ARTICLE

Sediment Distribution of the River Boundary Layer

Levent Yilmaz[*]

Nisantasi University, 1453 Neocampus, Civil Engineering Department, Hydraulic Division, 80626, Maslak, Istanbul, Turkey

ARTICLE INFO	ABSTRACT
	The movement of particles on the river boundary layer is a complex phenomena which can be never solved by a deterministic approach. The unsteady non uniform conditions in flow boundary layer show the result of water surface and bed stream changing with time and location of particles. To determine the movement of boundary layer particles other new theories about stochastic processes using the theory of probability and statistics in river alluvial channels will give better results.

1. Introduction

To determine several formulas for sediment transport discharge are useless which can be given as discharge of sediment as a function of flow properties because of its complexity. As a result, it was no universal formula determination foe sediment discharge. Different theories of probability have great potential in problem solution which are playing an important role for determination of particle movement duration on the alluvial river boundary layer [1]. In this aspect the laboratory observations can be used for verifying of the stochastic results.

Einstein [2] searched this phenomena using the probability density formulas solving the sediment transport length and rest of duration which are given as exponential functions. The travelling total distance of the particle is given as [1],

$$f(x,t) = k_1 \cdot e^{-k_1 x - k_2 t} \sum_{n=1}^{\sim} [(k_1 . x)^{n-1} / \Gamma(n)] \cdot [(k_2 . t)^n / \Gamma(n+1)] \qquad \ldots\ldots x > 0 \qquad (1)$$

in which $f(x,t)$ = gamma function

k_1, k_2 = constants of the mean step length and rest period, respectively.

Equation (1) is proportional to the sediment particle concentration with respect to longitudinal position of x as a function of time duration. It is derived two-dimensional model of stochastic interpretation of the bed-sediment particle layer height where the particles deposited.

In the model application, the function for probability density at the rest period is calculated in the deposition elevation as $f_{x_D}(x)$, the function for probability density function is given as $f_{T\backslash YD}(t\backslash y)$ which is given as the rest period at the elevation deposition.

Using cluster techniques of tracer particles [1], the statistics for the lengths and duration periods can be easily

Corresponding Author:
Levent Yilmaz,
Nisantasi University, 1453 Neocampus, Civil Engineering Department, Hydraulic Division, 80626, Maslak, Istanbul, Turkey;
Email: levent.yilmaz@nisantasi.edu.tr

calculated. Here k_1 and k_2 parameters are found from longitudinal concentration distribution curves [4]. It is reported that the theoretical and laboratory applications are overlapped. Model parameter estimations from different laboratory observations are difficult. Another experimental study is given by another research [3].

Different laboratory flume with two bed-material sizes were conducted by experiments were as bed configurations can be seen ripples and dunes. The step lengths and the rest can be determined by using single radioactive tracer movements where step lengths and the rest periods were measured directly. It is observed that the gamma distribution overlap with step lengths and the rest periods to be approximately exponentially distributed [1]. The bed elevation as a function of time records, it is given that the conditional probability density function of the rest periods can be observed by the exponential function [3],

$$f_{T/YD}(t/y) = k_y(y) e^{-k3 (y)t} \qquad (2)$$

in which $k_3(y)$ = the constant of the conditional mean rest period at elevation y. If significant further results is to be expected new solutions must be determined for estimating the probability distributions. The purpose of these experiments is to give the observations of different laboratory studies about flumes in which different data are collected.

2. Experiments about Flume Boundary Layer

In alluvial boundary layers the most expected bed forms are dunes. The shape of a dune is approximately triangular in long section with gentle upstream slope and a steep downstream slope. The upstream flow conditions determine the shape of a dune whereas the slope on dune is more dependent on the angle of repose of the bed material. The movement of dunes downstream gives the observation about erosion from the stoss or upstream side and deposition on the downstream face. Sediments particles of a dune at the upstream side must make a step in the downstream direction before being rest on the slip side. After deposition they rest until the dune has moved to another place whose sequence continues at the flume boundary layer. Particles show movement by erosion and rest on the bed boundary layer where its step length depends only on the height of bed slope from which it has moved. The number of dune crests shows deposition and shape and scale during the time of the erosion. If the number of particles per unit volume of the bed Ω is constant the accumulation is assumed in statistical sense stationary, both erosion and deposition cannot be observed at the same point and at the same time. The rest length of the particle

is given from the stochastic equation as $y_x(t)$ values. The value of sediment particles per unit area within the class intervals (η_j, η_{j41}) in step duration, given by $N_d (y_j)$ as [1],

$$N_d (y_j) = \Omega\Sigma_{x=1}^m \Delta y_{j,\pi} \qquad j = 1,2,\ldots\ldots,n \qquad (3)$$

where π = the value of class intervals for having of Y_D value;

$\Delta y_{j,k}$ = the elevation height of the bed in every class interval associated with y_j for the kth deposition period and m_j = the maximum value of bed forms contained in the $y_x(t)$ step and which also have some deposition in the class interval, having with y_1 for the kth deposition period.

