

REVIEW

The Coupled Operational Systems: A Linear Optimisation Review

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ABSTRACT

The purpose of this review is to summarise the existing literature on the operational systems as to explain the current state of understanding on the coupled operational systems. The review only considers the linear optimisation of the operational systems. Traditionally, the operational systems are classified as decoupled, tightly coupled, and loosely coupled. Lately, the coupled operational systems were classified as systems of time-sensitive and time-insensitive operational cycle, systems employing one mix and different mixes of factors of production, and systems of single-linear, single-linear-fractional, and multi-linear objective. These new classifications extend the knowledge about the linear optimisation of the coupled operational systems and reveal new objective-improving models and new state-of-the-art methodologies never discussed before. Business areas affected by these extensions include product assembly lines, cooperative farming, gas/oil reservoir development, maintenance service throughout multiple facilities, construction via different locations, flights traffic control in aviation, game reserves, and tramp shipping in maritime cargo transport.

1. Introduction

Business organisations have three basic management functions: operations, marketing, and finance. The operations function, which is the focus in this paper, is responsible for producing the goods and providing the services offered by the organisation. There are examples of these goods and services all surrounding us. Cars, TV sets, and refrigerators are examples of the goods, while medical care, internet access, and transportation are examples of the services. The creation of goods or services involves transforming or converting inputs to outputs. Inputs such as capital, labor, and information are used to create goods or services using one or more transformation processes and systems. If more than one

process or system is used, inputs may only be assigned to each system separately (decoupled processes or systems) or also be assigned to all systems collectively (coupled processes or systems). In the latter case, inputs cut across all systems. For example, in the BMW group, the V12 car engine has been input to two car operational systems of BMW and Rolls Royce brand names. The car engine, in this case, is said to be a ‘coupling resource’, and the car operational systems are said to be the ‘coupled systems.’ Other examples of the coupling resources include a main electrical power supply, the labor skilled to work for all systems, and the spare parts to be used for the production of all systems. Other examples of the coupled operational systems include the customised product assembly lines, the cooperative farming, the oil and gas reservoir devel-

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opment, the rendering of maintenance service throughout maintenance facilities, working via the different locations in construction, the flights traffic control in aviation, and the cargo transport by a fleet of cargo tramp ships in the maritime cargo transport. The optimisation of the operations function and its transformation processes and systems is required to ensure that the outputs are obtained within the optimal quantity and quality.

The introduction mentioned above defines the scope of the coupled operational systems and their optimisation this paper is tackling. The design optimisation of the coupled systems known as Multidisciplinary Design Optimisation (MDO) is considered out of scope in this paper. MDO allows design engineers to incorporate all the relevant engineering disciplines simultaneously. The optimisation of a simultaneous problem is superior to the design found by optimising each discipline sequentially since it can exploit the interactions between the disciplines. The problem is normally solved using appropriate techniques from the field of optimisation. These include gradient-based algorithms, population-based algorithms, or others. Very simple problems can sometimes be expressed linearly; in that case, the techniques of linear programming are applicable.

In the past, the coupled operational systems have been categorised as hierarchical and nonhierarchical according to the type of systems to which they apply. In hierarchical systems, child-systems are coupled only through their parent-system and not to each other. Nonhierarchical systems are more general since no restrictions are placed on how systems are coupled. Models of hierarchical systems are now being formulated as to apply to nonhierarchical models.

The coupled operational systems are also categorised as tightly and loosely coupled according to the number of coupling constraints cutting across all systems. Perhaps the simplest tightly-coupled model is the case of two stations working in series. Jobs flow through the system, first to station 1, then to station 2. The example for the loosely coupled model is the case of two stations working in parallel. Jobs are assigned to the two stations independently. The two stations employ different kinds of resources and a small number of common resources.

Recently, the coupled systems have been classified according to several attributes associated with the production and operation management (POM). From the operational cycle-time point of view, the coupled operational systems may be classified as time-sensitive or time-insensitive. In the time-sensitive coupled operational systems, the time varies considerably from one operational cycle to another. As in any customized product assembly lines, the time varies considerably from one operational cycle to another.

In time-insensitive coupled operational systems, the time is almost fixed from one operational cycle to another. Single non-customised product assembly lines are just one example for the latter systems.

