Journal of Mechanical Materials and Mechanics Research
https://ojs.bilpublishing.com/index.php/jmmmr/index

ARTICLE

Recognition Methods of Geometrical Images of Automata Models of Systems in Control Problem

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ARTICLE INFO

Article history
Received: 18 August 2021
Accepted: 23 September 2021
Published Online: 27 September 2021

Keywords:
Discrete deterministic dynamical system
Mathematical model
Automaton
Geometric image of an automaton mapping
Geometric curve
Sequence
Recognition of geometric images of automata

ABSTRACT

The laws of functioning of discrete deterministic dynamical systems are investigated, presented in the form of automata models defined by geometric images. Due to the use of the apparatus of geometric images of automata, developed by V.A. Tverdokhlebov, the analysis of automata models is carried out on the basis of the analysis of mathematical structures represented by geometric curves and numerical sequences. The purpose of present research is to further develop the mathematical apparatus of geometric images of automaton models of systems, including the development of new methods for recognizing automata by their geometric images, given both geometric curves and numerical sequences.

1. Introduction

In the class of discrete deterministic dynamical systems, finite deterministic automatons form the simplest, but rather studied subclass. Advanced methods of analysis, synthesis, recognition, etc. of finite state machines are effectively used in solving applied problems for real systems, the automaton models of which are explicitly specified by tables, matrices, graphs, logical equations, etc. After the introduction by McCulloch and Pitts (1943 [1]) of the basic provisions on which the concept of an automaton is built, the theory of automata was developed in [2-9] and many others etc. The research results presented in the works of these authors constitute the fundamental basis of the symbolic theory of automata, ie. theory, in which automata, as a rule, are not associated with classical numerical structures, which limits the application of the methods of classical mathematics in the automata theory.

In connection with the development of areas of application of the theory of automata, it turned out that for real systems of large dimension, the assignment of automata models with tables, matrices, graphs, and logical equations is practically ineffective. One of the ways to expand the field of application of the theory of automata was the research of Academician of the Russian Academy of Natural Sciences V.A. Tverdokhlebov, in which, since 1993, is considered the representation of the laws of functioning of automata, given by discrete symbolic next state and output functions, by continuous numerical structures.

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For this, the automata mapping is assumed to be a set of points with numerical coordinates and the laws of functioning are represented by broken lines, the vertices of which are located on analytically specified curves. This approach to defining of automata as well as some methods of analysis, synthesis and recognition of automata by their geometric images are used and developed.

The geometric approach used in this work to define the laws of the functioning of automata ideologically intersects with the research of other scientists [16-21], but has fundamental differences. For example, in [16] Benjamin Steinberg also considers a geometric approach to automata theory, but in the context of its use in combinatorial group theory, to solve various problems on the overlap between group theory and monoid theory. In [17] finite automatons are used in problems of encoding and compressing images, as well as in considering regular ω-languages (sets of infinite words defined by finite automata).

The work [18], which studies discrete-event systems, is of great interest, although it also uses symbolic forms of defining automata models of systems.

Quite interesting are the results of [19], which uses a geometric approach to describe such properties, within the framework of which the output function can be characterized by its polygon in the unit square [0, 1] × [0, 1], in [19] also investigated the properties of such polygons and their relationship with the properties of the used output function are investigated.

In this paper, we consider problems from the general problem of recognizing of finite deterministic automatons by properties and signs of their functioning.

In [10] it is shown that for a fixed number of input signals of the automaton and the order on the set of input words, the geometric image is in one-to-one correspondence with the sequence of the second coordinates of the points of the geometric image. From the geometric image γ_s of the automaton $A_s = (S, X, Y, \delta, \lambda)$, where S is the set of states, X is the set of input signals, Y is the set of output signals, $\delta : S\times X\rightarrow S$ is the next-state function, λ: $S\times X\rightarrow Y$ is the function of outputs and $s \in S$ is the initial state, a sequence of second coordinates of points of the geometric image is selected, which one-to-one corresponds to the complete geometric image (with a fixed order on the set X* and the value m = | X |).

As a result, the laws of the automaton functioning and the specific processes of the automata functioning (that is, the phase trajectories) turn out to be uniquely determined by the sequence of the second coordinates of the points of the geometric image.

This allows us to consider an arbitrary sequence of elements from a finite set as a sequence of second coordinates of points of a geometric image and, therefore, as definition of the laws of the automaton functioning, which, in turn, opens up opportunities for analyzing the processes of the functioning of systems through the analysis of the properties of sequences.

The fundamental novelty of this research consists in the development of new recognition methods, which use not the classical specification of automata models of systems by tables, matrices, graphs, systems of logical equations, but the representation of automatic models by their geometric images located on analytically specified curves (and, under certain conditions, extracted from such curves of numerical sequences). The novelty of this research is determined by the fact that such a representation, in contrast to the classical existing methods, in a number of cases allows one to effectively overcome the dimensionality barrier of automaton models of real systems.

