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ARTICLE

The Correlation of Gyroscope Axial Velocities

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ABSTRACT

In engineering, all movable expanse revolving objects manifest gyroscopic effects. These effects are created by the action of the outer load on the revolving items whose rotating mass originates eight inertial torques about two axes. Two torques of centrifugal forces, one torque of the Coriolis force originated by the rotating distributed mass, and the torque of the change in the angular momentum of the center mass act about each axis. The inertial torques activate rotations of the gyroscope by the determined correlation. Inertial torques depend on their geometry and orientation at the spatial coordinate system. The known analytical model for the rotation of the revolving disc about axes contains a mechanical error. This error was obtained by the incorrect integration of the centrifugal inertial torque. The corrected inertial torque yields the accurate expression for the interacted rotations of the revolving disc about axes.

Keywords: Gyroscope theory; Inertial torque; Correlation of angular velocities

1. Introduction

Physicists and mathematicians started studying the gyroscopic effects of the revolving disc, beginning from the time of intensive applications of engineering discoveries for human economics. They developed only one part of the mathematical model which is the inertial torque presented by the change in the angular momentum (L. Euler). The physics of other gyroscopic effects were explained intuitively as the operation of the inertial torques that are unacceptable for the analytical modelling of their processes [1-4]. The analytical formulation of the gyroscopic effects remained unsolved until our time [5-8].

The studies of gyroscopic effects show their foundation is based on several principles of classical mechanics that were developed over three centuries [5-8]. One of the physical principles of gyroscopic effects is the mechanical energy conservation law was for-

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mulated at the beginning of the 20 century. Over the last hundred years, researchers had the time and opportunities to derive mathematical models for the gyroscope theory but did not do it [9-13]. Gyroscopic effects were resolved finally in our time and published in several manuscripts and one book.

The gyroscope theory shows sophisticated rotations of the revolving disc based on several principles of physics mechanics. These principles are the centrifugal, and Coriolis forces, the change in the angular momentum, and kinetic energy conservation which were developed over three hundred years. The gyroscopic effects are the manifestation of the operation of principles pointed out above that is presented by the action of the sets of inertial torques and correlated rotations of the revolving disc [14]. One set presents four inertial torques operating about one axis, which are generated by two centrifugal and one Coriolis force, and the torque of the change in the angular momentum. One torque of centrifugal force act on two axes and the other torques act on one axis. The expressions of the gyroscopic inertial torques and their operations are shown in **Table 1** [15,16].

Table 1. Inertial torques operating on the revolving disc with the angle $\gamma = 0$.

Inertial torques generated by	Operation	Expression
Centrifugal forces	Opposition	$T_{ct.i} = (4\pi^2 / 9)J\omega\omega_i$
	Precession	
Coriolis forces	Opposition	$T_{cr.i} = (8/9)J\omega\omega_i$
Change in angular momentum	Precession	$T_{am.i} = J\omega\omega_i$

Table 1 contains expressions ω and ω_i which are the velocity of the revolving disc about axis *oz* and *i*, accordingly; *J* is the moment of inertia of the revolving disc. The operation of the inertial torques and rotations of the revolving disc about axes of the spatial coordinates system are shown in **Figure 1** [16].

Figure 1 contains the following expressions: *T*, $T_{ct.i}$, $T_{cr.i}$, $T_{am.i}$ are the external, centrifugal, Coriolis torque, and the torque of the change in the angular momentum, accordingly, γ is the angle of the revolving disc tilt, and other components are as specified above.

The mechanical energy conservation law for revolving items maintains the equality of the potential and kinetic energies about axes of rotation [16]. This canon enables defining the dependency for the correlated action of the inertial torques of the revolving disc and its rotations about two axes. The known analytical model for the correlation of the revolving disc rotations around axes contains errors [15]. The presented manuscript shows the origin of the incorrect solution of the correlation of the angular velocities for the revolving disc about axes of rotation.