From the employed factors-of-production point of view, the coupled operational systems may be classified as to whether they can employ one mix or different mixes of factors of production. In the BMW group, for example, most BMW brand cars are produced using the robot-intensive mix of factors of production, while the Rolls-Royce brand cars are produced using the labor-intensive mix. On the other hand, soft-drinks production lines are employing an automated mix of factors of production.

From the objective-function point of view, which can be mathematically formulated as linear, the coupled operational systems may be classified as systems of linear objective, systems of linear fractional objective, or systems of multi-linear-objective. The coupled systems of the time-sensitive operational cycle and the ones who can employ different mixes of factors of production are examples of the systems of a linear fractional objective. The game reserve is one example of the systems of multi-linear-objective.

The focus in this review is mainly on the linear optimisation of the coupled operational systems.

2. The Tightly-coupled Systems

Tight coupling is epitomised in the literature by the notion of just-in-time material flow. However, Baker^[1] has introduced a generic model of the tightly coupled systems as representing an automated production line that handles a diverse set of related products. In other words, the diversity within a general product family leads to small production lots and process-time variability in the requirements at each workstation. Alternatively, the model can be viewed as representing a traditional manual line, with variability induced by the human performance of repetitive tasks. His paper is organized into two main sections, one dealing with serial lines and the other with assembly systems, in which one may look at several common themes. The simplest models have two stages: in serial lines, which means an initial station and a final station; in assembly systems, which means a component stage (with parallel stations) and an assembly stage.

In the coupled two stations organized in series, Baker has discussed the case where the jobs are assigned in a synchronised way where the job at station 2 leaves the system, the job at station 1 moves to station 2, and a new job enters the system at station 1, all at the same time. To explain, let x_1 and x_2 denote the operation times at stations 1 and 2, respectively. In a synchronised system, jobs wait

until all operations are complete, and then all work proceeds simultaneously. In the two-station case, the job at station 2 leaves the system, the job at station 1 moves to station 2, and a new job enters the system at station 1, all at the same time. Let c denote the cycle time, which can be defined as the time between successive departures from the system. For the synchronised system, the cycle time is:

$$c = \max(x_1, x_2) \quad (1)$$

The throughput T is defined by:

$$T = [c]^{-1} = [\max(x_1, x_2)]^{-1} \quad (2)$$

In the assembly system, Baker discussed the case in which parts are produced in separate feeder lines and brought together at a final assembly station. The simplest assembly system is the three-station case, where the feeder lines each consist of one station and there are two feeder lines. In general, when the feeder lines may be longer than one station, it is assumed that the first station in each feeder line is never starved. A fundamental hypothesis about assembly systems is that their throughput performance can be evaluated by "unwrapping" them. In the three-station case, the model is unwrapped by treating the assembly station as the middle station in a serial line and the two feeder stations as the end stations of the serial line.

In assembly systems, there is an important distinction between two mechanisms for loading the assembly operation. Consider the three-station system and suppose that the feeder stations are busy and that the assembly station has just finished an operation. What happens when one of the feeder stations completes work on a component? Under simultaneous loading, the movement to assembly is synchronised. This means that the assembly station remains idle until both components are ready. The assembly station thus functions as if it is looking for two ready signals, one at each feeder station, in order to initiate the movement of components into the assembly process. By contrast, under independent loading, the movement to assembly is asynchronous. When the first component is ready, it moves to the assembly station and waits in an assembly workspace until its mate is ready. This means that the assembly station remains idle until both components are ready, but while it is waiting for the second component, there is no blocking of the station that completed the first component. One might think of the independent loading system as having a half-buffer at the assembly station.

Bowman^[2] has offered two forms of linear programming solutions to the assembly-line balancing problem.

Feasible solutions depend on work already presented on integer solutions to linear programming problems. As yet, the computation involved for a practical problem is quite large.

For a directly-coupled two-stage production system whose stochastic processing times may follow any distribution form, Lau^[3] presented a simple procedure for estimating its average cycle time. He also considered a new version of the time-unbalancing between the workstations that is more realistic than the classical version.

Balogun et al.^[4] have developed a linear programming model to derive the maximum profit from the production of soft drink for the Nigeria Bottling Company, Ilorin plant, Nigeria. The linear Programming model of the operations of the company was formulated, and the optimum results were derived using a software that employs the Simplex method.

Akpınar and Baykasoğlu^[5] have developed a mixed-integer linear mathematical programming (MILP) model for the mixed-model assembly line balancing problem with setups. The proposed MILP model considers some particular features of the real world problems such as parallel workstations, zoning constraints, and sequence-dependent setup times between tasks, which is an actual framework in assembly line balancing problems.