2. Mathematical Apparatus and Research Methods

Geometric images of the laws of the functioning of automata recognition of geometric images of automatic models of systems

Geometric image γ_s of the laws of operation (see [10-12]) (the next-state function $\delta : S\times X\rightarrow S$ and output functions λ: $S\times X\rightarrow Y$) of an initial finite state machine $A_s = (S, X, Y, \delta, \lambda)$ with sets of states S, input signals X and output signals Y is determined based on the introduction of a linear order ω in the automata mapping $\rho'_s = \bigcup_{p\in X^*}\{(p, \lambda'(s, p))\}$, where $\lambda'(s,p)=\lambda(\delta(s,p'),x)$, $p=p'x$. An automata mapping ρ'_s (set of pairs) is ordered by the linear order ω, defined based on the order ω_1 on X^* and given by the following rules:

Rule 1. A certain linear order ω_1 is introduced on the set X (which we will denote $\prec_1$);

Rule 2. The order ω_1 on X extends to a linear order on the set X^*, assuming that for any words $p_1,p_2\in X^*$ of unequal length ($|p_1|\neq|p_2|$) $|p_1|<|p_2|\rightarrow p_1 \prec_1 p_2$; for any words $p_1,p_2\in X^*$ for which $|p_1|=|p_2|$ and $p_1\neq p_2$, their ratio in the order ω_1 repeats the ratio of the nearest non-coinciding letters of the words p_1 and p_2 to the left.

The order ω'_2 on the set of words Y^* is defined similarly.

After introducing the linear order ω_1 on the set X^* we obtain a linearly ordered set $\rho_s = (\rho'_s, \omega'_1)$, where ω'_1 is the order on ρ'_s induced by the order ω_1 on X^*.

Defining the linear order ω_2 on the set Y and placing the set of points in the coordinate system with the abscissa axis (X^*, ω_1) and the ordinate axis (Y, ω_2), we obtain a geometric image of the laws of functioning of an initial finite state machine $A_s = (S, X, Y, \delta, \lambda, s)$. It should be noted that the linear orders ω_1 on X^* and ω_2 on Y are generally independent. This means that the specific form of the a geometric image of the laws of functioning of an initial finite state machine $A_s = (S, X, Y, \delta, \lambda, s)$ depends on the chosen orders ω_1 and ω_2. Other variants of linear orderings on X^* are also possible (see, for example, [10,12]). In this paper, the study of the laws functioning of an initial finite state machine is carried out using the order ω_1 on X^* defined above. Linear orders ω_1 and ω_2 allow to replace elements of the sets X^* and Y by their numbers $r_1(p)$ and $r_2(p)$ in these orders. As a result, two forms of geometric images are determined, firstly, as a symbolic structure in a coordinate system D_l, and secondly, as a numerical structure in a coordinate system with integer or real positive semiaxes.

From the geometric image γ_s of the automaton A_s is extracted sequence of second coordinates of points of the geometric image, which one-to-one corresponds to the complete geometric image (for a fixed order on the set X^* and the value m = |X|). As a result, the laws of the automaton functioning (that is, the phase picture) and the specific processes of the automaton functioning (that is, the phase trajectories) turn out to be one-to-one determined by the sequence of the second coordinates of the points of the geometric image. This allows us to consider an arbitrary sequence of elements from a finite set as a sequence of the second coordinates of points of a geometric image and, therefore, as setting the laws of the automaton functioning.

The representation of a geometric image γ_s as a numerical structure allows to use the apparatus of continuous mathematics in the formulations and methods of solving problems: setting the laws of the functioning of automata by numerical equations, using numerical procedures, interpolation and approximation of partially given laws of functioning, etc. The geometric image γ_s completely determines the laws of the automaton functioning, that is, the entire phase picture of the connections of the input sequences with the output signals. Specific variants of the functioning processes, that is, phase trajectories, have geometric images $\gamma_s(p), p \in X^*$, in the form of γ_s sections along individual points. Geometric images can also be defined by numerical, rather than symbolic, equations.

3. Results

Recognition of geometric images of automata models of systems

In [10] a formal apparatus for replacing symbolic automaton models in the form of tables, graphs, logical equations, numerical structures in the form of geometric figures, numerical equations and sequences is proposed and developed. This approach is intended to search for new ideas and methods for organizing technical diagnostics of complex systems. In works [10] V.A.Tverdokhlebov proposed and developed methods for the synthesis of an automaton by sequences and geometric curves. In [10], a new type of automaton is proposed - R (α, m, d (α)) - automaton. The laws of functioning of this type of automaton are specified by the numerical sequence α, which is assumed to be the sequence of the second coordinates of the points of the geometric image. An initial segment of length d (α) of the sequence α is considered. The value m is the number of input signals of the automaton.

In this paper, we propose a method for recognizing an automaton in a given finite family of automatons, based on the selection of a set of characteristic sequences from geometric images. The laws of functioning of a complex system in case of malfunctions from a set of accounted for malfunctions and in an operable state are represented by geometric images of automatons. The search for diagnostic sequences with this method of setting the laws of functioning is reduced to finding such intervals on the abscissa axis in which the geometric images corresponding to various faults do not coincide.

In [10] it is shown that for a fixed number of input signals of the automaton and the order on the set of input words, the geometric image is in one-to-one correspondence with the sequence of the second coordinates of the points of the geometric image. In view of this, an effective search for diagnostic influences is possible based on the analysis of numerical sequences. A numerical sequence (a sequence of the second coordinates of the points of the geometric image of the automaton) is associated with each fault from the set of considered and taken into account faults, and the problem of fault recognition is reduced to the problem of finding such numbers of elements in sequences whose values are different in each of the sequences under consideration.

