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REVIEW

A Comprehensive Overview of the ELECTRE Method in Multi-Criteria Decision-Making

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ABSTRACT

The ELECTRE (ELimination Et Choix Traduisant la REalite) method has gained widespread recognition as one of the most effective multi-criteria decision-making (MCDM) methods. Its versatility allows it to be applied in a wide range of areas such as engineering, economics, business, environmental management and many others. This paper aims to provide an overview of the ELECTRE method, including its fundamental concepts, applications, advantages, and limitations. At its core, the ELECTRE method is an outranking family of MCDM techniques, which allows for the direct comparison of alternatives based on a set of criteria. The method takes into account the preferences and importance of decision-makers and generates a ranking of the alternatives based on their relative strengths and weaknesses. The ELECTRE method is a powerful tool for decision-making, and its applicability to a wide range of fields demonstrates its versatility and adaptability. By understanding its concepts, applications, merits, and demerits, decision-makers can use the ELECTRE method to make informed and effective decisions in a variety of contexts.

Keywords: Decision making; Multi criteria decision making; ELECTRE method; ELimination Et Choix Traduisant la REalite method; Multi attribute decision making

1. Introduction

The ELECTRE (ELimination Et Choix Traduisant la REalite) method is a non-compensatory multi-attribute decision making (MADM) method that works based on alternatives' comparison considering individual criteria [1]. The main point that distinguishes the ELECTRE from compensatory methods

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such as simple additive weighting (SAW) [2] is that in the ELECTRE method, the weights are not criteria substitution rates instead the weights are coefficients of importance. Furthermore, in this method, it is not possible to offset a very bad value on a criterion with good ones on other criteria. It should be added that a few references consider the ELECTRE as a partially compensatory method and argued for placing it in the non-compensatory subgroup [3].

The initiated ELECTRE evaluation method was introduced in 1966 by Benayoun et al. [4]. This was the report of a real word problem in the European consultancy company SEMA, although the first journal article was published by Roy in 1968 to describe the method in detail and it was renamed later as ELECTRE I [5]. Then, different authors published manifold studies on this method based on similar theories to establish preference and outranking relations, and also to support decision-makers by making consistent exploration and analysis. Different ELECTRE versions were introduced by several scholars such as ELECTRE I, II, III, IV, IS, TRI, TRIB, and TRI-C. In this paper, the process steps of three methods that are between the most commonly used versions including ELECTRE I, II, and III are described in detail. ELECTRE I was developed as one of the earlier versions of ELECTRE methods, and ELECTRE II was developed in 1971 by Roy and Bertier as an improved and promoted version of ELECTRE I. ELECTRE III was developed based on the main principles of ELECTRE II, but instead of classical true criteria, it applies the pseudo-criteria and makes the presentations of the outranking relations in a fuzzy form possible [3,5,6]. These methods work based on selecting a desirable option between different alternatives meeting two separate demands including:

- Concordance preference above many evaluation benchmarks;
- Discordance preference under any optional benchmark.

The ELECTRE I method contains three concepts including a threshold value, concordance, and discordance indexes. In the ELECTRE II method, the concepts of concordance and discordance indexes incorporate weak and strong relationships that are extremely opposite relationships and result in two final rankings known as weak-ranking and strong-ranking [6]. On the other hand, ELECTRE III outperforms the former ones as it can deal with uncertain, imprecise, and inaccurate data using the fuzzy concept [7]. To sum up, although the ELECTRE method's versions differ in several aspects, for example some are designed for utilization in selection problems (such as ELECTRE I) and some are applicable for ranking purposes (such as ELECTRE II and III), and even for assignment problems (using ELECTRE TRI) all of them are included in the outranking multi-criteria decision making (MCDM) methods' family [1,3,5].

While there have been previous reviews on ELECTRE methods, this review provides a fresh perspective by consolidating and presenting up-to-date information on the topic. This review aims to provide an up-to-date understanding of the ELECTRE method and to discuss the basic concepts, principles, and steps of the ELECTRE method, along with variations and extensions proposed by researchers. Additionally, the review provides process steps of the ELECTRE method and suggests potential areas for future research. The following sections are provided to discuss the application areas of the ELECTRE methods, their advantages and disadvantages, and finally the ELECTRE principles and process steps. This comprehensive analysis provides a valuable resource for researchers, practitioners, and decision-makers seeking to explore the potential of ELECTRE methods in various domains and make informed choices about their implementation.

2. Application areas

The ELECTRE evaluation methods have manifold application areas and are widely used decision-making methods that can be applied in a vast range of areas from transport to environmental protection programs [6]. Here, to gain a better overview of the application areas, the results of a comprehensive review by Govindan and Jepsen [5] are summarized. They analyzed the literature worked based on the ELECTRE method considering different categories. The applied papers are considered here that focus on numerical research studies using the ELECTRE, developing algorithms, etc. **Table 1** shows a summary of the 13 categories of the ELECTRE application areas, and the description of the articles are included in the categories. In this study, they introduced "natural resources and environmental management (NRE)" as the most popular application area.