Barathwaj et al.^[6] have presented a mixed model assembly line (MMAL) in order to meet the variety of product demand. MMAL balancing helps in assembling products with similar characteristics in a random fashion. The objective of this work aims at reducing the number of workstations, workload index between stations and within each station.

Ritt et al.^[7] have proposed a stronger formulation of the precedence constraints and the station limits for the simple assembly line balancing problem. The linear relaxation of the improved integer program theoretically dominates all previous formulations using impulse variables and produces solutions of significantly better quality in practice.

Mohebalizadehgashti^[8] has studied the balancing and sequencing problems of the mixed model assembly line simultaneously. A mixed integer linear programming model is proposed to solve these problems simultaneously when the assembly line has the continuous motion and when the common tasks between different models of a product can be assigned to different workstations. Objectives in this research work are minimising the length of workstations, minimising the stations' cost, and minimising the tasks duplication cost. A branch and bound algorithm is exploited to solve the model.

In Alghazi^[9], the assembly line balancing problem

(ALBP) is discussed where the decision problem is to find a feasible line balance; in other words, assigning each task to one station such that the precedence constraints and any additional restrictions are fulfilled. For the case of the paced assembly line, the cycle time determines the maximum time a work piece can spend at each station. The line balance is feasible only if the total sum of task durations at any given station, named the station load, does not exceed the cycle time. Besides, if the station load is less than the cycle time then the idle time for each cycle would be the difference between the station load and the cycle time. The word “balancing” stemmed from the fact that minimising the total idle time across all station would yield a balanced workload across all stations which can be achieved by minimising the number of workstations. This problem was named the simple assembly line balancing problem (SALBP) where one product is produced in paced line with fixed cycle time, deterministic operation times, no assignment restrictions other than the precedence constraints, serial line layout with one-sided stations, and same stations in terms of machines and workers. If more than one model or version of the product is assembled on the same line, the assembly line is categorised either as mixed-model or multi-model depending on how these different models are intermixed on the assembly line. For random inter-mixing of models, the problem of balancing the line is referred to as the mixed model assembly line balancing problem (MALBP).

In managing the simple model assembly line, Alghazi has developed the following formulation for the line balancing:

Let x_{ijh} equal 1 if task h is assigned to worker j at station i , and equal 0 otherwise,

y_{ij} equal 1 if worker j is assigned at station i , and equal 0 otherwise,

v_{hl} equal 1 if task h is executed before task l , and equal 0 otherwise,

s_h equal the starting time of task h .

It is required to minimise the total workers assigned to all stations, given by:

$$\text{Minimise } Z = \sum_i \sum_j y_{ij} \quad (3)$$

Subject to:

- Task assignment to worker: each task has to be assigned to only one worker, given by:

$$\sum_{i \in F_h} \sum_j x_{ijh} = 1, \forall h, \quad (4)$$

where F_h is the set of feasible stations that task h is assignable to,

- Cycle time: the total weighted duration assigned

to a worker should not exceed the cycle time, given by:

$$\sum_h x_{ijh} \widehat{d}_h \leq cy_{ij}, \forall (i \in F_h, j), \quad (5)$$

where $\widehat{d}_h$ is the weighted duration of task h , and c is the cycle time,

- Station time: each task assigned to a worker should be scheduled between the worker's station start and finish times, given by:

$$s_h \geq \sum_{i \in F_h} \sum_j s_i x_{ijh}, \forall h, \quad (6)$$

where s_i is the worker's station start time,

$$s_h + \widehat{d}_h \leq \sum_{i \in F_h} \sum_j (s_i + c) x_{ijh}, \forall h, \quad (7)$$

- Precedence relations: a task can only start when all of its predecessors are finished, given by:

$$s_h + \widehat{d}_h \leq s_l, \forall h, l \in O_l, \quad (8)$$

where O_l is the set of immediate predecessors of task l ,

- Assignment restrictions: pairs of tasks with same (station/worker) or not the same (station/worker) restrictions, given by:

$$\sum_j x_{ijh} = \sum_j x_{ijl}, \forall i \in F_h \cap F_l \text{ and } (h, l) \in R^{ss}, \quad (9)$$

where R^{ss} is the set of tasks that must be done on the same station,

$$x_{ijh} = x_{ijl}, \forall (i \in F_h \cap F_l, j) \text{ and } (h, l) \in R^{sw}, \quad (10)$$

