

REVIEW**Convergence Proving of the Theoretical & True Elongation Inequalities by Derivation and Analogy****Run Xu***

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ABSTRACT

According to $LN\varepsilon$, theoretical & true elongation of tensile, and by adopting the increasing function of formulas with the derivation and analogy methods, the elongation formula of $0 < (1+\varepsilon)^{1/\varepsilon} < e$ & $0 < \varepsilon^{1/\varepsilon} < 1$ & four convergences are deduced too when $\varepsilon > 1$ and $0 < \varepsilon < 1$. The inequalities of $LN\varepsilon < \varepsilon$ and $LN(1+\varepsilon) < \varepsilon$ and $LN(1+\varepsilon) > LN\varepsilon$ are deduced if $\varepsilon > 1$ and $0 < \varepsilon < 1$ in material dynamics. Finally the conclusions of $LN\varepsilon < \varepsilon$ and $LN\varepsilon < LN(1+\varepsilon) < \varepsilon$ are deduced together if $\varepsilon > 1$ and $0 < \varepsilon < 1$.

1. Introduction

Under the condition of density invariance, the convergence of the real and theoretical elongation inequality in material mechanics is proved. When the elongation is greater than 0.15~0.2, there is no comparison between the true elongation and the theoretical elongation. [1-3] through the function analysis to judge their relationship of increasing function, so as to determine their comparison, in order to clarify their mathematical

comparison. Because we want to know the true elongation of $LN(1+\varepsilon)$ in the mechanics of materials. Although the phenomenon can be known from the experiment, it has not been proved from the mathematical relationship. Therefore, this paper abstracts the phenomenon and proves it mathematically, and finds that there is a certain relationship between them. That is, the true elongation is greater than the theoretical elongation, so that the experimental research becomes mathematical theory plasticity, which is a big proof of this paper. Prove "1 + 1 = ?" like goldbach's

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conjecture. In 1742, he wrote to Euler to talk about it. Its mean is what two prime numbers add? From the point of view of mathematics, finally "1 + 2" were proved by Chen Jingrun in a large chapter. It means that any bigger than two is presented by a prime number and multiplying two prime number. But it has some key points for elongation to prove successful, so from the increasing function and analogy is the key to this paper, prove that $\ln(1 + \epsilon) < \epsilon$ is the core of this article, the last two induced mathematical inequality can also induce even dozens of inequality. Its proof process is not as long as Chen jingrun wrote so deep. The same is true for shrinkage rates, which can be traced back to Chinese database natural science full text 2020.1 (3).

We use these formulas to understand the internal relation problem, carry on the derivation and the analogy method. We prove that $y = \ln(1+\epsilon)$ is an increasing function. Critical problems such as $\ln \epsilon < \ln(1+\epsilon) < \epsilon$ when $\epsilon > 0$ are proved.

2. The Proof of $0 < (1+\epsilon)^{1/\epsilon} < e$ & $0 < \epsilon^{1/\epsilon} < 1$

2.1 The Proof of $y = \ln \epsilon$ being Increasing Function

The discussion situation here is $\epsilon \geq 1$.

If $\epsilon > 1$ has $\ln \epsilon > 0$ (1)

Supposes $y = \ln x = \ln \epsilon$

has $\frac{dy}{dx} = \frac{d(\ln \epsilon)}{d\epsilon}$ (2)

since $y = \frac{d(\ln \epsilon)}{d\epsilon} = \frac{1}{\epsilon}$

Due to $\frac{d(\ln \epsilon)}{d\epsilon} > 0$ (3)

& $0 < 1/\epsilon < 1$ (4)

$y = \ln x$ is ncreasing function.
Its maximum is 1.

2.2 The Proof of $Y = \ln(1 + \epsilon)$ being Increasing Function

If $\epsilon > 1$ (5)

$\ln \epsilon > 0$ (6)

Has too $\ln(1+\epsilon) > \ln \epsilon$ (7)

supposes $y = \ln(1+x)$

so $\frac{dy}{dx} = \frac{d[\ln(1 + \epsilon)]}{d\epsilon} = \frac{1}{1 + \epsilon}$ (8)

since $\frac{dy}{dx} > 0$ (9)

So that $y = \ln(1+x)$ is increasing function.