3. Advantages and disadvantages

The ELECTRE method is known for its various advantages, although it also exhibits certain weaknesses. This section presents a concise list of the merits and demerits associated with the ELECTRE evaluation. Considered as one of the best Multiple Criteria Decision Making (MCDM) methods, ELECTRE possesses the following strengths: it employs a simple logic; it fully utilizes the information contained in the decision matrix; it incorporates a refined computational process [8]; it utilizes an outranking approach [9]; it models imperfect knowledge by considering indifference and preference thresholds; it avoids compensating for a very poor criterion value with good values on other criteria; and its non-compensatory nature is not as extreme as other methods [9].

On the other hand, the literature identifies several

Table 1. Application areas of ELECTRE methods [5].

No	Category	Description
1	Natural resources and environmental management	The articles are between for example geology and cartography; water, land, and waste management; forestry, ecotourism, and natural reserves sub-categories.
2	Business management	The sub-categories are for instance human resources, investment decisions, performance and benchmarking, etc.
3	Energy management	The articles are about energy management within a building or for a small set of customers, etc.
4	Design, mechanical engineering, and manufacturing systems	The articles focus on product design, equipment and material selection, the setup and maintenance of production lines, and manufacturing systems.
5	Structural, construction and transportation engineering	For example, includes articles on infrastructure and housing, transportation networks' management and development, etc.
6	Logistics and supply chain management	This application area includes different sub-categories for logistics and management of supply chain, selection of supplier and location, facility layout, etc.
7	Information technology	This includes four sub-categories including e-commerce and m-commerce, software evaluation, selection of the network, etc.
8	Financial management	The articles are about for example investment and portfolio management.
9	Policy, social, and education	This category includes mainly public planning and policy decision.
10	Chemical and biochemical engineering	This category includes different problems related to designing the chemical processes and bacteria identification, etc.
11	Agriculture & horticulture	This category includes different areas related to assessing the agricultural cropping systems, crops, land-use types; animal production, etc.
121	Health, safety, and medicine	The articles are about safety management and health sector problems.
13	Other areas and non-specific applications	Other application areas (excluded from the other 12 categories) such as destination assessment in the tourism industry, architectural design, selection of cars, evaluating movies, etc.

disadvantages of the ELECTRE method, which are as follows: The use of threshold values that may be arbitrary but significantly impact the final solution [8]; the time-consuming nature of the method [10]; the uncertainty in the accuracy of the ranking obtained through ELECTRE I [11]; the inability of the method to handle purely ordinal scales, as it requires a metric scale for the discordance index to compare differences. Consequently, ELECTRE I should only be employed when numerical scales are used to code the criteria; and the possibility of encountering the rank reversal phenomenon in ELECTRE II and III methods, wherein adding or removing an alternative can reverse the ranking between two alternatives [9].

4. Principle of ELECTRE methods

In this method, a decision-making problem is considered including *m* alternatives (known as $A_i : i = 1, 2, ..., m$) and *n* criteria ($X_j : j = 1, 2, ..., n$) and *n* weighting factors ($w_j : j = 1, 2, ..., n$). This can be shown as an $m \times n$ decision matrix (with x_{ij} elements). Also in these methods $\sum_{j=1}^{n} w_j = 1$. A decision problem aims to select the best alternative as a result. For this, the ELECTRE methods use outranking relations between each pair of alternatives. That is to say, these methods work based on preferring an alternative to another one. This concept is shown as:

$A_i \rightarrow A_k$ *(or* A_iSA_k*) means* A_i *is preferred to* A_k

For this, A_i should:

- Be at least as good as A_k for the majority of the criteria;
- Not be significantly worse when other criteria are considered.

The second factor can be examined by using a predefined threshold. This process aims to detect the dominated and non-dominated alternatives, although it is problematic in complex problems and when the matrix does not have crisp data due to their uncertainties. For this, two kinds of comparison sets are required among the criteria in which:

- $X_j(A_i)$ is superior to $X_j(A_k)$;
- $X_j(A_i)$ is not superior to $X_j(A_k)$.

Therefore, this method separately investigates the criteria that vote and veto the preference of A_i to A_k, and these sets are called concordance and discordance tests. Concordance tests are binary tests for ELECTRE I and II which use 0 and 1 index when the test is failed and passed; respectively. Consider a criterion that aims to minimize A_i, as an example. In this situation:

- *If* $A_i < A_k$, *results in a passed concordance test;*
- *If* $A_i > A_k$, *results in a failed concordance test.*

On the other hand, the outranking relations in the ELECTRE III method are fuzzy with the values between 0 and 1 based on how far an alternative is better than another when considering a criterion.

The second test known as discordance aims to evaluate the existence of a very high opposition between alternatives and to be intended for the criteria in which the performance of A_i is worse compared to A_k. This test also can be binary or fuzzy in nature.

