. [15], etc.

MESS can also be extended to the variable coefficient panel data model. Based on the existing studies, this paper studies the quasi-maximum likelihood estimation of the variable coefficient panel data model with fixed effects, and derives the large-sample properties of the resulting estimator. The rest of the paper is arranged as follows: Section 2 introduces the model and estimation method. Section 3 presents some assumptions and main results. Section 4 gives some related lemmas and proofs.

2. Model and Estimation

We consider the following MESS varying coefficient panel data model with fixed effects:

$$e^{\alpha W_n} y_{it} = \mathrm{x}_{it}^{\tau} \beta(u_{it}) + \gamma_i + v_{it}, i = 1, \cdots, N, t = 1, \cdots, T, \quad (1)$$

where y_{it} is the dependent variable, $\mathrm{x}_{it} = (x_{it1}, \cdots, x_{itp})^{\tau}$ is a p-dimensional independent variable, α is a scalar spatial dependence parameter, W_n is a known $N \times N$ spatial weight matrix with diagonal elements are zero, $\beta(u_{it}) = (\beta_1(u_{it}), \cdots, \beta_p(u_{it}))^{\tau}$ is a p-dimensional vector of unknown functions, and γ_i is the unobserved individual fixed effects. For convenience, we assume that u is a one-dimensional random variable, then $\beta_i(u)$ is an unknown function of one variable. We also assume that model holds with the restriction $\sum_{i=1}^{N} \gamma_i = 0$, and v_{it}'s are independent identically distributed (i.i.d.) of sequence of random errors with mean zero and variance σ^2.

Denoting

$$Y = (y_{11}, \cdots, y_{1T}, \cdots, y_{N1}, \cdots, y_{NT})^{\tau},$$

$$X = (\mathrm{x}_{11}^{\tau}, \cdots, \mathrm{x}_{1T}^{\tau}, \cdots, \mathrm{x}_{N1}^{\tau}, \cdots, \mathrm{x}_{NT}^{\tau})^{\tau},$$

$$v = (v_{11}, \cdots, v_{1T}, \cdots, v_{N1}, \cdots, v_{NT}),$$

$$\beta(u) = (\beta(u_{11}), \cdots, \beta(u_{1T}), \cdots, \beta(u_{N1}), \cdots, \beta(u_{NT}))^{\tau},$$

then model (1) can be rewritten in a matrix format yields

$$S(\alpha)Y = X\beta(u) + U_0\gamma_0 + v, \quad (1)$$

where $S(\alpha) = (I_T \otimes e^{\gamma W_n})$, $\gamma_0 = (\gamma_1, \cdots, \gamma_N)^{\tau}$, $U_0 = I_N \otimes i_T$. I_N denotes the $N \times N$ identity matrix, i_T denotes the $T \times 1$ column vector of ones, and $\otimes$ is the Kronecker product.

Let $\theta = (\alpha, \sigma^2)^{\tau}$, Let $\gamma = (\gamma_2, \cdots, \gamma_N)^{\tau}$, then model (2) can be converted to

$$S(\alpha)Y = X\beta(u) + U\gamma + v, \quad (3)$$

where $U = (-i_{N-1}, I_{N-1})^{\tau} \otimes i_T$ is $NT \times (N-1)$ matrix.

According to Chiu et al. [16], for any square matrix A, $|e^{\alpha A}| = e^{\mathrm{tr}(\alpha A)} = e^{\alpha \mathrm{tr}(A)}$. At this moment, suppose that $v_{it} \sim i.i.d(0, \sigma^2)$, then $E(vv^{\tau}) = \sigma^2 I_{NT}$, and as W_n has zero diagonal, the log-likelihood function of the model (3) based on the response vector Y is as follows:

$$\ln L(\theta, \gamma) = -\frac{NT}{2}\ln(2\pi) - \frac{NT}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}[S(\alpha)Y - X\beta(u) - U\gamma]^{\tau}[S(\alpha)Y - X\beta(u) - U\gamma]. \quad (4)$$

Take the partial derivative of (4) with respect to γ, and make it be 0, then the maximum likelihood estimate of γ is

$$\hat{\gamma}_L = (U^{\tau}U)^{-1}U^{\tau}[S(\alpha)Y - X\beta(u)].$$

Further, by substituting $\hat{\gamma}_L$ into (4), we obtain the log-likelihood function of θ

$$\ln L(\theta) = -\frac{NT}{2}\ln(2\pi) - \frac{NT}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}[S(\alpha)Y - X\beta(u)]^{\tau}H[S(\alpha)Y - X\beta(u)], \quad (5)$$

where $H = I_{NT} - U(U^{\tau}U)^{-1}U^{\tau}$. Take the partial derivative of (5) with respect to σ^2, and make it be 0, then the maximum likelihood estimate of σ^2 is

$$\hat{\sigma}_L^2 = (NT)^{-1}[S(\alpha)Y - X\beta(u)]^{\tau}H[S(\alpha)Y - X\beta(u)].$$