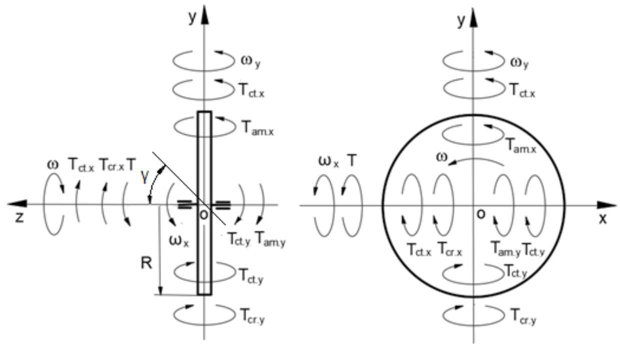

Figure 1. Outer and inertial torques operating on the revolving disc and its rotations.

2. Methodology

The mechanical energy of the body remains constant in any of its locations in the expanse by the mechanical energy conservation law. The gyroscopic effects express the operation of inertial torques of the revolving disc that contains the kinetic energies of their axial rotations. These energies are equal and express the canon of the mechanical energy conservation law [15]. The expressions of inertial torques operating on the revolving disc of the inclined axis on the angle γ reflect its kinetic energies about axes which are shown below and in **Figure 1**:

$$-T_{ct.x} - T_{cr.x} - T_{ct.y} - T_{am.y} = T_{ct.x}\cos\gamma + T_{am.x}\cos\gamma - T_{ct.y} - T_{cr.y} \quad (1)$$

where the expression of the torques is as specified in **Table 1** and **Figure 1**.

Equation (1) expresses torques operating about axes in the spatial coordinate system and differs from similar ones in publications [15]. The method of the causal investigatory correlation enables defining of the looped operation of the inertial torques. The revolving disc tilts on the angle *γ* in the counter clockwise direction by the operation of the outer torque T. The external torque T activates the inertial opposition torque $T_{r.x} = T_{ct.x} + T_{cr.x}$ of the centrifugal and Coriolis forces of the left side of Equation (1) originating from the rotating mass of the revolving disc. The opposition torque T_{rx} of axis ox originates the precession torques T_{px} of the centrifugal force and the change in the angular momentum of axis oy. The torques T_{px} of the right side of Equation (1) is multiplied by cosγ, $T_{p.x} = T_{ct.x}\cos\gamma + T_{am.x}\cos\gamma$ which originates the opposition torques $T_{ct.y}$ and $T_{cr.y}$ of the centrifugal and Coriolis forces without cosγ. The opposition torques $T_{ct.y}$ and $T_{cr.y}$ are produced by the torque $T_{p.x}$ operating on axis oy. The resulting torque $T_{res.y} = T_{ct.x}\cos\gamma + T_{am.x}\cos\gamma - T_{ct.y} - T_{cr.y}$ of the right side of Equation (1) operating on axis oy originates the precession torque $T_{p.y} = T_{ct.y} + T_{am.y}$ of the centrifugal force and the change in the angular momentum of axis ox. The torques $T_{p.y}$ is combined with the opposition torque $T_{rx} = T_{ct.x} + T_{cr.x}$. Then the resulting torque of axis ox is presented by the expression $T_{res.x} = T_{ct.x} + T_{am.x} + T_{ct.y} + T_{cr.y}$, which formulates with the external torque T, the opposition torque $T_{r.x}$, and precession torques $T_{r.x}$ of axes ox and oy, accordingly. The looped system of the inertial torques of the revolving item shows the equality of its kinetic energies of axis ox and oy. The expressions of the inertial torques of each axis contain the velocities ω_x and ω_y of two axes ox and oy that demonstrate their correlations. The expressions of the inertial torques (**Table 1**) are substituted into Equation (1) that yields:

$$-\frac{4\pi^2}{9}J\omega\omega_x - \frac{8}{9}J\omega\omega_x - \frac{4\pi^2}{9}J\omega\omega_y - J\omega\omega_y = \frac{4\pi^2}{9}J\omega\omega_x\cos\gamma + J\omega\omega_x\cos\gamma - \frac{4\pi^2}{9}J\omega\omega_y - \frac{8}{9}J\omega\omega_y \tag{2}$$

Simplification of Equation (2) yields:

$$\omega_\gamma = -[4\pi^2 + 8 + (4\pi^2 + 9)\cos\gamma]\omega_x \tag{3}$$

where the sign (-) is removed because shows the direction of the inertial torque and does not relate to the rotations of the revolving disc.

Equation (3) is the correlation of the velocities for the revolving disc as a function of the angle γ of the inclined disposition about axis ox. For the angle $\gamma = 0$, Equation (3) is $\omega_y = (8\pi^2 + 17)\omega_x$. The correlation of the disc velocities ω_y/ω_x of two axes by Equation (3) is shown in **Figure 2**.