2.3 Proves $\ln(1+\epsilon) < \ln \epsilon$

since $\ln(1+\epsilon)/(1+\epsilon) < \ln(1+\epsilon)/\epsilon$ (10)

If $\epsilon > 1$ according to $\epsilon t = \ln(1+\epsilon) > \ln \epsilon$ (11)

has $\ln(1+\epsilon) / \ln \epsilon > \ln \epsilon / \ln \epsilon = 1$. (12)

because $\ln(1+\epsilon)$ and $\ln \epsilon$ is increasing function.
From above it has
from (11) it has

$\ln(1+\epsilon) / \ln \epsilon > 1$ (13)

So that $\ln(1+\epsilon) > \ln \epsilon$ (14)

That is the first result in this paper.

2.4 Proves $\ln(1+\epsilon) < \epsilon$

2.4.1 When $\epsilon > 1$

From $1/\epsilon < 1$ (15)

According to the above increasing function it has

$\ln(1+\epsilon)/\epsilon < \epsilon / \ln \epsilon$ (16)

This is the comparison among elongation and true stress when elongation $\epsilon \geq 1$, the above inequality(15) is gained upon inequalities comparison. The relationship may be seen in Figure 1, the biggest difference among $\ln \epsilon$ & $\ln(1+\epsilon)$ is 25.

$\epsilon / \ln \epsilon < \epsilon / \epsilon = 1$ (17)

According to equations (15) & (16) it has

$1/\epsilon \ln(1+\epsilon) < 1$ (18)

ie. $\ln(1+\epsilon) < \epsilon$ (19)

This is the important inequality result in this paper among true stress and engineering one when $\epsilon > 1$.

2.4.2 When $0 < \epsilon \leq 1$

Since $\epsilon t = LN(1+\epsilon) > LN\epsilon$ (23)

has $LN(1+\epsilon) / LN\epsilon > LN\epsilon / LN\epsilon = 1$ (24)

Has too $LN(1+\epsilon) > LN\epsilon$ (25)

If $0 < \epsilon < 1$ because $1/\epsilon > 1$

in equation(12) it is known that $1/\epsilon > \epsilon$ (26)

This is the relationship between true and engineering elongation when $\epsilon > 1$. The above inequality (26) and below (27) is obtained from this search for plasticity to compare which result the inequality (31). the relationships is in Figure 2, the biggest difference is 25.

According to equation, it has been known that when $0 < \epsilon < 1$ it has $LN\epsilon < 0$ ie. $1/\epsilon > 1$,

hence $1/\epsilon LN\epsilon < 1$ (27)

has $LN\epsilon < \epsilon$ (28)

According to $1/\epsilon LN(1+\epsilon) < LN2 < 1$ (29)

and $1/\epsilon > \epsilon$ since $1/\epsilon > 1 > \epsilon, LN(1+\epsilon) > 0$

$0 = LN1 < 1/\epsilon LN(1+\epsilon) < 1/\epsilon LN2 < 1$ (30)

$LN(1+\epsilon) > 1/\epsilon$

Hence $LN(1+\epsilon) < \epsilon$ (31)

It is a important result deduced in this paper when $0 < \epsilon \leq 1$.

From equation (25) & $LN\epsilon < 0$ it has been known

$LN(1+\epsilon)/\epsilon < -LN(1+\epsilon)/LN\epsilon < \epsilon / -LN\epsilon < \epsilon / \epsilon = 1$ (31)

$-LN(1+\epsilon)/LN\epsilon < -\epsilon / LN\epsilon < \epsilon / \epsilon = 1$ (32)

Hence according to (25) it has always

$LN(1+\epsilon) > LN\epsilon$ (33)

This is the results between $LN\epsilon$ and $LN(1+\epsilon)$ mathematics when $0 < \epsilon \leq 1$.