The total value of sedimentation per unit boundary layer area rests over all intervals which is deposited by N_d and is obtained by summing them [1].

$$N_d = \Sigma_{j=1}^n N_d (y_j) = \Omega\Sigma_{j=1}^n\Sigma_{k=1}^{mj} \Delta y_{j,k} \qquad (4)$$

This equation was approximated by us of the sample probability mass function, given as

$$P Y_D (y_i) = P [\eta_j < Y_D < \eta_{j+1}] = N_d (y_i) / N_d \text{ for a large m} \qquad (5)$$

The particle probability of erosion within a particular class interval can be observed in the same time where the erosion periods instead of the deposition periods must be taken. The probability function will be given to either deposition periods or erosion periods, the bed height probability function in deposition and erosion must be identical [4].

3. Probability of Particle Rest Periods

The particle rest duration is given as the rest time between the deposition and erosion at the flume boundary layer where the $y_x(t)$ value defines the probability estimation of particle density function at duration on the height of bed elevation. This probability is given by [3]. Measurement the time difference between a down crossing and the previous up crossing values ($t_{j,k}$, j= 1,2,…..., n and k = 1,2, ,,,,,, m_n) for a relative frequency analysis of the statistic ($t_{i,k}$) is given for a sample conditional probability mass function of the rest period definition [3].

$$P_{T,YD}(t_\alpha|y_j) = P [\tau_\alpha < \tau < \tau_{\alpha+1} / \eta_j < Y_D < \eta_{j+1}]$$
$$j=1,2,..,n \qquad \alpha = 1, 2,\ldots\ldots.r \qquad (6)$$

where τ = the random variable describing the rest periods, t_α, and y_i = the class properties for τ and Y_D.

τ_α and $\tau_{\alpha+1}$ = the lower and upper class limits of τ_α, and r = the number of class intervals for τ.

This function of probability is used to determine the mass function for the sediment deposition durations

$P\tau_n(t\alpha) = P [\tau_\alpha < \tau < \tau_{\alpha+1}] = \Sigma_{p=1}^{n} P_\tau ; Y_D (t_\alpha|y_i) P_{YD} (y_t)$
$\alpha = 1,2,..r$ (7)

The duration of probability function for the deposition of sedimentation on the flume boundary layer can be collected by distribution of bed height value at a fixed location with time interval. Other observations can be given as [3].

(1) Noting that both sediment transport at the bed boundary layer with the other word erosion and rest of particles (deposition) do not see at the same time interval,

(2) The sedimentation at the bed height is assumed stationary in view of statistics where its results must show the same application in the field and at the experimental-set-up.
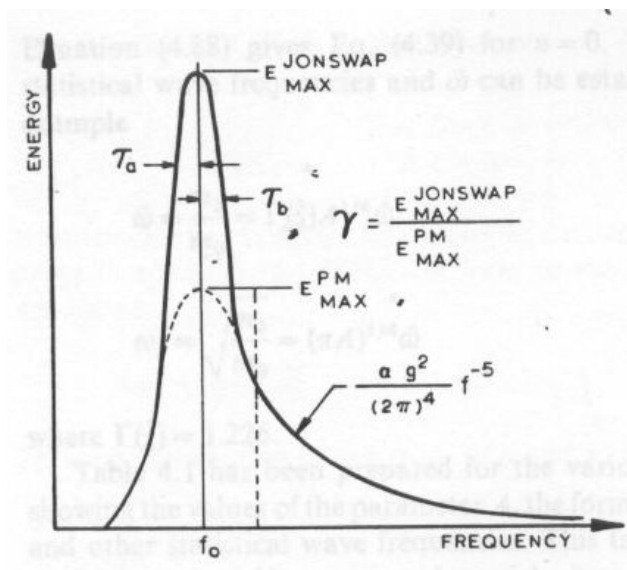

