Macro Management & Public Policies
https://ojs.bilpublishing.com/index.php/mmpp

ARTICLE

A Markov Model for Production and Maintenance Decision

Mohammad Saber Fallahnezhad* **Alie Ranjbar** **Faeze Zahmatkesh Sredorahi**

Industrial Engineering, Yazd University, Yazd, Iran

ARTICLE INFO

Article history
Received: 1 April 2019
Accepted: 10 October 2019
Published Online: 15 November 2019

Keywords:
Markov chain
Cost function
Maintenance policy
Production process

ABSTRACT

In this paper, we consider a production machine which may fail and it is necessary to repair the machine after each failure and there are two statuses for each repair; in one case, we should replace the machine because of catastrophic failure and in the other case, only small repairs are needed. Times between failures and repair and replace times are random and demands are satisfied by inventory during repairing and replacing the machine; shortage level is limited. We model described system as a Markov chain and develop an algorithm to compute the expected number of transitions among states.

1. Introduction

Breakdown of machines in industrial environments influences on different items and since breakdown or failure is a random process so some researchers have focus on modeling reliability and breakdown of machines by Markov models.

In real world most of phenomenon is not deterministic meaning that under specified conditions, there are a set of possible outcomes which are occurred by their probabilities. Markov analysis is one of the main tools for describing stochastic processes. Markov analysis methods consist of two basic methods: Markov chain and Markov process.

For any given system, a Markov model consists of a list of the possible states of that system, the transition probability between those states. Markov chain assumes that states are discrete while states in Markov process are continuous. A Markov chain can be homogeneous or non- homogeneous. In a homogeneous Markov chain transition probabilities between states are constant while in a non- homogeneous Markov chain these probabilities are not fixed. Markov chains have been successfully used in modeling system behavior especially in systems with stochastic and multi state conditions. One of the important applications of Markov chains is to model production processes and reliability.

Definition of reliability is the probability that the system will perform its operation under specified working condition for a specified period of time. The most important techniques in reliability analysis are reliability block diagrams, network diagrams, fault tree analysis, Monte Carlo simulation and Markov model. In the reliability analysis, transitions in Markov analysis usually consist of failures and repairs.

Many researchers have analyzed behavior of systems reliability with Markov chains. Abboud [1] has modeled machine produces an item at a constant rate, which is assumed to be greater than the demand rate, and the demand

**Corresponding Author:*
Mohammad Saber Fallahnezhad,
Associate Professor of Industrial Engineering, Yazd University, Yazd, Iran;
Email: fallahnezhad@yazd.ac.ir

 DOI: https://doi.org/10.30564/mmpp.v2i1.655

is assumed to be known and constant. While operating, the machine can fail, and upon failure it requires service. The machine times-to-failure and repair times are random, and during repairs, demand is backordered as long as the backordering level does not exceed a prescribed amount, after which demand is lost. By considering time to be of discrete units and the times-to-failure and repair times to be geometrically distributed, he models the production-inventory system as a Markov chain and develops an efficient algorithm to compute the potentials that are used to formulate the cost function. He has considered one type of failure and as the best of author's knowledge, modeling two types of failures with Markov chains has not been addressed before.

Grabski [2,4] has modeled the properties of the reliability function of an object with the failure rate by a semi-Markov process. Grabski has supposed that random load of an object can be modeled by a random process. It is assumed that failure rate of the object depends on its random load and the failure rate is the random process too.

Prowell and Poore [3] have computed system reliability using Markov chain models. The system under their study may be a single module, or may be composed of several modules.

Veeramany and Pandey [5] have carried out reliability analysis of nuclear component cooling water (NCCW) system. In their analysis they have used semi-Markov process model. The objective of their study is to determine system failure probability under assumptions like Weibull distribution for the failure time of components. Finally Monte Carlo simulation is used to validate the model result.

Wang and Liu [7] have studied on reliability of Air Traffic Control Automatic System Software based on Markov chain. First 36 month failure data of Air Traffic Control Automatic System are collected and then based on Markov chain reliability is predicted.

Lisnianski et.al [6] have presented a multi-state Markov model for a coal power generating unit. They proposed using Markov chain for the estimation of transition rates between the various generating capacity levels of the unit based on field observation.

Liu et.al [8] have modeled a system consisting of stochastic supply and stochastic demand by Markov processes and then used a measurement of performance of the system. The performance measure considered is the probability that the demand is met by the supply during given time interval.