where R^{sw} is the set of tasks that must be done by the same worker,

$$\sum_j (x_{ijh} + x_{ijl}) \leq 1, \forall i \in F_h \cap F_l \text{ and } (h, l) \in R^{ns}, \quad (11)$$

where R^{ns} is the set of tasks that cannot be done on the same station,

$$x_{ijh} + x_{ijl} \leq 1, \forall (i \in F_h \cap F_l, j) \text{ and } (h, l) \in R^{nw}, \quad (12)$$

where R^{nw} is the set of tasks that cannot be done by the same worker,

- Variable domain, given by:

$$x_{ijh}, y_{ij}, v_{hl} \in \{0, 1\}, \forall i, j, h \text{ and } l, \quad (13)$$

$$s_h \in Z^+, \forall h, \quad (14)$$

where Z^+ is the set of permissible times.

Alghazi has also added other constraints to include tasks overlap, worker interference, ergonomic risk, and resource requirement. The model is solved by integer linear programming (ILP) algorithm given by Hillier^[10].

3. The Loosely-coupled Systems

The loosely coupled systems, same as the decoupled systems, are a complex of multiple operational systems epitomised in the literature by the notions of facility location, production distribution, resource allocation, and assignment problems.

The only difference between the complex of the loosely-coupled type and the decoupled type of operational systems is that the former has a small number of resources and constraints cutting across all its systems, while the latter has no such resources and constraints at all. Perhaps it is a good idea to review some papers of the decoupled type of systems (majority) followed by a few number of papers of the loosely-coupled type of systems.

For the decoupled type of operational systems, Owen and Daskin^[11] have discussed a wide range of model formulations and solution approaches of facility location with applications ranging across numerous industries.

Sarmiento and Nagi^[12] have reviewed current work on the integrated analysis of production-distribution systems, and identified important areas where further research is needed. Integrated analysis is described as the analysis performed on models that integrate decisions of different production and distribution functions for a simultaneous optimisation. They reviewed work that explicitly considered the transportation system in the analysis, since they are interested in the following questions: (i) how have logistics aspects been included in the integrated analysis? And (ii) what competitive advantages, if any, have been obtained from the integration of the distribution function to other production functions within a company and among different companies? In their review they also mentioned whether the work had been done at the strategic level, i.e., if it concerns the design of the distribution system, or at the tactical level, and if it concerns optimisation problems for which the characteristics of the distribution system are provided.

Sarmiento and Nagi concluded that a clear classification of the Inventory/Distribution and Production/Distribution problems is hard to make, given the diversity and number of assumptions that such problems can take into consideration. The Inventory/ Routing problem, on the other hand, is somehow a better-defined problem which has received increased attention in recent years. The survey of research done on integrated analysis shows that, in some cases, the integration of the logistics function into the analysis of previously isolated production functions (e.g., inventory control, facilities location and production planning) has the potential of providing significant benefits to companies, in the form of costs savings and efficiency

improvement. However, many aspects of the integrated analysis have not been covered yet, in particular, the characterisation of systems for which integrated analyses are most beneficial. They believe that, given the relevance of logistics costs in overall operational costs, the integrated analysis of production/distribution systems can provide a significant competitive advantage to companies that adopt it.

Humayd^[13] presents a comprehensive planning framework for the distribution system from the distribution company perspective. It incorporates distributed generation (DG) units as an option for local distribution companies (LDCs) and determines the sizing, placement and upgrade plans for feeders and substations.

In Omu et al.^[14], a model is created for the design (i.e., technology selection, unit sizing, unit location, and distribution network structure) of a distributed energy system that meets the electricity and heating demands of a cluster of commercial and residential buildings while minimising annual investment and operating cost. The model is used to analyze the economic and environmental impacts of distributed energy systems at the neighborhood scale in comparison to conventional centralized energy generation systems. The model is solved by mixed integer linear programming (MILP) algorithm.

Gupta^[15] have studied the use of linear programming models for many different purposes. Airline companies apply these models to optimize their use of planes and staff. NASA has been using them for many years to optimise their use of limited resources. Oil companies use them to optimise their refinery operations. Small and medium-sized businesses use linear programming to solve a vast variety of problems, often involving resource allocation.