At the large number of sequences and a large length of the sequences themselves, this problem has a complex solution and requires large amounts of computational resources. In view of this, it is proposed to carry out the recognition of the original (numerical) sequences based on the analysis of characteristic (binary) sequences reflecting the location of the values of the elements in the original sequences.

The diagnostic method based on the decomposition of geometric images of automatons includes the following stages:

(1) Construction of a mathematical model of the system in a working state and mathematical models of the laws of the system's functioning in case of malfunctions (faults) in the form of partially specified geometric images of

automatons;

(2) Regularization of partially given geometric images to complete ones based on the use of classical interpolation methods (see [12,14]);

(3) Extraction of numerical sequences (sequences of second coordinates of points) from complete geometric images obtained on the basis of interpolation;

(4) Decomposition of sequences into a set of characteristic sequences without loss of information;

(5) Analysis of the obtained set of characteristic sequences in order to identify such a minimum set of characteristic sequences that covers all faults from the set of considered and taken into account system faults.

To illustrate the described diagnostic method with a reduction in the generality of reasoning, let us assume that the set of considered and taken into account system faults consists of 999 faults, and as a result of the implementation of stages (1)-(3) of the proposed diagnostic method, the following family of numerical sequences { H_0, H_1, H_2, …, H_{999}}, where H_0 is the sequence of the second coordinates of points of the geometric image of the automaton model of an operable system, and H_1, H_2, …, H_{999} are the sequences of the second coordinates of the points of the geometric image of the automaton models of the system in case of malfunctions.

With a reduction in the generality of reasoning, as the sequences H_0, H_1, H_2, …, H_{999}, numerical sequences of length 1000 characters are considered, extracted from the initial segment of a number π with a length of 1,000,000 characters according to the following rule: the first character of the sequence H_i , $0 \leq i \leq 999$, has a number $(i+1)\cdot 1000$ in number π, and the last sign of the sequence H_i has a number $(i+1)\cdot 1000$, i.e. the initial segment of number π with a length of 1,000,000 characters is sequentially divided into 1,000 subsequences of the same length 1,000 characters each.

As a result of the implementation of stage (4), a set of characteristic sequences was built, consisting of 10,000 sequences of length 1000. A computational experiment that implements stage 5 of the proposed diagnostic method revealed the following specific properties of the considerated set of characteristic sequences under. Recognition of all 1000 sequences H_0, H_1, H_2, …, H_{999} (in the case when all 1000 characters are analyzed) is possible using any of 10 characteristic sequences, i.e. each of the sequences H_0, H_1, H_2, …, H_{999} has a unique distribution of all ten digits.

Thus, the amount of necessary diagnostic information can be reduced by at least 10 times (for each malfunction, instead of storing the entire original sequence, it is sufficient to store any of 10 characteristic sequences).

In addition, the following properties are noted, which make it possible to more significantly reduce the amount of diagnostic information:

(1) When using only the first 10 points of the characteristic sequences, 85% of the sequences are recognized;

(2) When using the first 20 points of the characteristic sequences, more than 97% of the sequences are recognized;

(3) When using the first 25 points of the characteristic sequences, more than 99% of the sequences are recognized;

(4) To recognize all 1000 sequences, the first 60 points of the characteristic sequences are sufficient, highlighting the location of the digit 8 in the sequences H_0, H_1, H_2, …, H_{999};

(5) The use of any of the 10 sections of the geometric image for recognizing all 1000 sequences is possible when using the first 90 points of the characteristic sequences.

The performed computational experiment shows that the use of the diagnostic method using the decomposition of geometric images of automatons (using the example of the class (π, m, d (π)) - automatons constructed from the first 1,000,000 digits of π) can significantly reduce the amount of diagnostic information.

The effectiveness of the proposed method for reducing diagnostic information was investigated for another 9 classes of automatons constructed from 9 sequences by length of 1 million characters, specifying the approximations of the following mathematical quantities: e, $\varphi = \frac{1+\sqrt{5}}{2}$ (so-called golden ratio), $\sqrt{2}$, $\sqrt[3]{2}$, ln (2), ln(10), $\zeta(3) = \sum_{x=1}^{\infty} \frac{1}{x^3}$, Catalan's constants $C = \sum_{n=0}^{\infty} \frac{(-1^n)}{(2n+1)^2}$, Euler's constants $\gamma = \lim_{n\to\infty}(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}-\ln(n))$ (see [15]).

Specific properties are defined for each class. For example, the largest number of recognizable sequences (88%) using only the first 10 points of characteristic sequences is noted in the class of automata constructed from the first million digits of a number $\sqrt[3]{2}$. In the class (e, m, d(e)) - automatons for recognizing all 1000 sequences, it is enough to know the first 54 points in the characteristic (binary) sequence that highlights the location of the digit 1.

The used geometric approach allows us to research the properties of the laws of functioning of discrete deterministic dynamic systems of large dimension based on the analysis of the properties of geometric curves and numerical sequences. In this paper, classes of discrete deterministic automatons are constructed and analyzed, which are determined on the basis of the mathematical properties of geometric images that define the laws of the functioning of automatons.

The results presented in the paragraph show the possibility of practical use of the apparatus of geometric images for specifying and studying the properties of the laws of functioning of discrete deterministic dynamical systems. A method for reducing diagnostic information based on the decomposition of geometric images is proposed. An illustration of the method is given on the example of the class (π, m, d (π)) - automatons constructed from the first million digits of the sequence specifying the approximation of the number π.