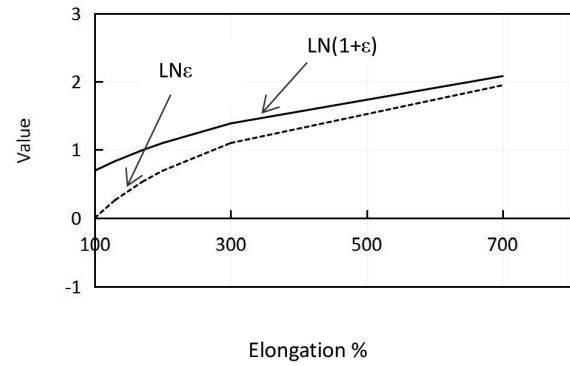


Figure 1. The relations of function of ϵ , $LN\epsilon$ & $LN(1+\epsilon)$ if $\epsilon > 1$

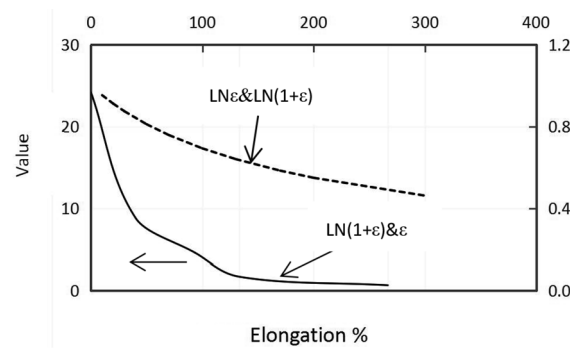


Figure 2. The relations of function of ratio with ϵ , $LN\epsilon$ & $LN(1+\epsilon)$ If $\epsilon > 1$

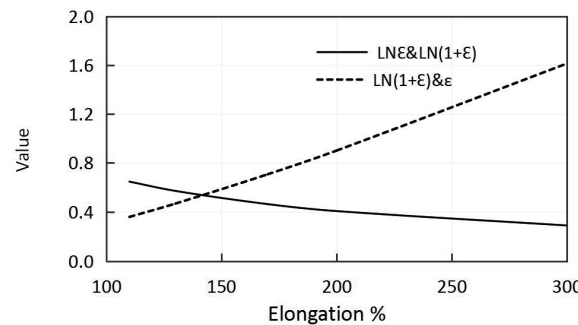


Figure 3. The relations of function of difference with ϵ , $LN\epsilon$ & $LN(1+\epsilon)$ If $\epsilon > 1$

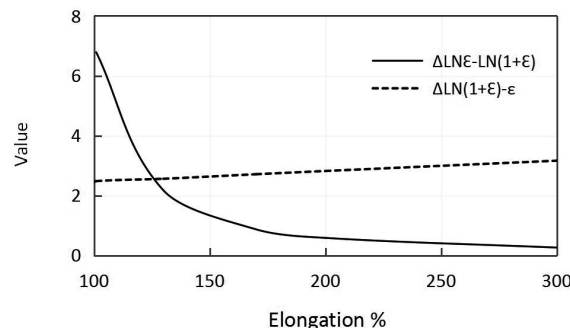


Figure 4. The relations of function of deviation with ϵ , $LN\epsilon$ & $LN(1+\epsilon)$ If $\epsilon > 1$

2.5 Prove $0 < (1+\epsilon)^{1/\epsilon} < e$ (49)
 $LN(1+\epsilon)^{1/\epsilon} < 1 = LNe$

Since $LN(1+\epsilon)/|LN\epsilon| > \epsilon/|LN\epsilon|$ (34) *i.e.*
 $(1+\epsilon)^{1/\epsilon} < e$ (50)

From equation(13) it has

$1/|LN\epsilon| > 1/\epsilon$ (35) $LN(1+\epsilon)/\epsilon > LN1/\epsilon = 0$ (51)

hence $|LN\epsilon| > \epsilon$ (36) So $0 < (1+\epsilon)^{1/\epsilon} < e$ (52)

Ie $LN(1+\epsilon) < -LN\epsilon$ (37)

It has that $(1+\epsilon)^{1/\epsilon} < e$, when $1 > \epsilon > 0$ it has $LN\epsilon < 0$; when $\epsilon > 1$ it has $LN\epsilon > 0$. This is discussed above. when $0 < \epsilon < 1$ it has in(12)and(13)

$(1+\epsilon)^{1/\epsilon} < 1$ (38)

From equation(10) it has known that

$\epsilon^{1/\epsilon} < (1+\epsilon)^{1/\epsilon} < 1$ (39)

When $0 < \epsilon < 1$ it has known that $y' = \epsilon^x LN\epsilon > 0$, so that $y = \epsilon^x$ is increasing function. hence

$0 < \epsilon^x$ (40)

Hence it is been proven as below

$0 < \epsilon^{1/\epsilon} < 1$ (41)

From (12) it has know that

$LN(1+\epsilon)^{1/\epsilon} < 1 = LNe$ (42)

i.e.