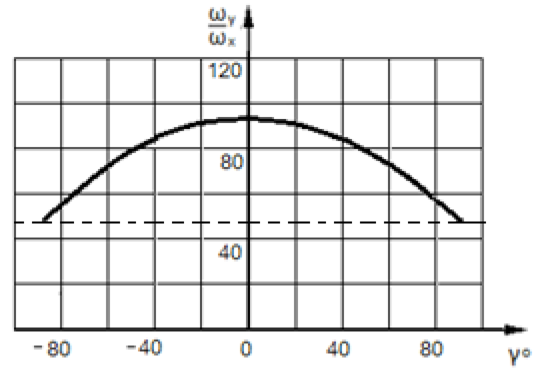

Figure 2. The correlation of the disc axial velocities ω_y/ω_x as a function of the angle γ.

Equation (3) does not maintain when the revolving disc axis has a vertical disposition ($\gamma = \pm 90^{o}$) that gives $\omega_y = 0$ because inertial precession torques act about axis oy. The small tilt of the revolving disc of horizontal disposition on axis ox yields the turn on 90^{o} about axis oy. The expression $\varphi = -(8\pi^2 + 17)\gamma$ yields $\gamma = 0.938^{o}$ = 58'22''. This correlation is validated in the rotations of the gyroscopic frames that call gimbals. The small swing of the gyroscope's outer frame on $\gamma = 0.938^{o}$ yields the high rotation on $\varphi = 90^{o}$ of the inner frame which measurement is problematic on the movable parts. The laboratory gyroscope can validate the rotation of the inner frames from $\varphi = -90^{o}$ to $\varphi = 90^{o}$ and the rotation on the angle $\gamma = 1{,}876^{o} = 1^{o}52'33''$ of the outer frames. The rotations of the gyroscope frames confirm the correctness of Equations (2) and (3).

Figure 3 shows the change in the angular dispositions of the inner and the outer gyroscope frames. The theoretical and practical tests discover the nature of gyroscopic frame rotations that were one in the series of former unsolved gyroscopic effects.

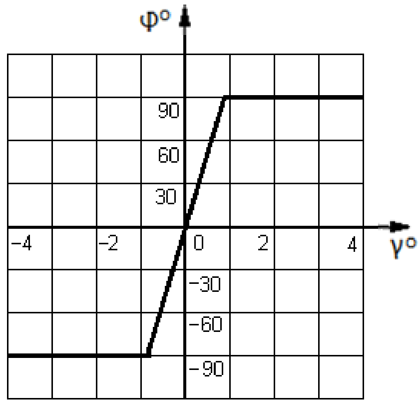

Figure 3. The turn of the outer frame to γ versus the inner one on φ for the revolving disc.

3. Results and discussion

The first publications related to gyroscopic effects contain unpleasant mistakes of the pioneering work for complex problems. The mistakes are presented by the incorrect mathematical processing of a complex integral equation for the inertial torque originated by the centrifugal forces of the rotating mass of the revolving disc [14]. This mathematical mistake yielded the incorrect solution for the correlation of gyroscope axial rotations. The exact expression for inertial centrifugal torque gives the twice value for the published incorrect expression. The exact centrifugal torque decreases twice the velocity of the gyroscope about axis *ox* and increases twice the velocity about axis *oy*. The recorded tests of the gyroscope rotation about axis *oy* give the same result as for the test with the incorrect expression of the centrifugal torque. The gyroscope angular velocity about axis *ox* did not measure because of the too-small turn which was problematic technically for the movable parts. The correction of the theory of gyroscopic effects enables avoiding criticism from readers.

4. Conclusions

The breakthrough theory of gyroscopic effects for revolving items can solve all problems for gyroscopic devices. This theory yields a new chapter in the mechanics of dynamics for rotating bodies and closes one gap in science. Engineering science received a new method for an analysis of the inertial torques originated by the rotating items and computing dynamical parameters of gyroscopic effects. The first publications of the theory of gyroscopic effects contain the incorrect expressions for the centrifugal torque and the related correlations. The corrected expressions will be positively used in practice and educational processes.

Conflict of Interest

There is no conflict of interest.

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