A method for recognizing automata, the vertices of the geometric images of the laws of functioning of which are located on analytically specified geometric curves

Let the automaton A_0 be a mathematical model of a workable technical system and the family of automatons $\alpha = \{A_i\}_{i\in I}$ represent the set of faults I of the technical system. Suppose that these automata are given by a geometric image γ_0 and , respectively, by a family of geometric images $\beta = \{\gamma_i\}_{i\in I}$. In the developed method of technical diagnostics, geometric images are assumed to be located on an analytically specified geometric curve L_0 and a family of analytically specified geometric curves $L = \{L_i\}_{i\in I}$. Then equality $L_0 \cap \bigcup_{i\in I} L_i = \varnothing$ is determined by the solution of the control problem using a simple unconditional experiment.

Definition 1. Let L be a geometric curve and Δ - a segment on the abscissa axis, on which the curve or part of the curve L is defined. This part of the curve will be denoted by L (Δ).

Theorem 1. Let: - to the initial automaton A_0 = (S, X, Y, δ, λ, s_0), $s_0 \in S$, there is a one-to-one correspondence with the geometric image $\gamma_0 \subset X^* \times Y$ located on the geometric curve L_0; - the family of initial automatons $\alpha = \{A_i\}_{i\in I}$, where $A_i = (S_i, X, Y, \delta_i, \lambda_i, S_{0i})$, $S_{0i} \in S_i$, is in one-to-one correspondence with a family of geometric images $\beta = \{\gamma_i\}_{i\in I}$ located respectively on geometric curves $L = \{L_i\}_{i\in I}$.

For the proof, see, for example, [12,14].

On the basis of Theorem 1, is proposed a method of recognizing of automatons, the laws of functioning of which are defined by geometric images located on analytically given curves. The method consists of the following steps:

Stage 1. Construction (selection) of a family $L = \{L_i\}_{i\in I}$ of geometric curves and the location on them of geometric images of the laws of functioning of automatons from the family of automatons $\alpha = \{A_i\}_{i\in I}$.

Stage 2. For the system of inequalities $\{L_i\} \cap \{L_j\} = \varnothing$, $i, j \in I$, $i \neq j$, is determined the family of solutions $\{\Delta_{ij}\}$, $i, j \in I$, $i \neq j$.

Stage 3. A segment $\Delta = \bigcap_{i\neq j} \Delta_{ij}$ is determined, which, by construction, satisfies the following conditions:

(1) If $\bigcap_{i\neq j} \Delta_{ij} \neq \varnothing$, then each point of the segment Δ , which is the first coordinate of points of geometric images of automatons from a family of automatons $\alpha = \{A_i\}_{i\in I}$, determines the solution of the problem of recognizing an automaton in a family of automatons by a simple unconditional experiment.

(2) If $\bigcap_{i\neq j} \Delta_{ij} = \varnothing$, then for the selected specific geometric curves L_i, $i \in I$, and the arrangement of geometric images of the laws of the functioning of automata on these curves, the solution of the problem of recognizing an automaton in a family of automatons does not exist by a simple unconditional experiment.

Stage 4. In accordance with the conditions $\Delta \neq \varnothing$ or $\Delta = \varnothing$ a specific solution to the problem of recognizing an automaton in a family of automatons is determined by a simple unconditional experiment, or it is concluded that for a family of automatons $\alpha = \{A_i\}_{i\in I}$, a selected family of geometric curves $L = \{L_i\}_{i\in I}$ and a chosen arrangement of geometric images on curves, the solution to the problem recognition of an automaton in a family of automata does not exist by a simple unconditional experiment.

Remark to a method. In the developed method, symbolic structures - input and output sequences - are determined not by their numbers according to the linear orders ω_1 and ω_2, but by the numbers associated with the numbers. The numbers are matched to the numbers of the input sequences based on their alignment with points equally spaced on the abscissa axis. To the numbers of the output sequences are mapped half-intervals on the ordinate axis.

4. Discussion

Consider an example of constructing a solution to the problem of recognizing an automaton in a family of automatons whose points of geometric images of the laws of functioning are located on the following geometric curves (see Figure 1): $y_1 = e^{(x-5.5)}$ (for an automaton $A_1 = (S_1, X, Y, \delta_1, \lambda_1, s_{01})$), $y_2 = 1 + \cos(\frac{x}{1.8} + 1.75)$ (for the automaton $A_2 = (S_2, X, Y, \delta_2, \lambda_2, s_{02})$), $y_3 = \left(\frac{x+1}{6}\right)^3$ (for the automaton $A_3 = (S_3, X, Y, \delta_3, \lambda_3, s_{03})$), $y_4 = \left(\frac{x - 2.8}{3}\right)^2$ (for the automaton $A_4 = (S_4, X, Y, \delta_4, \lambda_4, s_{04})$).

For the example under consideration, we will restrict ourselves to 16 points of the geometric image. The relationship between input sequences, numbers of input sequences and numbers associated with numbers of input sequences is shown in Table 1 (with | X | = 2). For output signals, each signal is associated with a half-interval of the form (α, β]. This allows each point in the selected area of the plane with the first coordinate, which is the number associated with the input sequence number, to determine the semi-interval (to which the second coordinate of the point belongs).