$(1+\epsilon)^{1/\epsilon} < e$ (43)

In terms of the above proven $LN(1+\epsilon)$ being increasing function it has

$LN(1+\epsilon)/\epsilon > LN1/\epsilon = 0$ (44)

Due to $LN1/\epsilon = 0$ (45)

It has $0 < (1+\epsilon)^{1/\epsilon} < e$ (46)

This is a mathematical inequality in this paper, it expresses that below relationship $(1+\epsilon)^{1/\epsilon} < e$ and $0 < (1+\epsilon)^{1/\epsilon}$.

It expresses that $(1+\epsilon)^{1/\epsilon}$ mathematical meaning in the scope of 0 & $e=2.7$.

$0 < 1/(1+\epsilon)$ (47)

So that $LN(1+\epsilon) < \epsilon$ (48)

i.e.

2.6 Prove $0 < \epsilon^{1/\epsilon} < 1$

It is known above that $(1+\epsilon)^{1/\epsilon} < e$, if $1 > \epsilon > 0$, $LN\epsilon < 0$; If $\epsilon > 1$, $LN\epsilon > 0$. from(12)and(10)it has

$(1+\epsilon)^{1/\epsilon} < 1$ (53)

From(10)& (12) it has

$\epsilon^{1/\epsilon} < (1+\epsilon)^{1/\epsilon} < 1$ (54)

If $\epsilon \geq 1$ $dy/dx = \epsilon^x LN\epsilon > 0$, so $y = \epsilon^x$ is increasing function. hence

$0 < \epsilon^x$ ----- (55)

It proves the relationship of $0 < \epsilon^{1/\epsilon} < 1$ (56)

3. Discussion

Figure 1~4 is the comparison of the true elongation and the elongation when the is not less than 1. The inequality (13) above and the inequality (14) below are the plastic relations obtained by comparing the inequality (17). Their sizes are shown in figure 1, and the comparison between the two is shown in figure 2. The maximum ratio between $LN\epsilon$ and $LN(1+\epsilon)$ is over 25. According to figure 1, $LN(1+\epsilon)$ is greater than $LN\epsilon$ and the convergence trend is 700%.

The biggest difference is shown in Figure 3 for $LN(1+\epsilon)$ and ϵ is 4 for $LN\epsilon$ and $LN(1+\epsilon)$ is 2. As shown in figure 4 the maximum deviation for $LN(1+\epsilon)$ and ϵ is 7 for $LN\epsilon$ and $LN(1+\epsilon)$ is 1. Therefore, if the elongation is greater than 1, the value drops rapidly. When the elongation is greater than 130%, the trend is flat.

Figure 5~8 shows the comparison of the true elongation and the elongation when the is less than 1. The inequality (13) above and the inequality (14) below are the plastic relations obtained by comparing the inequality (17). Their sizes are shown in figure 5, and the comparison between the two is shown in figure 6. The maximum ratio for $LN\epsilon$ and $LN(\epsilon)$ is 25. According to Figure 5, $LN(1+\epsilon)$ is greater than $LN\epsilon$.

The biggest difference is shown in Figure 7 for $LN(1$

+ ϵ) was 0.5 for $\ln(1+\epsilon)$ and $\ln \epsilon$ and $\ln(1+\epsilon)$ is 2.5. As shown in figure 8 the maximum deviation for $\ln(1+\epsilon)$ and $\ln \epsilon$ is 7 for $\ln \epsilon$ and $\ln(1+\epsilon)$ is 2. So if the elongation is less than 1, the value drops sharply. When the elongation is less than 70%, the trend is flat.

Because $dy/dx = 1/\epsilon x^{1/\epsilon-1} > 0$, when $\epsilon > 1$ $y = x^{1/\epsilon}$ is increasing function. at the same time to prove the inequation (56).