Sofi et al.^[16] have concluded that linear programming (LP) technique is relevant in the optimization of resource allocation and achieving efficiency in production planning particularly in achieving increased agriculture production of food crops (Rice, Maize, wheat, Pulses and other crops). In this paper, a Linear programming technique is applied to determine the optimum land allocation of 5 food crops by using agriculture data, concerning various factors viz. Daily wages of labor and machine has charged for the period 2004-2011. The proposed LP model is solved by the standard simplex algorithm. It is observed that the proposed LP model is appropriate for finding the optimal land allocation to the major food crops.

Khor et al.^[17] have surveyed the widespread use of numerical optimisation or mathematical programming approaches to develop and produce petroleum fields for design and operations; lift gas and rate allocation; and

reservoir development, planning, and management. Early applications adopted linear programming alongside heuristics. With continuous advancements in computing speed and algorithms, they have been able to formulate more complex and meaningful models including nonlinear programs and mixed-integer linear and nonlinear programs.

For the loosely-coupled type of operational systems, Rios and Ross ^[18] have used the Dantzig-Wolfe decomposition algorithm for assigning delays to flights. Traffic Flow Management (TFM) of the National Airspace System (NAS) endeavors to deliver flights from their origins to their destinations while minimising delays and respecting all capabilities.

Kondoh et al. ^[19] proposed a resource sharing method among multiple production systems to reduce initial investment for inverse manufacturing. To this end, this paper introduced a transferability benefit index (TBI), the ratio of the benefits to difficulties, to identify the most promising resources for sharing among multiple production systems.

Torgnes et al. ^[20] discuss the optimisation of a petroleum production allocation problem through a parallel Dantzig-Wolfe algorithm. Petroleum production allocation problems are problems in which the determination of optimal production rates, lift gas rates and well connections are the central decisions. The motivation for modeling and solving such optimisation problems stems from the value that lies in an increased production rate and the current lack of integrated software that considers petroleum production systems as a whole.

Abrache et al. ^[21] were motivated by the development of iterative auction mechanisms that induce the bidders into progressively revealing their private preferences. In this paper, they consider a model that abstracts in a reasonable general way an exchange of interdependent goods and they propose an iterative auction mechanism based on the well-known Dantzig-Wolfe decomposition principle, where the bidders reveal parts of their preferences through straightforward, utility-maximising bids.

In El Noshokaty ^[22], the mathematical model of the loosely-coupled systems has been formulated as follows:

Let x_j^k be the quantity of product j produced in production system k ,

p_j^k be the j^{th} product unit profit in production system k ,

c_o^k be the operational cycle fixed cost for production system k ,

a_{ij}^k be the j^{th} product unit consumption from production-system- k resource constraint i ; b_i^k , or from coupling resource constraint i ; b_i , respectively,

' n_k ' be the number of variables in production system k ,

' m_k ' and ' m_o ' be the number of production-system- k

constraints and the number of coupling

constraints, respectively,

' l ' be the number of production systems.

Then the objective function can be formulated as follows:

Maximise

$$Z = \sum_{k=1}^l \left(\sum_{j=1}^{n_k} p_j^k x_j^k - c_o^k \right) \quad (15)$$

Subject to:

- the coupling-constraints, given by:

$$\sum_{k=1}^l \sum_{j=1}^{n_k} a_{ij}^k x_j^k \leq b_i, i = 1, \dots, m_o, \quad (16)$$

- the k -constraints, given by:

$$\sum_{j=1}^{n_k} a_{ij}^k x_j^k \leq b_i, i = (m_o + 1), \dots, (m_o + m_k), k = 1, \dots, l, \quad (17)$$

where

$$m_{ko} = \sum_{i=0}^{k-1} m_i,$$

- the non-negativity constraints, given by:

$$x_j^k \geq 0, j = 1, \dots, n_k, k = 1, \dots, l. \quad (18)$$

The model may be solved by Linear Programming (Hillier ^[10]) for small problems and by Dantzig-Wolfe Decomposition Principle (Dantzig and Wolfe ^[23]) for massive problems.