In the example under consideration, the number of output signals of the automaton is 74 and each output signal $y_i \in Y$, where $1 \le i \le 74$, is compared to a semi-interval of the form (α_i, β_i], where $\alpha_i = \frac{(i-1)}{10}$ and $\beta_i = \frac{i}{10}$. Table 2 shows the values of the functions $y_1 = e^{(x-5.5)}$, $y_2 = 1+\cos(\frac{x}{1.8}+1.75)$, $y_3 = \left(\frac{x+1}{6}\right)^3$, $y_4 = \left(\frac{x-2.8}{3}\right)^2$ in points, the first coordinates of which are mapped to the numbers of the input sequences (see Table 1).

As a result of the specified choice of the family of geometric curves y_1, y_2, y_3 and y_4 (respectively, for the automatons A_1, A_2, A_3, A_4) and the chosen arrangement of the geometric images of the automatons on the curves (the numbers are corresponded to the numbers of the input sequences based on their alignment with equidistant on the abscissa axis points) and the analysis of the curves, 7 solutions of the problem of recognizing an automaton in a family of automatons $\alpha=\{A_1, A_2, A_3, A_4\}$ were found by a simple unconditional experiment.

Table 1. Relationship of input sequences, number of input sequences and numbers associated with numbers of input sequences.

Numbers of input sequences	Input sequences	Numerical value
1	x_1	0
2	x_2	0,5
3	$x_1 x_1$	1
4	$x_1 x_2$	1,5
5	$x_2 x_1$	2
6	$x_2 x_2$	2,5
7	$x_1 x_1 x_1$	3
8	$x_1 x_1 x_2$	3,5
9	$x_1 x_2 x_1$	4
10	$x_1 x_2 x_2$	4,5
11	$x_2 x_1 x_1$	5
12	$x_2 x_1 x_2$	5,5
13	$x_2 x_2 x_1$	6
14	$x_2 x_2 x_2$	6,5
15	$x_1 x_1 x_1 x_1$	7
16	$x_1 x_1 x_1 x_2$	7,5

The solutions obtained using the developed method are the input sequences $p_1= x_1 x_2 x_1$, $p_2= x_2 x_1 x_1$, $p_3 = x_2 x_1 x_2$, $p_4= x_2 x_2 x_1$, $p_5= x_2 x_2 x_2$, $p_6= x_1 x_1 x_1 x_1$, $p_7= x_1 x_1 x_1 x_2$.

Table 3 shows the reactions of the automatons A_1, A_2, A_3, A_4 on 16 input sequences (based on the comparison to the numbers of the output signals $y_i \in Y$, where $1 \le i \le 74$, a semi-interval of the form (α_i, β_i], where $\alpha_i = \frac{(i-1)}{10}$ and $\beta_i = \frac{i}{10}$).

Table 2. Values of functions y_1, y_2, y_3, y_4 .

Numbers of input sequences	Numerical value	$y_1=e^{(x-5.5)}$	$y_2 = 1+\cos(\frac{x}{1.8}+1.75)$	$y_3 = \left(\frac{x+1}{6}\right)^3$	$y_4 = \left(\frac{x-2.8}{3}\right)^2$
1	0	0.004	0.821	0.004	0.871
2	0,5	0.006	0.558	0.015	0.587
3	1	0.011	0.329	0.037	0.36
4	1,5	0.018	0.151	0.072	0.187
5	2	0.030	0.039	0.125	0.071
6	2,5	0.049	0	0.198	0.01
7	3	0.082	0.037	0.296	0.004
8	3,5	0.135	0.148	0.421	0.054
9	4	0.223	0.325	0.578	0.16
10	4,5	0.367	0.553	0.770	0.321
11	5	0.606	0.816	1	0.537
12	5,5	1	1.093	1.271	0.81
13	6	1.648	1.362	1.587	1.137
14	6,5	2.718	1.604	1.953	1.521
15	7	4.481	1.799	2.370	1.96
16	7,5	7.389	1.933	2.843	2.454