Both tests are necessary to gain a true outranking relation between pairs of alternatives. The results can be finalized as:

- A_i *is preferred to* A_k ($A_i \rightarrow A_k$ *or* A_iSA_k): *If both tests are passed*;
- A_i *is incomparable to* A_k ($A_i\ R\ A_k$): *Neighter* $A_i \rightarrow A_k$ *nor* $A_k \rightarrow A_i$;
- A_i *is indifferent to* A_k ($A_i\ I\ A_k$): *One is not preferred over another alternative.*

The main and fundamental concept of the ELECTRE methods is discussed above. However, there are some differences in the steps of conducting different versions of the ELECTRE due to the differences in their application of them in decision-making. For example, ELECTRE I is used for the selection problems, and ELECTRE II and III are designed for ranking purposes. The following section aims to discuss the process steps of the different ELECTRE methods more specifically [3,7].

5. Process steps

Different ELECTRE methods can differ from each other in different aspects such as the process of determination of concordance and discordance matrices, the type of outranking relations (for example binary or fuzzy), etc.

5.1 ELECTRE I process steps

Step 1. Normalizing the Decision Matrix

The $n \times m$ decision matrix X is shown in Equation (1):

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{bmatrix}_{n\times m} \quad (1)$$

This step is similar to the first step in The TOPSIS method, and the attributes with various scales are transformed into comparable scales using Equation (2). This makes all the attributes similar in terms of the unit length of the vector.

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{m} x_{ij}{}^2}} \quad (2)$$

Step 2. Weighting the Normalized Matrix (V)

This step also is similar to TOPSIS, and aims to add the effect of weight to the decision matrix using the following method:

$$V = (v_{ij})\ \mathrm{n} \times \mathrm{m} \quad (3)$$

where $v_{ij} = w_j r_{ij}$. (4)

Step 3. Determining the Concordance and Discordance Sets

This step aims to separate the decision criteria set into two distinct subsets (C_{kl} (Concordance) *and* D_{lk} (Discordance) : $l \neq k$) for each pair of alternatives. That is to say, when two alternatives are considered, the alternatives are compared considering the type of criteria (cost or benefit types), and then based on which one is better or worse, the concordance and discordance sets are determined. For example, in a 4 × 6 matric if all attributes are positive (benefit attributes), when $C_{12} = [3,4,5,6]$, and $D_{12} = [1,2]$, then it means:

- $v_{11} < v_{21}$; $v_{12} < v_{22}$; $v_{13} \geq v_{23}$; $v_{14} \geq v_{24}$; $v_{15} \geq v_{25}$; $v_{16} \geq v_{26}$

Generally, the concordance and discordance sets are:

$$C_{kl} = \{j | v_{kj} \geq v_{lj}\}$$
$$D_{kl} = \{j | v_{kj} < v_{lj}\} = J - C_{kl}$$

Step 4. Building the Concordance Matrix

After recognizing the concordance and discordance sets, in this step, the sum values of the weights associated with the concordance set are considered as the concordance index. Other equations are also recommended by different authors. Here a simple method is introduced:

$$c_{kl} = \sum_{J \epsilon C_{kl}} w_j \quad (5)$$

considering the example in the previous step, for instance, the sum of w_2, w_3, w_4, w_5 must be placed in the c_{12} in the concordance matrix (C). This general (not symmetric) concordance matrix is shown as:

$$C = \begin{bmatrix} - & c_{12} & \dots & c_{1n} \\ c_{21} & - & \dots & c_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & \dots & c_{m(m-1)} & - \end{bmatrix} \quad (6)$$

Step 5. Building the Discordance Matrix

In this step, the focus is especially on the degree to which an alternative is worse than the other one (considering the pairs of alternatives). For this the discordance index should be calculated using the following equation:

$$d_{kl} = \frac{\max_{j \epsilon D_{kl}} |v_{kj} - v_{lj}|}{\max_{j \epsilon J} |v_{kj} - v_{lj}|} \quad (7)$$

and then the discordance (D) matrix is built similarly to the concordance matrix:

$$D = \begin{bmatrix} - & d_{12} & \dots & d_{1n} \\ d_{21} & - & \dots & d_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & \dots & d_{m(m-1)} & - \end{bmatrix} \quad (8)$$

In this matrix, higher values of d_{kl} show that A_k is less favorable than A_l for the discordance criteria. In contrast, a lower value implies that A_k is favorable to A_l. In this matrix, the d_{kl} values are between 0 and 1.

Step 6. Determining the Concordance Index (Dominance) Matrix

In this step, a threshold value is considered for the concordance index. That is to say, based on a threshold value, it can be determined whether A_k have the chance of dominating A_l, when C_{kl} exceeds at least a certain value for the threshold ($\overline{c}$). That is to say:

- $c_{kl} \geq \overline{c}$

where the $\overline{c}$ can be determined in different ways. For example, it can be calculated as follows:

$$\bar{c} = \sum_{\substack{k=1\\k\neq l}}^{m} \sum_{\substack{l=1\\l\neq k}}^{m} \frac{c_{kl}}{m(m-1)} \tag{9}$$

But the threshold can be assumed 0.7 usually. Then a bloomlean matrix (*F*) must be constructed with the following elements:

- $f_{kl} = 1; if\ c_{kl} \geq \bar{c};$
- $f_{kl} = 0; if\ c_{kl} < \bar{c}.$

Each "one value" for the matrix elements means "the dominance of one alternative with respect to another alternative".