Substituting $\hat{\sigma}_L^2$ into (5), one can show that the following concentrated log-likelihood function respect to γ:

$$\ln L(\alpha) = -\frac{NT}{2}[\ln(2\pi) + 1] - \frac{NT}{2}\ln(\hat{\sigma}_L^2). \quad (6)$$

Due to the varying coefficient functions $\beta(u_{it})$ is unknown, the estimate of α cannot be obtained directly by maximizing (6). Hence, we need to apply a local linear regression technique to estimate the varying coefficient functions $\{\beta_j(u_{it}), j = 1, \cdots, p\}$.

Firstly, for given θ, replace γ with $\hat{\gamma}_L$, then the weighted least-squares estimation method is used to obtain the feasible initial estimation $\hat{\beta}_{IN}(u)$ of $\beta(u)$. For u in a small neighborhood of u_0, one can approximate $\beta_j(u)$ locally by a linear function

$$\beta_j(u) \approx \beta_j(u_0) + \beta_j'(u_0)(u - u_0) \equiv a_{1j} + a_{2j}(u - u_0), \ j = 1, \cdots, p,$$

where $\beta_j'(u)$ is the derivative of $\beta_j(u)$ at u. This leads to the following weighted local least-squares problem: find $\{(a_{1j}, a_{2j}), j = 1, \cdots, p\}$ to minimize

$$\sum_{i=1}^{N}\sum_{t=1}^{T}[e^{\alpha W_n}y_{it} - \hat{\gamma}_{Li} - \sum_{j=1}^{p}(a_{1j} + a_{2j}(u_{it} - u_0))x_{itj}]^2 K_h(u_{it} - u_0), \quad (7)$$

Where K is a kernel function, h is a bandwidth and $K_h(u_{it} - u_0) = h^{-1}K((u_{it} - u_0)/h)$..

Denote $NT \times NT$ diagonal matrix $W_u = diag(K_h(u_{11}$

$-u),\cdots,K_h(u_{NT}-u))$, $\delta=(\beta_1(u),\cdots,\beta_p(u),h\beta_1'(u),\cdots,h\beta_p'(u))^{\tau}$, and

$$D_u=\begin{pmatrix} x_{11}^{\tau} & \frac{u_{11}-u}{h}x_{11}^{\tau} \\ \vdots & \vdots \\ x_{1T}^{\tau} & \frac{u_{1T}-u}{h}x_{1T}^{\tau} \\ \vdots & \vdots \\ x_{NT}^{\tau} & \frac{u_{NT}-u}{h}x_{NT}^{\tau} \end{pmatrix},$$

then the matrix form of (7) is

$$\underset{\delta}{\operatorname{argmin}}[S(\alpha)Y-D_u\delta]^{\tau}HW_uH[S(\alpha)Y-D_u\delta].$$

The solution to (7) is given by

$$\hat{\delta}=[D_u^{\tau}HW_uHD_u]^{-1}D_u^{\tau}HW_uHS(\alpha)Y.$$

Then the feasible initial estimation of $\beta(u)$ is

$$\hat{\beta}_{IN}(u)=(\beta_1(u),\cdots,\beta_p(u))^{\tau}=(I_p,0_p)^{\tau}\hat{\delta}=(I_p,0_p)^{\tau}A_u^{-1}Q_u, \quad (8)$$

where $A_u=(NT)^{-1}D_u^{\tau}HW_uHD_u$, $Q_u=(NT)^{-1}D_u^{\tau}HW_uHS(\alpha)Y$. Thus, the estimator for $X\beta(u)$ is

$$X\hat{\beta}_{IN}(u)=\begin{pmatrix} (x_{11}^{\tau}\quad 0)(D_{u_{11}}^{\tau}HW_{u_{11}}HD_{u_{11}})^{-1}D_{u_{11}}^{\tau}HW_{u_{11}}H \\ \vdots \\ (x_{NT}^{\tau}\quad 0)(D_{u_{NT}}^{\tau}HW_{u_{NT}}HD_{u_{NT}})^{-1}D_{u_{NT}}^{\tau}HW_{u_{NT}}H \end{pmatrix}S(\alpha)Y=MS(\alpha)Y,$$

where the matrix M is a smoothing matrix and depends only on the observations $\{(u_{it},x_{it}^{\tau}),i=1,\cdots,N,t=1,\cdots,T\}$.