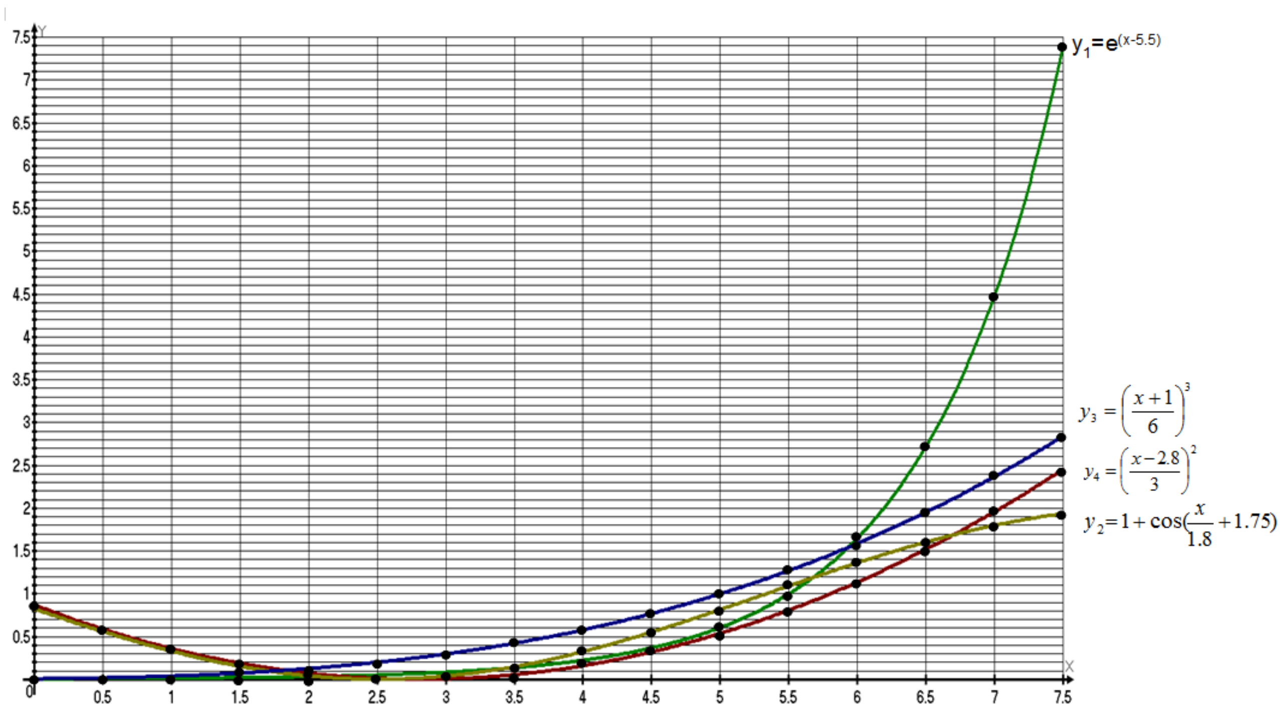

Figure 1. Geometric curves on which the geometric images of the laws of functioning of the automatons A_1, A_2, A_3, A_4 are located

Table 3. Reactions of automatons A_1 , A_2 , A_3 , A_4 .

Number of sequence	Numerical value	Input sequences	A_1	A_2	A_3	A_4
1	0	x_1	y_1	y_9	y_1	y_9
2	0,5	x_2	y_1	y_6	y_1	y_6
3	1	$x_1\ x_1$	y_1y_1	y_9y_4	y_1y_1	y_9y_4
4	1,5	$x_1\ x_2$	y_1y_1	y_9y_2	y_1y_1	y_9y_2
5	2	$x_2\ x_1$	y_1y_1	y_6y_1	y_1y_2	y_6y_1
6	2,5	$x_2\ x_2$	y_1y_1	y_6y_1	y_1y_2	y_6y_1
7	3	$x_1\ x_1\ x_1$	$y_1y_1y_1$	$y_9y_4y_1$	$y_1y_1y_3$	$y_9y_4y_1$
8	3,5	$x_1\ x_1\ x_2$	$y_1y_1y_2$	$y_9y_4y_2$	$y_1y_1y_5$	$y_9y_4y_1$
9	4	$x_1\ x_2\ x_1$	$y_1y_1y_3$	$y_9y_2y_4$	$y_1y_1y_6$	$y_9y_2y_2$
10	4,5	$x_1\ x_2\ x_2$	$y_1y_1y_4$	$y_9y_2y_6$	$y_1y_1y_8$	$y_9y_2y_4$
11	5	$x_2\ x_1\ x_1$	$y_1y_1y_7$	$y_6y_1y_9$	$y_1y_2y_{10}$	$y_6y_1y_6$
12	5,5	$x_2\ x_1\ x_2$	$y_1y_1y_{10}$	$y_6y_1y_{11}$	$y_1y_2y_{13}$	$y_6y_1y_9$
13	6	$x_2\ x_2\ x_1$	$y_1y_1y_{17}$	$y_6y_1y_{14}$	$y_1y_2y_{16}$	$y_6y_1y_{12}$
14	6,5	$x_2\ x_2\ x_2$	$y_1y_1y_{28}$	$y_6y_1y_{17}$	$y_1y_2y_{20}$	$y_6y_1y_{16}$
15	7	$x_1\ x_1\ x_1\ x_1$	$y_1y_1y_1y_{45}$	$y_9y_4y_1y_{18}$	$y_1y_1y_3y_{24}$	$y_9y_4y_1y_{20}$
16	7,5	$x_1\ x_1\ x_1\ x_2$	$y_1y_1y_1y_{74}$	$y_9y_4y_1y_{20}$	$y_1y_1y_3y_{29}$	$y_9y_4y_1y_{25}$

5. Practical Use

Construction and analysis of automata models of FPGA

Field Programmable Gate Array (FPGA) concern to a class of complex technical devices with large dimensions of a set of signals and memories. FPGA is complex digital integrated microcircuits consisting of programmed logic blocks and programmed connections between these blocks. FPGA are widely used not only for realization of simple controllers and the interface units, but also for digital processing signals, complex intellectual controllers, neurochips and in systems of mobile communication.

Mathematical models of integrated chips are the basic information for the decision of problems of development of chips, the analysis and technical diagnosing, optimization of structures and laws of functioning of integrated circuit. Field Programmable Gate Array (FPGA) concern to technical devices with enough complex structure, complex laws of functioning and the big dimension of sets of signals and memories (at present the industry lets out models of FPGA with more than 1000 pins). In this connection mathematical models of FPGA available now, particularly, the finite state machines, are not sufficient not only for the decision of problems, but even for explicit representation of FPGA.

In this parer is considered construction of mathematical model of FPGA in the form of symbolical and numerical graph (located on analytically set curves) and also a basic provisions of diagnosing of FPGA based on use of the apparatus of geometrical images of laws of functioning of state machines (automatons). Without loss of generality as an example in article is analyzed one of the most widespread classes of algorithms of digital processing of signals, realizing on FPGA– the algorithms based on application of orthogonal transformations.