Figure 1. Comparison of the JONSWAP and Pierson-Moskowitz Normal Distributions at particle movement energy [5]

4. Step Length Determination with Probability Distributions

For step length distribution first we must assume that a particle has also energy by deposition and erosion in view of bed load distribution on the river boundary layer [5]. Using Model theory we can easily have the same experimental-set-up on the laboratory. The suspended bed load is given as that material which is transported from upstream and deposited on the downstream part of the bed material formation like a dune from which it shows a movement. The suspended bed material must be that sediment material which is not deposited on the downstream part of the same bed formation. If we observe the bed load as previously defined the distance between the transporting at elevation y_1, and the transportation to the bed height y_1

of the k-th bed formation in the $y_t(x)$ location. This observation is a definition of the step distance of a sediment particle which is transported from the bed elevation y_i and is resting on the bed height y_i on the same bed form. The frequency analysis of the observations (x, β) determines a sample conditional probability mass function which is defined as:

$P_{x|YE, YD} (X_{i\beta}|Y_t,Y_j)=$ (8)

in which X = the random step length value,

$P_{x|YE, YD} (X_{i\beta}|Y_t,Y_j)= P [\lambda_\beta < X < \lambda_{\beta+1}] [\eta_i < Y_E < \eta_{i+1} , \eta_j < Y_D < \eta_{\beta+1}]$ (9)

$\beta = 1,2,....., s$ and $i,j = 1,2,....., n$

X_β = the class property for the observation of $X_i\lambda_\beta$, and $\lambda_{\beta+1}$ the lower and upper class limits of X_β , and s = the number of class intervals for the realizations of X variable.

If Y_E and Y_D are assuming independent for a uniformly sized boundary layer sedimentation, it defines that a particle passing a bed form crest has no memory of the height of its movement where the sample mass function of the rest length given that a particle is lying at elevation y_j, defined as [3]

$P_{x|YD} (X_\beta|Y_i)= P [\lambda_\beta < X < \lambda_{\beta+1} | \eta_j < Y_D < \eta_{j+1}]$ (10)

The step length function of the bed load sedimentation can be determined by combining the observations taken in the $y_x (t)$ and $y_t(x)$ measurements. The other conditions are given as [5]:

(1) No accumulation can be observed on the upstream sides of dunes and no sediment transportation occurs on the downstream parts of the bed forms.

(2) The height of accumulation, Y_E and the elevation of particle deposits, Y_D, are independent.

(3) The flow is in Froude number as transition part [6]. The first assumptions may not be strictly true due to flow separation, in the dune environment where both deposition and erosion may occur at the same point. For dune flow conditions, laboratory observations show that such an area is small. The second assumption seems to be reasonable because as a sediment particle passes a dune crest it is likely to lose any memory of where it came from. Bed load is defined as that portion of the total load which passes only one dune crest per step

5. Sediment Discharge Definition

The transport rate of sedimentation at elevation $y_p v_B$ (j) is given as,

$$vB(j) = \frac{E\left[X|YD = Yj\right]}{E\left[\tau|YD = yp\right]} \qquad (11)$$

in which $E[X|Y_D=y_j]$ = th mean step length of the sample, given that the height of deposition is at y_p, and $E[\tau|Y_D=y_p]$ = the mean rest period of the sample given the elevation of deposition is at y_p. This transport rate can be determined at each zone of the bed contributes to the total bed-load.

Using the continuity equation the total bed-load distribution is given as [6]

$$Q_B = \gamma_B (1- \theta) \Sigma_{i=1}^{n} v_n (j)\xi_j \Delta y_p \qquad (12)$$

where $Q_{B=}$ the bed-load discharge in weight per unit time and width,

γ_B = the specific weight of the bed particles,

θ = the porosity of the bed material,

Δy_p = the class width which remarks at the bed elevation, y_j

In the bed discharge formula, the bed-material particles must have identical transport properties. We can change the above expression as

$$Q_B = \gamma_B (1- \theta) h\, v_B \qquad (13)$$

where h= the mean depth of the layer in which bed-load particles transported

v_B = the average transport velocity of a bed-load transportation.

6. Analysis of Experimental Evaluation

Different experiments were observed in a recirculating flume of rectangular cross-section [6]. The bed transportation observed in the experiments, was a uniformly sized river accumulation with uniform distribution. After an uniform discharge was observed, the y_x (t) and y_t(x) values, the bed material discharge, and the hydraulic units were evaluated. The methods and procedures of evaluation have been given [7].