Step 7. Determining the Discordance Index (Dominance) Matrix

On the other hand, a similar process is conducted to build the discordance dominance matrix (*G*). A threshold value is considered here as well such as:

$$\bar{d} = \sum_{\substack{k=1\\k\neq l}}^{m} \sum_{\substack{l=1\\l\neq k}}^{m} \frac{d_{kl}}{m(m-1)} \tag{10}$$

Also, $\bar{d}$ can be assumed 0.3 usually. The elements of the matrix are:

- $g_{kl} = 1; if\ d_{kl} \leq \bar{d};$
- $g_{kl} = 0; if\ d_{kl} > \bar{d}.$

Step 8. Determining the Aggregate Dominance Matrix

This step aims to combine *f* and *G* matrices, and calculate the intersection of the *F* and *G* matrices called Aggregate Dominance Matrix (*E*) with e_{kl} elements that is gained by the multiplication of f_{kl} and g_{kl} elements in the *f* and *G* matrices:

$$e_{kl} = f_{kl} \times g_{kl} \tag{11}$$

So, the elements of the aggregate dominance matrix can have just zero or one value for $k \neq l$.

Step 9. Eliminating the Less Favorable Alternatives

Now it is time to eliminate the less favorable alternatives. The alternatives' partial-preference ordering is given in matrix *E*. That is to say:

- If $e_{kl} = 1$; for both concordance and discordance criteria A_k are preferred to A_i.

But it is important to note that still A_k can be dominated by other alternatives. Therefore, more comprehensive conditions must be considered to conclude that A_k is not dominated in the whole ELECTRE procedure. These conditions are as follows:

- $e_{kl} = 1$; *for at least one l*;
- $e_{il} = 0$; *for all i.*

In these conditions: $k \neq l, i \neq k, i \neq l, l\&i = 1,2,...m.$ Although applying this condition can appear difficult, *E* matrix can be easily used to identify the alternatives with the following method:

If at least one "element of 1" is existed in any column of the *E* matrix; it means that "the column is 'ELECTREcally' dominated by the corresponding row(s)". Therefore, the column(s) with at least one element of 1 should be eliminated. In this step, a graphical representation of the over-ranking relationships can be also helpful to illustrate the relationships better. Consider the following $E_{example}$ matrix as the result of Step 8:

$$E_{example} = \begin{bmatrix} - & 1 & 0 & 1 \\ 0 & - & 0 & 0 \\ 0 & 1 & - & 1 \\ 0 & 1 & 0 & - \end{bmatrix}$$

In this example, the graphical representation can be shown in **Figure 1** considering the following relationships given as the result of $E_{example}$ matrix elements:

$A_1 \rightarrow A_2,\ A_1 \rightarrow A_4,\ A_3 \rightarrow A_2,\ A_3 \rightarrow A_4,\ A_4 \rightarrow A_2$

It can be concluded from the figure that A_2 and A_4 are dominated by A_1 and A_3, although the preference relationship between A_1 and A_3 cannot be obtained from this. These results also can be taken from the columns of the $E_{example}$ matrix. As A_1 and A_3 do not possess any element of 1 [3,6,8,11,12].

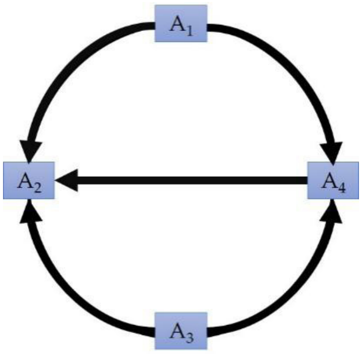

Figure 1. An example for over-ranking relationships [8].

5.2 ELECTRE II process steps

ELECTRE II is similar to ELECTRE I, but the differences are related to the definitions of two outranking relations called "strong outranking" and "weak outranking". Strong outranking and weak outranking have solid bases and questionable grounds, respectively.

Strong outranking relies on solid bases, whereas weak outranking has questionable grounds. Therefore, the result is given as an outranking graph dichotomy in weak and strong outranking graphs. The process steps of the ELECTRE I were defined in the previous sub-section in detail. Here, as some steps are similar to the ELECTRE I (such as decision matrix formation), just the main steps of the ELECTRE II are listed. The main process steps are listed following:

Step 1. Outranking Relationships Definition

This method compares the alternatives pair to pair as discussed in the ELECTRE I. The aim of this step is to define whether A_k outranks $A_l$strongly. Three concordance thresholds (α): high, medium, and low and two non-concordance thresholds (D)are used in this method. For this, the following conditions must be met:

$$\begin{cases} c_{kl} \geq \alpha^{+} \\ and \\ X_j(A_k) - X_j(A_l) \leq D_1(\mathrm{j}), \forall \mathrm{j} \\ \text{and} \\ \dfrac{P^{+}(k,l)}{P^{-}(k,l)} \geq 1 \end{cases} \tag{12}$$

and/or

$$\begin{cases} c_{kl} \geq \alpha^{0} \\ and \\ X_j(A_k) - X_j(A_l) \leq D_2(\mathrm{j}), \forall \mathrm{j} \\ \text{and} \\ \dfrac{P^{+}(k,l)}{P^{-}(k,l)} \geq 1 \end{cases} \tag{13}$$

where:

- P^{+} (k, l) is "the sum of criteria weighting of the k when it is preferable to l (considering the type of the criteria)";
- P^{-} (k, l) is "the sum of criteria weighting of the l when it is preferable to k (considering the type of the criteria)";
- $P^{=}$ (k, l) is "the summation of criteria weighting of the k when the k equals to l".
- $\alpha^{+} \geq \alpha^{0}$ (threshold values) and D_2 (j) $\leq D_1$ (j) (thresholds of discordance).