Secondly, taking $X\hat{\beta}_{IN}(u)$ in to (5), we get the approximation of $\ln(\theta)$:

$$\ln\tilde{L}(\theta)=-\frac{NT}{2}\ln(2\pi)-\frac{NT}{2}\ln(\sigma^2)-\frac{1}{2\sigma^2}[S(\alpha)Y-X\hat{\beta}_{IN}(u)]^{\tau}H[S(\alpha)Y-\hat{\beta}_{IN}(u)]. \quad (9)$$

Maximizing $\ln\tilde{L}(\theta)$ with respect to θ, we have the estimate of θ, i.e.,

$$\hat{\theta}=\underset{\theta}{\operatorname{argmax}}\ln\tilde{L}(\theta).$$

In solving maximum (9) with respect to σ^2, we obtain the following initial estimate:

$$\hat{\sigma}_{IN}^2=(NT)^{-1}[S(\alpha)Y-X\hat{\beta}_{IN}(u)]^{\tau}H[S(\alpha)Y-X\hat{\beta}_{IN}(u)]. \quad (10)$$

Replace σ^2 in the (9) with $\hat{\sigma}_{IN}^2$, then the concentrated log-likelihood function respect to α is:

$$\ln\tilde{L}(\alpha)=-\frac{NT}{2}[\ln(2\pi)+1]-\frac{NT}{2}\ln(\hat{\sigma}_{IN}^2). \quad (11)$$

Further, α is estimated to be

$$\hat{\alpha}=\underset{\alpha}{\operatorname{argmax}}\ln\tilde{L}(\alpha)$$

The above nonlinear optimization problems can be solved by iterative method.

Next, according to (8) and (9) and $\hat{\alpha}$, we obtain the final estimates of β_u and σ^2, respectively:

$$\hat{\beta}(u)=(I_p,0_p)^{\tau}[D_u^{\tau}HW_uHD_u]^{-1}D_u^{\tau}HW_uHS(\hat{\alpha})Y,$$

$$\hat{\sigma}^2=(NT)^{-1}[S(\hat{\alpha})Y-X\hat{\beta}(u)]^{\tau}H[S(\hat{\alpha})Y-X\hat{\beta}(u)].$$

Finally, γ is estimated to be

$$\hat{\gamma}=(U^{\tau}U)^{-1}U^{\tau}[S(\hat{\alpha})Y-X\hat{\beta}(u)].$$

3. Assumptions and Main Results

Denote $\rho_j=\int u^jK(u)du,\varphi_j=\int u^jK^2(u)du$. Let $\theta_0=(\alpha_0,\sigma_0^2)^{\tau}$ be the true valve of the parameter θ. To obtain the consistency and asymptotic normality of the resulted estimators of θ and function coefficient $\beta(u)$, we need following assumptions:

A1 $\{x_{it},y_{it},u_{it},v_{it},i=1,\cdots,N;t=1,\cdots,T\}$ are i.i.d. random sequences. They satisfy the following conditions: (i) The random variable u has a bound support $\mathcal{U}$, and the edge density function $f_t(u)$ is continuous differentiable and bound away from 0 on its support. (ii) The matrix $\Omega(u)=E(x_{it_1}x_{it_2}^{\tau}|u_{it_1}=u,u_{it_2}=u)$ is non-singular and every element is second-order continuous differentiable. The matrix $E(x_{it}x_{it}^{\tau})$ is a non-singular constant that satisfies $E(x_{it}v_{it}|u_{it})=0$ and $E(x_{it}b_i|u_{it})=0$. (iii) There exists $r=\max 4,s$ such that $E(\|x_{it}\|^r<\infty)$, $E(\|b_i\|^r<\infty),E(\|v_{it}\|^r<\infty)$ and for some $\epsilon<2-s^{-1}$such that $(NT)^{2\epsilon-1}h<\infty$.

A2 $\{\beta_i(u),i=1,\cdots,p\}$ have continuous second derivative in $u\in\Omega$ and for any u, $|\beta_i(u)|\le c$ is a positive constant.

A3 The kernel $K(\cdot)$ is a symmetric density function with a continuous derivative on its compact support [-1,1].

A4 (i) The row and column sums of W_n and S^{-1} are uniformly bounded in absolute value. (ii) $S^{-1}(\alpha)$ are uniformly in α in a compact convex parameter space Θ. The true α_0 is an interior point in Θ.

A5 $NTh\to 0$ as $N\to\infty,T\to\infty$ and $h\to 0$.

A6 There is a unique θ that makes model (2) true.

A7 Either (i) $\lim_{NT\to\infty}(NT\sigma_0^2)^{-1}[X\beta(u)+U\gamma]^{\tau}\Lambda_0^{\tau}H\Lambda_0[X\beta(u)+U\gamma]$ exists and is non-singular, where $\Lambda_0=W_nS^{-1}(\alpha_0)$; or (ii) $\lim_{NT\to\infty}\{(NT)^{-1}\mathrm{tr}(\Lambda_0^2)+(NT)^{-1}\mathrm{tr}(\Lambda_0^{\tau}H\Lambda_0)-2(NT)^{-2}\mathrm{tr}^2(\Lambda_0^{\tau}H)\}>0.$.