Now FPGA is used for construction various on complexity and opportunities digital devices. Existing mathematical models of FPGA, in particular, the finite state machines, do not allow to represent obviously FPGA in view of their big dimension (existing FPGA contain up to 10 million logic gates, have more than 1000 pins and tens mbyte of the built in block memory). In this work is offered to use the mathematical apparatus of geometrical images of state machine (see [10]) for the representation and research of laws of functioning of FPGA, including for the decision of problems of technical diagnosing. In work [10] Tverdohlebov V.A. is shown, that, technical diagnosing of systems, which are characterized as large-scale or complex systems carried out in conditions of essential restrictions on mathematical models and means of diagnosing. FPGA as object of diagnosing does not suppose enough a full and exact intuitive review and formal representation by traditional means: tables, columns, the logic equations. Besides opportunities of means of diagnosing in each used interval of time are limited by supervision only parts of structure of object and supervision only some functions of object. The analysis of working capacity and localization of defects can be demanded on the interval of time somehow removed from the beginning of functioning of object. Technical diagnosing of FPGA cannot be carried out by homogeneous means of diagnosing. Only overlapping of testing, measurement of physical parameters, the analysis of processes of "decision" objects of diagnostic problems, optical survey and the signal system, etc. should form means of diagnosing.

The method of technical diagnosing of complex systems with use of the apparatus of geometrical images [10] includes construction of mathematical models of means of technical diagnosing in the form of communication of diagnostic interactions with reactions to them of object of diagnosing; construction of mathematical models in the form of geometrical images for object of the diagnosing, including development of geometrical images on the basis of interpolation and extrapolation; development of strategy of carrying out of diagnostic experiment on the basis of the analysis of geometrical images and realization of diagnostic experiment according to the developed strategy.

To illustrate the possible practical application of the apparatus of geometric images of automata in this paper is considered construction of mathematical model in the form of a geometrical image of the FPGA of family Xilinx Spartan II (see [22]), including development of geometrical images on the basis of classical methods of interpolation of Newton, Lagrange and Gauss (a detailed description of the method of synthesizing an automaton model of an FPGA in the form of a geometric image is not given due to limitations on the size of the article, it can be found, for example, in work [12]). One of the basic classes of algorithms of digital processing signals implementated by FPGA – the algorithms based on application of orthogonal transformations is considered: fast Fourier transformation (FFT), Hartly, EWT, Hadamard, Karhunene-Loev expansion etc. (Figure 2 represent as an example the algorithm based on orthogonal transformation - algorithm of 16-dot fast Fourier transformation (FFT)).

FPGA is programmed on realization of the specified algorithms with use of system Xilinx ISE 9.2i. Efficiency of the specified methods of interpolation for restoration of partially set geometrical images of laws of functioning is analyzed at a various arrangement and number of units of interpolation. In Figure 3 is represented the example (it is schematically shown) of comparison on an initial interval of an initial geometrical image and the geometrical image constructed with use of a method of interpolation of Newton (in the given example at programming of FPGA as off pins are used only 4 pin). Lack of a method of interpolation of Gauss unlike the considered methods of interpolation of Newton and Lagrange is restriction of an opportunity of its use only for a case of equidistant units of interpolation. (A detailed description of the research of the effectiveness of various interpolation methods as a means of completing the definition of automatons models can be found in work [12]). The given property imposes additional restrictions on allocation of units

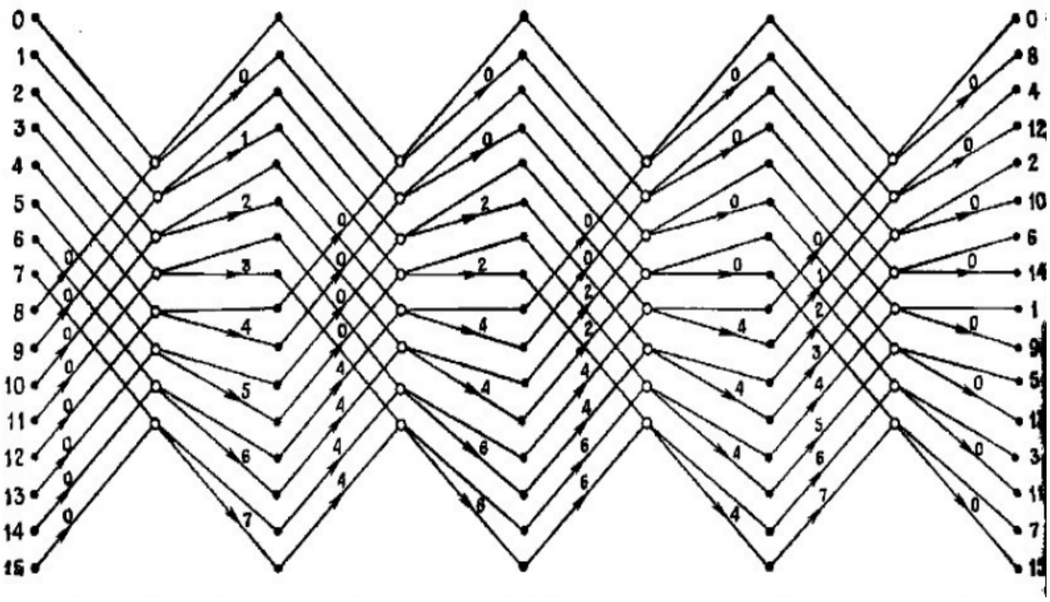

Figure 2. Algorithm 16-dot FFT on the basis 2 with constant structure, without replacement, with normal order and binary-inverse on an output (multipliers for decimation on time and frequency are shown).