The above definitions for $P(k, l)$ are similar to Step 4 in the ELECTRE I. Considering these relations A_k strongly outranks A_l if:

- In Equation (12), the outranking equation is "strongly" concordant and "fairly" discordant;
- In Equation (13), it is "fairly" concordant and "weakly" discordant.

On the other hand, the following equation is used as the weak outranking relation to determine A_k weak outranks A_l:

$$\begin{cases} c_{kl} \geq \alpha^{-} \\ and \\ X_j(A_k) - X_j(A_l) \leq D_2(\mathrm{j}), \forall \mathrm{j} \\ \text{and} \\ \dfrac{P^{+}(k,l)}{P^{-}(k,l)} \geq 1 \end{cases} \tag{14}$$

where $X_j(A_k)$ is the evaluation of A_k on criterion j. In Equation (14), $\alpha^{-} \leq \alpha^{0}$, and A_k weakly outranks A_l, if:

- The relation is weakly concordant and fairly discordant.

To discuss the relations more, considering the hypothesis of the problem "A_k outranks A_l" it can be noted that:

- If $X_j(A_k) - X_j(A_l) \leq D_1(\mathrm{j})$, then the criterion j does not possess a major opposition to the considered hypothesis, and the discordance is weak in this situation;
- If $D_2(\mathrm{j}) \leq X_j(A_k) - X_j(A_l) \leq D_1(\mathrm{j})$, then the criterion j does not possess a major opposition to the considered hypothesis, and the discordance is mean in this situation.

Step 2. The Concordance Coefficient Definition

Consider cases A_k and A_l are compared. The concordance coefficient/index can be determined by the following equation (similar to ELECTRE I, but here another equation is used which can be also in ELECTRE I):

$$c_{kl}=\frac{P^{+}(k,l)+P^{=}(k,l)}{P^{+}(k,l)+P^{=}(k,l)+P^{-}(k,l)} \tag{15}$$

Conditions are as follows to accept the concordance test:

$$\begin{cases} \dfrac{P^{+}(k,l)}{P^{-}(k,l)} \geq 1 \\ and \\ c_{kl} \geq \alpha^{+} \ or \ c_{kl} \geq \alpha^{0} \ or \ c_{kl} \geq \alpha^{-} \end{cases} \tag{16}$$

After defining the main concepts in the ELECTRE II, now it is time to enter the classification process.

Step 3. Constructing the Graphs

First, two separate graphs are used to show the relationships between alternatives: Strong and weak outranking graphs.

- In a strong outranking graph (G_F), the nodes of alternatives are liked, if there is a strong outranking relation between them. G_F includes Y as the set of nodes and U_F as the set of arcs between the nodes, and can be shown as $G_F(Y, U_F)$.
- In weak outranking graph (G_f), the nodes of alternatives are liked, if there is a weak outranking relation between them.

The graphs should be modified to remove the circuits which are defined as "A circuit is a set of actions in which we have, for example, action A dominates action B, action B dominates action C and action C dominates action A". In the circuits, finding the best alternatives is not possible, therefore these actions are defined as equivalent for the decision makers. After removing the circuits, a modified graph is shaped. In this graph, the circuit is replaced by one of the fictitious actions(for example instead of the ABC circuit, *A'* is used).

Step 4. Conducting the Direct Classification (Forward Ranking)

Then, the direct classification can be conducted in several iterations. the process simply is shown in **Figure 2**.