Remark 1. A1 to A3 and are the basic assumptions for nonparametric models, which can be used in Fan and Huang [17]. A4 concerns the essential features of spatial weights matrix, which can be found in the Assumptions 4, 5 and 7 in Lee [18]. A5 is easily satisfied. A6 is the unique identification condition of the parameter. A7 guarantees the asymptotic normality of parameter estimation (see e.g., Chen et al.[15]).

Denote $\Theta_0=\{\alpha|\alpha\in\Theta,\|\alpha-\alpha_0\|\le\eta_1(NT)^{-1/2}\}$, where η_1 is a given positive constant. We now sate the main results.

Theorem 1. *Suppose that Assumptions A1-A6 are satisfied.* $\forall u\in\mathcal{U},\forall\alpha\in\Theta_0$, *then (i)* $\hat{\theta}-\theta_0=o_p(1)$, *and (ii)* $\hat{\beta}(u)-\beta(u)=o_p(1)$.

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Theorem 2. *Suppose that Assumptions A1-A7(i) or A1-A6 and A7(ii) are satisfied. Then* $\sqrt{NT}(\hat{\theta}-\theta_0) \xrightarrow{d} N(0,\Sigma^{-1})$,

where $\Sigma = -\lim_{NT\to\infty} E\left[\frac{1}{NT}\frac{\partial^2 \ln L(\theta)}{\partial\theta\partial\theta^{\tau}}|_{\theta=\theta_0}\right]$.

Theorem 3. *Suppose Assumptions A1-A6 hold, for any* $u\in\mathcal{U}, \alpha\in\Theta_0$,

$$\sqrt{NTh}(\widehat{\beta}(u)-\beta(u)-\phi(u)) \xrightarrow{d} N\left(0,\sigma_{\beta}^2(u)\right),$$

where $\phi(u)=h^2\rho_2\beta''(u)/2$, $\sigma_{\beta}^2(u)=\varphi_0\sigma_0^2[F(u)\mathbf{\Omega}]^{-1}$, $\beta''(u)$ *is the second derivative of* $\beta(u)$, $F(u)=\lim$ $F(u)=\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T} f_t(u)$ *and* $\mathbf{\Omega}=E\left(\mathrm{x}_{i_1t_1}\mathrm{x}_{i_2t_2}^{\tau}\right)$.

Further, as $NTh^5\to 0$, *then*

$\sqrt{NTh}(\widehat{\beta}(u)-\beta(u)) \xrightarrow{d} N\left(0,\sigma_{\beta}^2(u)\right)$.

4. Lemmas and Proofs

Lemma 1. *Suppose that Assumptions A1-A5 hold true, then*

$A_u=F_u\begin{pmatrix}\mathbf{\Omega} & 0\\ 0 & \mu_2\mathbf{\Omega}\end{pmatrix}\{1+O_p(c)\}, Q_u=F_u\begin{pmatrix}\mathbf{\Omega}\beta(u)\\ 0\end{pmatrix}\{1+O_p(c)\}$,

where $F(u)=\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T} f_t(u), \mathbf{\Omega}=E\left(\mathrm{x}_{i_1t_1}\mathrm{x}_{i_2t_2}^{\tau}\right), i_1, i_2 = 1,\cdots,N, t_1,t_2=1,\cdots,T$.

Proof. See Lemma 1 in Chen et al. (2019).

Lemma 2. *Suppose that Assumptions A1 - A5 hold true, then* $\widehat{\beta}_{IN}(u)-\beta_u=o_p(1)$.

Proof. Note that

$$\widehat{\beta}_{IN}(u)=\left(I_p,0_p\right)^{\tau}A_u^{-1}Q_u,$$

and from Lemma 1, one can show that $\widehat{\beta}_{IN}(u)\xrightarrow{p}\beta_u$. Then the Lemma 2 is proved.

Proof of Theorem 1 to 3. Using Lemma 1, Lemma 2 and following the proof of Theorem 1 to 5 in Chen et al. [15], one can easily show that Theorem 1 to 3 hold true.

5. Conclusions

This paper combines the advantages of MESS models and the characteristics of fixed effects and variable coefficient models to change the spatial correlation from geometric decay to exponential decay, which makes the theoretical modeling simpler. We study matrix exponential spatial specification varying coefficient panel data models with fixed effects. Firstly, the dummy variable method is used to deal with the fixed effect, then the weighted least square estimation method is used to estimate the variable coefficient function, and the quasi-maximum likelihood estimation method is used to estimate the model parameters. Finally, the consistency and asymptotic normality of model parameter estimation and function coefficient estimation are deduced.

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