4. The Coupled Systems of Time-sensitive Operational Cycle (El Noshokaty ^[22] and El Noshokaty ^[24])

The operational cycle refers to a set of the operation processes or systems that begins with inputs and ends with finished outputs. The operation time is the time taken to complete an operational cycle. The operational cycle is said to be less sensitive to time if operation time does not vary considerably from one cycle to another. Examples are crop harvesting, car manufacturing and assembly lines, and liner shipping in maritime cargo transport (El Noshokaty ^[25]). Operation cycle is said to be sensitive to time if operation time varies considerably from one cycle to another. Examples are customized product assembly lines, cooperative farming, gas or oil reservoir development, and tramp shipping in maritime cargo transport (El Noshokaty ^[26]). The less-sensitive operational cycle may become sensitive when it is subject to a change in the amount of each production factor employed in the cycle. While mathematical models which represent the less-sensitive production cycle have a gross profit objective, the ones representing the time-sensitive operational cycle have a gross-profit-per-day objective. Gross-profit-per-day objective cares

for the higher gross profit it yields and the less number of days it takes to generate such profit. By repeating the cycle the year around, the latter objective gives more gross profit by the end of the year. In the product capacity and the gross profit planning, the objective is not to set a maximum level of operation output or gross profit, but to set a maximum rate of the output or gross profit. One such rate is to relate output or gross profit to time.

Based on the above-mentioned arguments, the mathematical model of the time-sensitive operational systems can be formulated as follows:

Let x_j^k be the quantity of product j produced in production system k ,

p_j^k and t_j^k be the j^{th} product unit profit and its share in the operational cycle time of production system k , respectively,

c_o^k and t_o^k be the operational cycle fixed cost and time for production system k , respectively,

a_{ij}^k be the j^{th} product unit consumption from production-system- k resource constraint i ; b_i^k , or from coupling resource constraint i ; b_i , respectively,

' n_k ' be the number of variables in production system k ,

' m_k ' and ' m_o ' be the number of production-system- k constraints and the number of coupling constraints, respectively,

' I ' be the number of production systems.

Then the objective function can be formulated as follows:

Maximise

$$Z = \sum_{k=1}^I \left(\sum_{j=1}^{n_k} p_j^k x_j^k - c_o^k \right) / \left(\sum_{j=1}^{n_k} t_j^k x_j^k + t_o^k \right) \quad (19)$$

The denominator in formula (19) is the time taken to complete the operational cycle in system k , where the cycle has more than one product or product version in succession (mixed production model), e.g., customised product assembly lines and tramp shipping. If the cycle contains one product, e.g., cooperative farming and oil reservoir development, formula (19) is reduced to:

Maximise

$$Z = \sum_{k=1}^I \left(1 / t_o^k \right) \left(\sum_{j=1}^{n_k} \left(p_j^k / t_j^k \right) x_j^k - c_o^k \right)$$

Subject to the coupling-constraints, given by:

$$\sum_{k=1}^I \sum_{j=1}^{n_k} a_{ij}^k x_j^k \leq b_i, i = 1, \dots, m_o, \quad (20)$$

the k -constraints, given by:

$$\sum_{j=1}^{n_k} a_{ij}^k x_j^k \leq b_i^k, i = (m_{ko} + 1), \dots, (m_{ko} + m_k), k = 1, \dots, I, \quad (21)$$

where

$$m_{ko} = \sum_{i=0}^{k-1} m_i$$

and the non-negativity constraints, given by:

$$x_j^k \geq 0, j = 1, \dots, n_k, k = 1, \dots, I. \quad (22)$$

The model is a new extension to the linear optimisation developed to solve the coupled operational systems of a time-sensitive operational cycle. It may be solved by Block-Angular Linear Ratio Programming (El Noshokaty ^[27]) if the operational cycle has more than one product or product version in succession, and by Dantzig-Wolfe Decomposition Principle (Dantzig and Wolfe ^[23]) if the cycle has one product. Refer to El Noshokaty ^[22] for a detailed description for the examples of customized product assembly lines, cooperative farming, and gas or oil reservoir development, and for the application of tramp shipping mode of maritime cargo transport.

A group of systems having the following characteristics is a candidate user of the model (19) to (22):

(1) The operations take place in a complex of multiple systems.

(2) The operational systems are coupled with some coupling resources.

(3) The operational cycle in each system is time-sensitive.

5. The Coupled Systems Employing Different Mixes of Factors of Production (El Noshokaty ^[22] and El Noshokaty ^[28])

Capital intensity is sometimes defined as a measure of how much quantity of robots, machines, equipment, and other organisation's assets. Whereas, the capital intensity ratio of an organisation is a measure of the amount of capital needed per dollar of revenue. It is calculated by dividing total assets of a company by its sales. It is reciprocal of total asset turnover ratio. A higher capital intensity ratio for a company means that the company needs more assets than a company with a lower ratio to generate an equal amount of sales. A high capital intensity ratio may be due to lower utilization of the organisation's assets, or it may be because the organisation's business is more capital intensive and less labor intensive, as in most automated industries. However, for organisations in the same industry and following similar business model and production processes, the organisation with lower capital intensity is better because it generates more revenue using fewer assets.