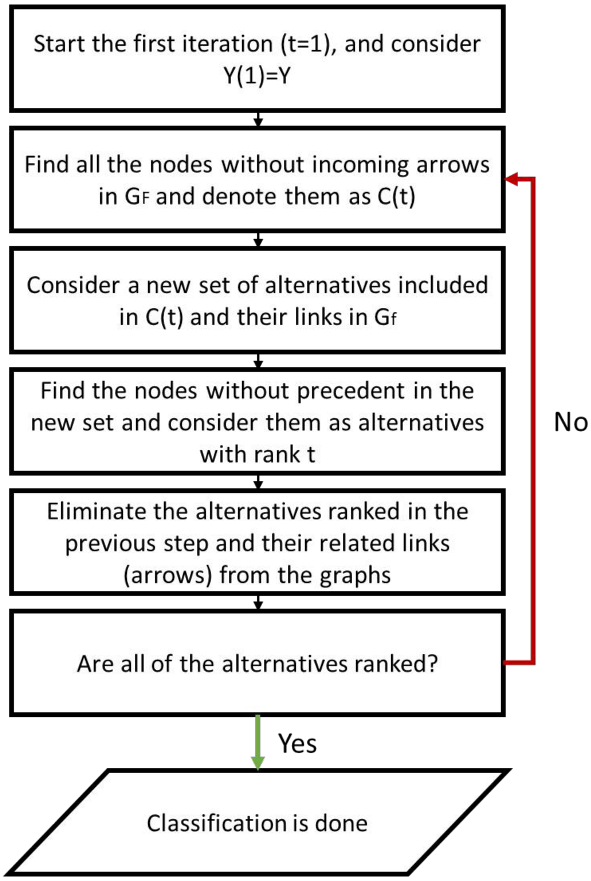

Figure 2. Direct classification process.

Consider **Table 2** for ranking the alternatives completed by the following steps:

1) Firstly, in $t = 1$ consider $Y(1) = Y$;

2) In G_F find all the nodes (alternatives) without incoming arrows (or not having a precedent). Denote them as $C(1)$.

3) Go to G_f, see the nodes in $C(1)$, and remove all the ties between these nodes and the system (you just consider the connections between the alternatives included in $C(1)$), and add the sets of links as $\overline{U_f}(t)$. Then, construct the $(C(1), \overline{U_f}(t))$ graph, and the nodes that have no precedent in this graph are the set of selected nodes in this iteration ($A(1)$) that is the non-dominated set at the first iteration. It shows the alternatives without precedent in either strong or weak outranking graphs.

4) Eliminate all the nodes and arrows related to the $A(1)$ in $Y(1)$, and consider it as $Y(2)$. Therefore in each iteration $Y(t) = Y(t-1) - A(t-1)$. That is to say, in a new iteration t (step t), the classified actions from the previous steps should be removed from the outranking graph.

5) Continue the iterations until all nodes are

ranked. In this table, lower rank means better performance, for example if *rank* (A_k) < **rank** (A_l) then A_k is better than A_l. Therefore, the action related to r_1 is the best.

Table 2. Process of ELECTRE II.

Step/Iteration (*t*)	***Y*(*k*)**	***C*(*t*)**	$\overline{U_f}(t)$	***A*(*t*)**	r_t
1	$Y(1)$				r_1
2	$Y(1) - A(1)$				r_2
⋮	⋮	⋮	⋮	⋮	⋮
Until all alternatives are ranked and eliminated	$\emptyset$				r_t

Step 5. Conducting the Reverse Classification

In this step rank the actions similar to the previous step (direct ranking), but first reverse the direction of the all arrows in the strong outranking and weak outranking graphs. The results of the ranking table also must be adjusted. For example, the following equation is used for this purpose:

$$Rank\ reverse\ (A_i) = 1 + Rank(A_i)_{max} - Rank(A_i) \quad (17)$$

Step 6. Determining the Partial Preorder

Using the ranks determined in Steps 4 and 5 a partial preorder should be determined. The following points can be concluded from two classifications:

- A_k is preferred to A_l in the final preorder, if this trend can be seen in both direct and reverse preorders;
- A_k is preferred to A_l in the final classification, if it is preferred in one of the preorders, and the actions are equal in the other one;
- A_k and A_l are incomparable, when A_k is preferred to A_l in one of the preorders, and A_l is preferred to A_k in another one.

Also, special charts can be beneficial to gain global classification based on the reverse and direct classifications. In these charts x-axis and y-axis performs the inverse and direct ranks [3,6,12,13].

5.3 ELECTRE III process steps

This method is similar to the ELECTRE II method; the main difference is using the concept of pseudo-criteria instead of applying classic criteria. In this method two different thresholds are used which are indifference and strict preference thresholds. On the other hand, both concordance index and discordance index are defined in fuzzy form. According to this advantage, the ELECTRE III outperforms previous versions (I and II) in addressing imprecise, inaccurate, and uncertain data. In this method, the concept of a credibility degree or reliability of the outranking also is introduced. The result is not binary, and I is not just a choice between accepting or rejecting the alternatives. The steps are as follows:

Step 1. Determining the Required Values

For criterion *j*, determine three thresholds known as 1) indifference (*q*), 2) preference (*p*), and 3) veto (*v*) and consider $v \geq q \geq p$. Furthermore, similar to other methods, importance weights (w_j) should be also determined for the criteria. Using these thresholds, strict, indifferent, and weak relations can be identified.

Step 2. Determining Concordance Index

In this step, the concordance index should be determined first. For this the following equation which has a fuzzy form is used for criterion *j*, and between A_k and A_l alternatives:

$$\begin{cases} c_j(k,l) = \dfrac{X_j(A_k) + p_j - X_j(A_l)}{p_j - q_j}\ if\ q_j < X_j(A_k) - X_j(A_l) \leq p_j \\ c_j(k,l) = 1\ if X_j(A_k) - X_j(A_l) \leq q_j \\ c_j(k,l) = 0\ if\ p_j < X_j(A_k) - X_j(A_l) \end{cases} \quad (18)$$

After calculating all $c(k,l)$ values for all criteria, a global concordance index is calculated using the following equation:

$$C_{kl} = \frac{\sum_j p_j . c_j(k,l)}{\sum_j p_j} \quad (19)$$

Then this process should be applied to all pairs of alternatives, and the result is placed in the elements of a matrix called Concordance Matrix. The elements of this matrix are defined as "the percentage of criteria where one alternative is at least as good as the other". For example, if C_{kl} is 0.5, then half of the criteria A_k is at least as good as A_l.