The y_t (x) evaluation were given by mounting a sonic depth sounder on an instrument carriage such that the ultra sounder was over the center line of the flume and then moving the sounder and carriage in the upstream direction. The evaluation time was approximately 5 min., the y_t(x) evaluation is continuously.

The y_x (t) evaluation was held by putting a sonic depth sounder at the canal centerline, downstream of the instrumentation. Both the y_t(x) and y_x(t) evaluations were digitized with an analog-to-digital converter at the lag intervals [8]. The lag interval on the y_t(x) evaluations was not constant because the speed of the carriage was somewhat

different for each lag.

The probability mass function samples were obtained by the given above formulas by accumulation height of sedimentation at the flume boundary layer and sediment transport [8].

The y_x(t) evaluation of each run was standardized so that the class intervals y_t, measures the heights of deposition or erosion in terms of the standard deviation about the mean bed height.

The class width of 0,4 standard deviations was used for all class evaluations. The histograms of frequency for accumulation and sediment transport are given [1]. The Gaussian density function obtained from data evaluation shows to fit the values for different experimental-set-ups very well. For flume stationary condition continuity needs that the probability of erosion from any bed height is equal to the accumulation probability. Therefore, the density functions for the height of accumulation, and sediment transport must be identically distributed. The mean and variance of evaluation data histograms are also given. The total number of points are available for analysis, Σm_j. In the low sedimentation processes the slow transport rates give the limited number of occurrences. The deposition periods were computed by determining the difference between the sediment transport from the bed formations and the time of deposited which have defined as each observed event as m_{ti} . Also given in this figure the overlapping of occurrence into the two-parameter gamma probability density function.

7. Summary and Results

(1) For deposition or sediment transport at the flow boundary layer evaluation by the sediment probability density function it has the shape of standard normal density function with +- 2.4 standard deviations.

(2) The observation of mean rest period of sedimentation shows with decreasing bed height an increase. The property of mean deposition duration seems to be not a function of upstream continuum mechanics

(3) Noting that both sediment transport at the bed boundary layer with the other word erosion and rest of particles (deposition) do not see at the same time interval,

(4) The mean deposit period depends on accumulation of boundary layer properties which can be larger than the mean deposition duration of particles at the boundary layer height.

(5)The exponential density function overlaps with the measured deposition duration distributions reasonably well.

(6) The mean step length of a sedimentation particle increases nearly linearly with a decrease in the boundary

layer height. The mean step length of a sedimentation particle seems nearly 45 % of the mea bed form length for different experimentation procedure.

(7) The Gamma Density Function fits the measured deposition length distribution reasonably well.

References

[1] Lee, K. Baum. Stochastic Analysis of Dune bed Profiles, ASCE, Journal of the hydraulics Division, HY7,10557, 1974: 849-857.

[2] Einstein,H. A.. The Bedload Movement as a Probability Problem. Mitteilung der Versuchsanstalt fur Wasserbau, an der EidgenossischeTechnischeHochschule in Zurich, VerlagRascher and Co., Zurich, Switzerland, 1937.

[3] Hubbell, D.W., Sayre, W.W.. Closure in "Sand Transport Studies with Radioactive Tracers". Journal of the Hydraulics Division, ASCE, Proc. Paper 4464, 1965, 91(HY5): 139-149.

[4] Sediment. Transportation Mechanics: Introduction and properties of Sediment. by the Task Committee on preparation of Sedimentation, Manual of the Committee on Sedimentation, Vita A. Vanoni. Journal of the Hydraulics Division, ASCE, Proc. Paper 3194, 1962, 88(Hy 4): 77-107.

[5] Yilmaz, L.. Stochastic Sedimentation Process Distribution at Particle Energy. Nisantasi University, 1453, Neocampus, Istanbul, Turkey, (unpublished research) 2019.

[6] Shen, H. W., Todorovic, P.N.. A General Stochastic Model for the Transport of Sediment Bed Material. Proceedings, 1st International Symposium on Stochastic Hydraulics, Chao – Lin Chiu ed., Pittsburg, Pa.,1971: 426-448.

[7] Williams , G.P.. Aids in designing Laboratory Flumes. Open-File report, United States Geological Survey, 1971.

[8] Lee, B. K.. Laboratory Study of an alluvial stream at one Foot Depth. Thesis presented to Colorado State University, at Fort Collins, Colo., in partial fulfillment of the requirements for the degree of Master of Science, 1969.