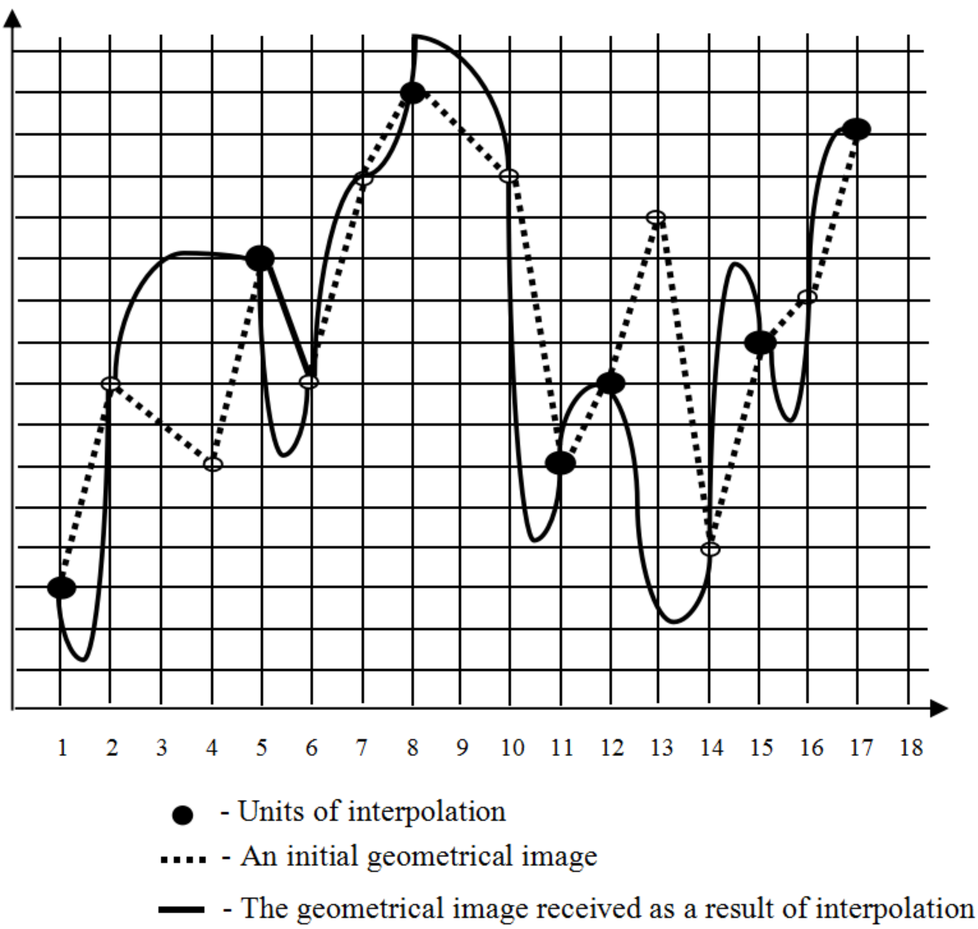

Figure 3. Initial pieces of an initial geometrical image and restored by means of a method of interpolation of Lagrange.

of interpolation. On the basis of the lead computing experiment it is shown, that for a considered class of algorithms the method of interpolation of Gauss yields the best results, than methods of interpolation of Newton and Lagrange. For each of the considered algorithms from 2 methods of interpolation (Newton and Lagrange) the most effective method is certain.

Base model of the discrete determined dynamic systems are the finite state machines. The analysis of such state machines can be made on the basis of research of the mathematical structures representing specificity of laws of functioning of the state machines. As such structures can be used geometrical images of laws of functioning of automata. Use of the apparatus of geometrical images for the definition of laws of functioning of the discrete determined dynamic systems allows to carry out search of diagnostic sequences effectively. For this purpose from set of considered malfunctions and to an efficient condition of system the mathematical model in the form of a geometrical image of laws of functioning of the state machine is compared with each malfunction. Points of geometrical images are assumed located on the curves which can be set analytically. At such way of the definition of diagnostic model search of diagnostic sequences is reduced to a finding of intervals on an axis of abscissa in which the geometrical curves representing mathematical models of malfunctions, have no even points. Effective search of such intervals can be carried out on the basis of the decision of systems of inequalities (or equalities).

6. Conclusions

In this paper we propose methods for recognizing automata models of systems defined by analytically specified geometric curves (on which points are located, interpreted by using the apparatus of geometric images of automata as a pairs of automata mappings). It is proposed to carry out the recognition of automata by their geometric images based on the decomposition of the original (numerical) sequences into a set of characteristic sequences and their subsequent analysis. In a number of cases, the combined use of the proposed methods for recognizing automata makes it possible to effectively overcome the dimensionality barrier of automaton models of systems. One of the problems of interest for further research is the problem of developing a search method under the conditions of specific variants of restrictions on the use of means of control and diagnostics of variants of specific sets of observation sites of control and diagnostic information.

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