Step 3. Determining Discordance Index

Discordance index calculation needs to define a veto (*v*) threshold for each criterion (v_j). When decision makers give no credibility to the outranking of alternative A_k with respect to A_l, the v_j is $X_j(A_k) - X_j(A_l)$. The index of discordance is obtained using fuzzy concept by the following equation:

$$\begin{cases} d_j(k,l) = 1 \; if \;\; v_j < X_j(A_k) - X_j(A_l) \\ d_j(k,l) = \dfrac{X_j(A_k) - X_j(A_l) - p_j}{v_j - p_j} \; if \; p_j \le X_j(A_k) - X_j(A_l) \le v_j \\ d_j(k,l) = 0 \; if \; X_j(A_k) - X_j(A_l) < p_j \end{cases} \quad (20)$$

In this step, all discordance indices should be calculated for all pairs of alternatives considering all criteria.

Step 4. Determining the Outranking Credibility Degree and Building Credibility Matrix

Now a concordance and discordance measure is available for each pair of alternatives for each criterion. But, these measures must be combined to gain an outranking degree and assess the reliability of the hypothesis criteria A_k is at least as good as A_l (A_k SA_l). This is possible using a credibility degree concept. Credibility is calculated as follows:

$$S(k,l) = \begin{cases} C_{kl} \; if \; d_j(k,l) \le C_{kl} \\ C_{kl} \cdot \prod\limits_{j \in \overline{F}} \dfrac{1 - d_j(k,l)}{1 - C_{kl}} \end{cases} \quad (21)$$

$\overline{F}$ is the set of criteria by which $d_j(k,l) > C_{kl}$. According to the credibility, it is assumed that when $d_j(k,l) \le C_{kl}$, the C_{kl} should not be modified, but else the effect of $d_j(k,l)$ should be considered for outranking as the hypothesis should be questioned, and C_{kl} needs a modification. On the other hand, if $d_j(k,l) = 1$, then there is no confidence to say that criteria A_k is at least as good as A_l, and credibility for this criterion and pair of alternatives is zero. The values of $S(k,l)$ can be put into a matrix called Credibility Matrix in this step as well. This matrix then will be used for ranking step.

Step 5. Descending and Ascending Distillations (Exploitation Step)

Similar to the ELECTRE II method, two preorders (descending and ascending distillations) are determined in this step.

Descending distillation process

Similar to the direct outranking in the ELECTRE II, this process is a step-by-step which starts with the first iteration and ends when all the alternatives are ranked. Also, descending distillation determines the best alternative first, and ends with finding the worst one. But, the basic here is to set a qualification score for the alternatives. For this, a discrimination threshold ($s(\lambda)$, $0 < \lambda < 1$) should be set. For example, the following equation can be used for determining $s(\lambda)$:

$$s(\lambda) = 03 - 0.15\lambda \quad (22)$$

Now, Use the following process (shown simply in **Figure 3**) to define the qualifications of alternatives:

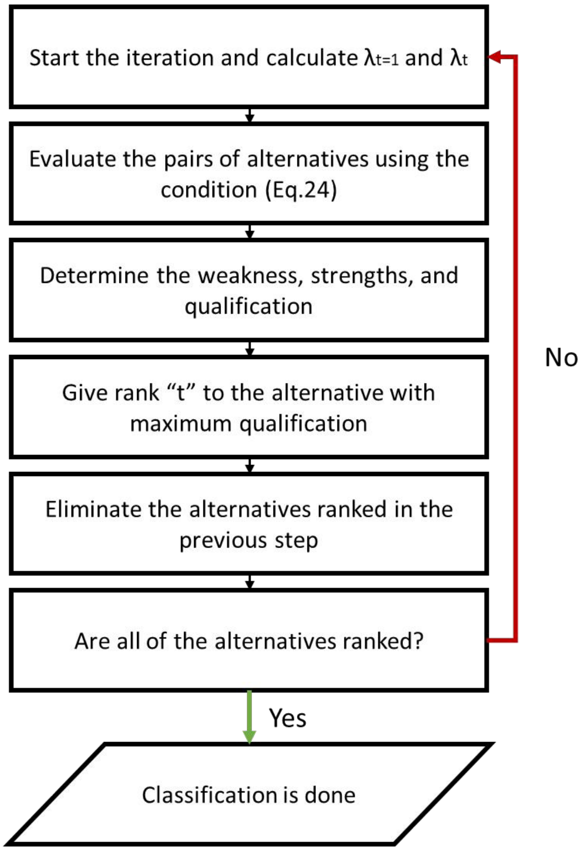

Figure 3. Descending distillation process.