On the other hand, labor intensity is defined as a measure of human resource. It may be quantified by taking a ratio of the cost of labor (i.e., wages and salaries) as a proportion of the total capital cost of producing the good or service.

Capital intensity is sometimes defined as the mix of equipment and human resource in the system; the greater the relative cost of equipment, the greater is the capital intensity. As the capabilities of technology increase and its costs decrease, managers face an ever-widening range of choices, from operations utilizing very little automation to those requiring task specific equipment and little human intervention. This paper will consider the latter definition of capital intensity but within an economic context rather than the former definition, which is formulated in a financial context.

In the traditional economics, the factors of production include capital, land, labor, and entrepreneur. Information is added as being the fifth factor of production. In the operational cycle, several factors of production may be employed in fixed or variable amounts. If they are employed in fixed amounts, the operation cycle employs the same amount of each factor of production each time the operation cycle is run. While if they are employed in variable amounts, the operation cycle may be planned to use different mixes of factors of production; each mix has its amounts of factors of production. The soft drink production system represents an example close to the former, while the car production system presents an example close to the latter. The car may be assembled using more robots and little labor intervention and may be assembled employing more labor and little robots intervention.

In case the multiple operational systems can employ different mixes of factors of production, the gross-profit objective becomes inappropriate because it ignores the production time taken by each mix. The entrepreneur is more interested in the mix which brings him more profit and takes less production time. If the entrepreneur uses a gross-profit-per-day objective, he can then repeat the more profit-per-day production cycle more times as to collect more profit by the end of the year.

For the planning of multiple production systems which can employ different mixes of factors of production, global maximisation of the gross profit per day for all systems may be achieved by using a linear ratio model. In this model, the objective equals the sum of the gross profit per day of the selected mix of the production factors, for all products of all production systems. The objective nominator of each production system is its total gross profit of the selected mix for all products, and the denominator is its production time taken to produce these products.

The mathematical model can be formulated as follows, assuming that each production system produces its products at full production capacity within the product demand:

Let x_{ij}^k be the decision variable, where $x_{ij}^k = 1$ if product i is produced by the production system k employing

mix j of factors of production and $x_{ij}^k = 0$ otherwise, where $k = 1, 2, \dots, K$; $i = 1, 2, \dots, I$; and $j = 1, 2, \dots, J$.

q_{ij}^k be the quantity of product i produced in the operational-cycle of system k employing mix j ,

p_{ij}^k and t_{ij}^k be the gross profit and the operational-cycle time taken to produce the quantity q_{ij}^k , respectively,

f_{rij}^k be the amount of coupling factor of production r employed by system k in producing q_{ij}^k .

F_r be the total amount of factor of production r cutting across all systems, where $r = 1, 2, \dots, R$.

c_o^k and d_o^k be the operational cycle fixed cost and time of system k , respectively,

Then the optimal-product-quantity with global maximal profit-per-day can be given by:

Maximise

$$Z = \sum_{k=1}^K \left(\sum_{i=1}^I \sum_{j=1}^J p_{ij}^k x_{ij}^k - c_o^k \right) / \left(\sum_{i=1}^I \sum_{j=1}^J t_{ij}^k x_{ij}^k - t_o^k \right) \quad (23)$$

The denominator in formula (23) is the time taken to complete the operational-cycle in system k , where the cycle has more than one product or product version in succession (mixed model). If the cycle has one product, formula (23) becomes:

Maximise

$$Z = \sum_{k=1}^K \left(1 / t_o^k \right) \left(\sum_{i=1}^I \sum_{j=1}^J \left(p_{ij}^k / t_{ij}^k \right) x_{ij}^k - c_o^k \right)$$

Subject to:

The coupling-constraints, which limit the amount of coupling factors of production employed in all systems to an upper limit, are given by:

$$\sum_{k=1}^K \sum_{i=1}^I \sum_{j=1}^J f_{rij}^k x_{ij}^k \leq F_r, r = 1, \dots, R, \quad (24)$$

the i -constraints, which limit the number of mixes of each product to one mix, are given by (other k -constraints may also be added):

$$\sum_{j=1}^J x_{ij}^k = 1, i = 1, \dots, I, k = 1, \dots, K, \quad (25)$$

and the integrality constraints, given by:

$$x_{ij}^k = 0 \text{ or } 1, i = 1, \dots, I, j = 1, \dots, J, k = 1, \dots, K. \quad (26)$$