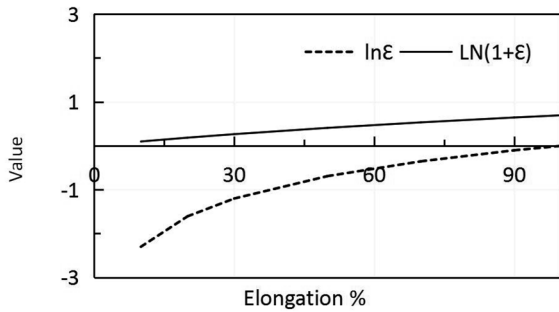


Figure 5. The relations of function of ϵ , $\ln \epsilon$ & $\ln(1+\epsilon)$ if $\epsilon < 1$

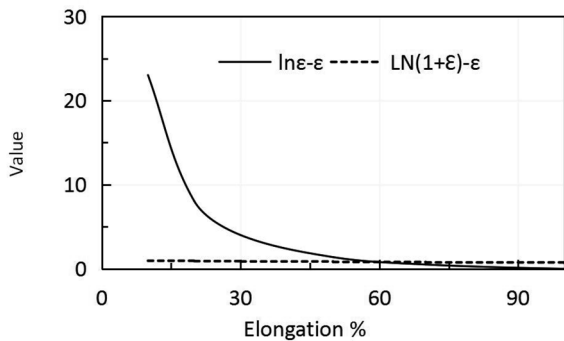


Figure 6. The relations of function of ratio with ϵ , $\ln \epsilon$ & $\ln(1+\epsilon)$ if $\epsilon < 1$

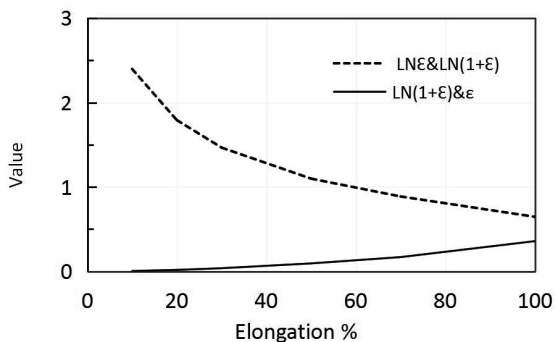


Figure 7. The relations of function of difference with ϵ & $\ln \epsilon$, $\ln \epsilon$ & $\ln(1+\epsilon)$ if $\epsilon < 1$

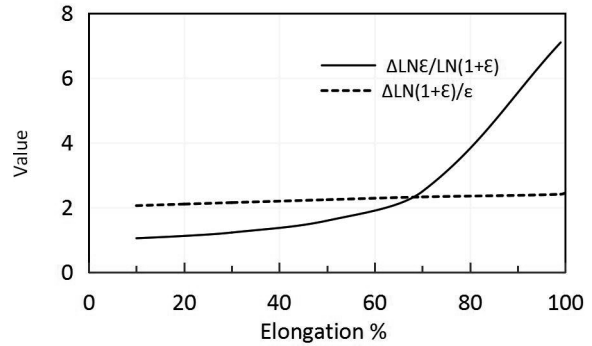


Figure 8. The relations of function of deviation with ϵ & $\ln \epsilon$, $\ln \epsilon$ & $\ln(1+\epsilon)$ if $\epsilon < 1$

4. Conclusions

1. The inequality between true and theoretical elongation in material mechanics is proved. The inequality between $\ln(1+\epsilon) > \ln \epsilon$ and $\ln(1+\epsilon) < \epsilon$ is proved when $0 < \epsilon$.

2. The convergence of $0 < (1+\epsilon)^{1/\epsilon} < e$ & $0 < \epsilon^{1/\epsilon} < 1$ is derived from $y = \epsilon^x$ & $y = x^{1/\epsilon}$ is an increasing function. In mechanics of materials prove $\ln(1+\epsilon) < \ln \epsilon$, $\ln(1+\epsilon) < \epsilon$ and $\ln(1+\epsilon)/(1+\epsilon) < \ln(1+\epsilon)/\epsilon$, $\ln(1+\epsilon)/\epsilon < \ln(1+\epsilon)/\ln \epsilon < \epsilon/\ln \epsilon < \epsilon/\epsilon$ is established.

3. the greatest difference in $\ln(1+\epsilon)$ and ϵ is 4 while $\ln \epsilon$ and $\ln(1+\epsilon)$ is 2. The greatest deviation in $\ln(1+\epsilon)$ and ϵ is 7 while $\ln \epsilon$ and $\ln(1+\epsilon)$ is 1. The maximum ratio between $\ln \epsilon$ and $\ln(1+\epsilon)$ is more than 25.

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