- For the first iteration, set $\lambda_0 = \max\limits_{k,l \in A} S(k,l)$, then use the following equation to gain a cutoff level of outranking (λ_1):

$$\lambda_1 = \max_{k,l \in A} S(k,l) \; where \; S(k,l) < \lambda_0 - s(\lambda_0) \quad (23)$$

That is to say, λ_1 is "the largest outranking score which is just less than the maximum outranking score minus the discrimination threshold".

- Then, consider the following equation for the pair of alternatives in the first iteration:

$$A_k S A_l \; iff \begin{cases} S(k,l) > \lambda_1 \\ and \\ S(k,l) - S(l,k) > s((k,l)) \end{cases} \quad (24)$$

Both conditions should be applied for all pairs of alternatives in this step. For the result of these evaluations, give 1 value (strength) to an alternative every time it outranks another alternative, and –1 value (weakness) when it is outranked by another alternative. The sum of these values in each iteration gives the qualification of the alternatives. The alternative with the maximum qualification is the ranked alternative in this iteration. It must be noted that if there are more than one alternative with similar maximum values, this step must be repeated (but considering a matrix just with the alternatives have the maximum qualification) to choose the best between them as the ranked alternative in the iteration one. Although, if they gained similar qualifications again, they can be considered as alternatives with similar ranks.

Another similar method can be also used for describing the qualification values in each iteration (t). The following indicators can be used to show the number of alternatives that are outranked by A_k $((p_A^{\lambda_t}(A_k))$, the number of alternatives that outrank $A_k((f_A^{\lambda_t}(A_k))$, and the qualification of A_k $(q_A^{\lambda_t}(A_k) = p_A^{\lambda_t})$; respectively:

$$p_A^{\lambda_t}(A_k) = card(\{A_l \in A | A_k S_A^{\lambda_t} A_l\}) \tag{25}$$

$$f_A^{\lambda_t}(A_k) = card(\{A_l \in A | A_l S_A^{\lambda_t} A_k\}) \tag{26}$$

$$q_A^{\lambda_t}(A_k) = p_A^{\lambda_t}(A_k) - f_A^{\lambda_t}(A_k) \tag{27}$$

- For the second iteration, eliminate the ranked alternative/alternatives from the credibility matrix. Similar to the previous step, calculate $\lambda_1 = \max_{k,l \in A} S(k,l)$, and then gain the cutoff level of outranking (λ_2) (apply Equation (23) for λ_1 and λ_2). Then, compare the pairs of alternatives using the conditions presented in Equation (24) similar to the previous step. Gain the second ranked alternative/s.
- Repeat the process until all alternatives are ranked, and gain the list of alternatives' ranking considering in the descending distillation process the alternative/s ranked in the first iteration is/are the best.

Ascending distillation process

This process is similar to the descending distillation, but the difference is that in each iteration the alternative with the minimum qualification is chosen, and eliminated in the next iteration. Therefore, alternatives are ranked from the worst to the best.

Step 6. Final Preorder

In the final step, the ranking results of two distillation processes must be combined to gain a unit ranking list (similar to the ELECTRE II method) [7,12].

6. Conclusions

In conclusion, the ELECTRE method is a versatile and widely-used multi-criteria decision-making method with various applications across different fields. The advantages of this method include its ability to handle imprecise and uncertain data, its flexibility in accommodating various decision-making scenarios, and its robustness in dealing with complex and conflicting criteria. However, like any other MCDM method, the ELECTRE method also has its limitations, such as the need for a considerable amount of input data, the subjectivity of the decision-maker's preferences, and the potential for inconsistent results.

The process steps of the ELECTRE method involve careful consideration and analysis, including defining the problem, identifying the criteria and alternatives, assessing the criteria's importance, evaluating the alternatives' performance, and generating the ranking of alternatives based on the outranking principle. There are different versions of the ELECTRE method specifically designed for different purposes, such as selection and ranking problems, but they all share the same core strategy and process steps.

This paper provides a comprehensive overview of the ELECTRE method, its applications, and its main characteristics. Additionally, it aims to provide step-by-step processes to describe the ELECTRE I, II, and III methods in a simple and easy-to-understand manner. By understanding the fundamental concepts, applications, advantages, and limitations of the ELECTRE method, decision-makers can make informed and effective decisions in various contexts. While the review provides a general overview of its applications, the absence of detailed real-world examples limits the ability to assess the method's effectiveness and identify potential challenges or areas for improvement in specific contexts. Including more case studies or empirical evidence could provide valuable insights and enhance the manuscript's practical relevance. Future research can focus on improving the ELECTRE method by addressing its limitations and developing more efficient and accurate approaches for decision-making.

Author Contributions

Conceptualization, H.T.; methodology, H.T.; validation, H.T.; formal analysis, H.T. and M.M.; resources, H.T. and M.M.; data curation, H.T.; writing—original draft preparation, M.M. and H.T.; writing—review and editing, M.M.; visualization, H.T. and M.M.; supervision, H.T. All authors have read and agreed to the published version of the manuscript.

Conflict of Interest

There is no conflict of interest.

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