The model is a new extension to the linear optimisation developed to solve the coupled operational systems employing different factors of production. It may be solved by 0-1 Integer Linear programming (Hillier^[10], El Noshokaty^[28]). At each branching node, the problem may be solved by Block-Angular Linear Ratio Programming (El Noshokaty^[27]) if the operational-cycle has more than one product or product version in succession, and by Dantzig-Wolfe Decomposition Principle (Dantzig and Wolfe^[23]) if the cycle contains one product. Refer to El Noshokaty^[22] for the detailed descrip-

tion of the examples of the car production systems,

A group of systems having the following characteristics is a candidate user of the model (23) to (26):

- (1) The operations take place in a complex of multiple systems.
- (2) The operational systems are coupled with some coupling resources.
- (3) The operational-cycle in each system can employ different mixes of factors of production.

6. The Coupled Systems of Multi-objective (El Noshokaty)^[22]

Many real situations require the operational systems to have more than one objective to express the influence of not only the quantity of output, but also other factors like preserving the workforce, satisfying the customers, and avoiding the increasing power of the competitors. An example of such systems is the game reserve.

The mathematical model of the systems of multi-linear-objective can be formulated as follows:

Let x_j^k be the quantity of product j produced in production system k ,

p_{jr}^k be the j^{th} product unit profit in linear objective r for the production system k , $r=1, \dots, R$,

a_{ij}^k be the j^{th} product unit consumption from production-system- k resource constraint i ; b_i^k , or from coupling resource constraint i ; b_i , respectively

' n_k ' be the number of variables in production system k ,

' m_k ' and ' m_o ' be the number of production-system- k constraints and the number of coupling constraints, respectively,

' l ' be the number of production systems.

Then the set of objective functions to be maximised can be formulated as follows:

Maximise

$$Z = \left\{ \left(\sum_{j=1}^{n_k} p_{jr}^k x_j^k \right), r=1, \dots, R, k=1, \dots, K \right\} \quad (27)$$

Subject to the coupling-constraints, given by:

$$\sum_{k=1}^l \sum_{j=1}^{n_k} a_{ij}^k x_j^k \leq b_i, i=1, \dots, m_o, \quad (28)$$

the k -constraints, given by:

$$\sum_{j=1}^{n_k} a_{ij}^k x_j^k \leq b_i^k, i=(m_{ko}+1), \dots, (m_{ko}+m_k), k=1, \dots, l, \quad (29)$$

where

$$m_{ko} = \sum_{i=0}^{k-1} m_i$$

and the non-negativity constraints, given by:

$$x_j^k \geq 0, j=1, \dots, n_k, k=1, \dots, l. \quad (30)$$

The model is a new extension to the linear optimization developed to solve the coupled operational systems of multi-linear-objective. It may be solved by Dantzig-Wolfe Decomposition Principle (Dantzig and Wolfe^[23]) with the sub-problem being solved by Linear Goal Programming (Hillier^[10]). Refer to El Noshokaty^[22] for the detailed description for the example of the game reserve.

7. Conclusion

The purpose of this review is to summarise the existing literature on the operational systems as to explain the current state of understanding on the coupled operational systems. The review only considers the linear optimisation of these systems.

Traditionally, the operational systems were classified as decoupled and coupled systems. The coupled operational systems were again classified as loosely and tightly coupled systems. The name 'tightly coupled' is used to describe the production assembly lines where the product or product versions are subject to series of assembly operations tightly related. The name 'multidisciplinary' is not used to describe the coupled operational systems since it is purely an engineering name used to describe tightly related design components of an engineering product.

Lately, the coupled operational systems were classified as systems of time-sensitive and time-insensitive operational cycle, systems employing one mix and different mixes of factors of production, and systems of single-linear, single-linear-fractional, and multi-linear objective. These new classifications extend the knowledge about the linear optimisation of the coupled operational systems and reveal new objective-improving models and new state-of-the-art methodologies never discussed before. Business areas affected by these extensions include product assembly lines, cooperative farming, gas/oil reservoir development, maintenance service throughout multiple facilities, construction via different locations, flights traffic control in aviation, game reserves, and tramp shipping in maritime cargo transport.

Declaration

The Author has reused the words of the authors of the cited papers to describe their work,

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