The remainder of this paper is organized as follows. In Section 2, we present a system description and state all the assumptions that our system adheres to. In Section 3, we present transition probability which denotes the probability of changing of different states. In Section 3, we propose a simple and efficient procedure for computing expected number of visiting states. In Section 4, we construct the cost model and in Section 5, we present numerical examples that demonstrate the behavior of the model. Finally, in Section 6 we state our findings.

2. Problem Statement

In this paper, we propose a model that generalizes from Abboud [1]'s model. Our assumption are the same of Abboud [1]. We change Abboud [1]'s status to 4 different status. In this paper, we assume that the system considers a production machine which can produce (p) items per unit time. The demand rate for produced items is d, such that p > d. While producing, the machine may fail and it is necessary to repair it; failures are two types, in one case we should replace it with another machine and in the other case we should only repair the machine. When number of produced items equals to (I) then we stop production and allow the machine to be idle and supply demand by on-hand inventory and we start production again after using all of them. Times between failures and repair times are random. Demands are supplied by inventory in the time of repairing machine; note that maximum number of backorder is (B) units and it cannot be more than this level.

We use (i,j) for describing the state of the system; represents the level of the inventory and j represents the status of the machine. Note that the machine can be in 4 different statuses shown with $j = 1, 2, 3, 4$ which represents that the machine is operating, idle, under repair and under replacing respectively. Also the inventory can vary from $-B$ to I. So we have a Markov process with state space $L=\{(i,j):-B \le i \le \mathrm{I}, j=1,2,3,4\}$.

Assumptions about related probabilities come in following and we determine the transition flow diagram and the one step transition probability matrix of Markov process based on them. In section 3, we formulate the expected number of visiting state (i,j) in described system and provide an algorithm for computing the expected values of transitions among states. The expected value of different cost functions in the production- inventory system in comes section 4.

2.1 Notations

Basic assumption on repaired probabilities are summarizes as follows,

α_1=1-P[no failures which require only repair will be in $\{(0,\frac{1}{p-d})$machine is operating$\}]$

α_2=1-P[no failures which require replace will be in

DOI: https://doi.org/10.30564/mmpp.v2i1.655

$\{(0,\dfrac{1}{p-d}$)machine is operating$\}$]

α=1-P[no failures will be in $\{(0,\dfrac{1}{p-d}$)machine is operating$\}$]

β=1-P[Machine will be repaired in $\{(0,\dfrac{1}{p-d}$)machine is operating$\}$]

γ=1-P[Machine will be replaced in $\{(0,\dfrac{1}{p-d}$)machine is operating$\}$]

Note that: $a=a_1+a_2$

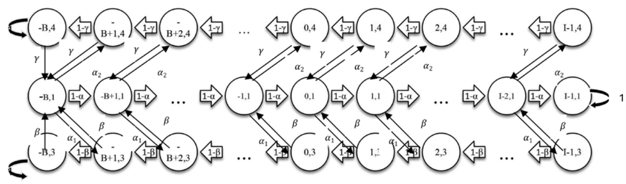

Figure 1. The transition flow diagram of the Markov chain

2.2 Markov Model

One- step transition probability which denotes the probability of going from state (i,j) to state (m,n) is shown by $P_{(i,j)(m,n)}$. They are obtained as follows.

$$P_{(i,1)(i+1,4)}=\alpha_2 \qquad for\ i=-B,\ldots,I-2$$
$$P_{(i,1)(i+1,3)}=\alpha_1 \qquad for\ i=-B,\ldots,I-2$$
$$P_{(i,1)(i+1,1)}=1-\alpha \qquad for\ i=-B,\ldots,I-2$$
$$P_{(i,3)(i-1,1)}=P_{(-B,3)(-B,1)}=1-\beta \qquad for\ i=-B+1,\ldots,I-1$$
$$P_{(i,3)(i-1,3)}=\beta \qquad for\ i=-B+1,\ldots,I-1$$
$$P_{(i,4)(i-1,1)}=P_{(-B,4)(-B,1)}=1-\gamma \qquad for\ i=-B+1,\ldots,I-1$$
$$P_{(i,4)(i-1,4)}=\gamma \qquad for\ i=-B+1,\ldots,I-1$$
$$P_{(1,2)(0,1)}=1$$
$$P_{(i,2)(i-1,2)}=1 \qquad for\ i=2,\ldots,I$$

All other transition probabilities are zero.

3. Expected Number of State (i,j)

The expected number of visiting state (i,j) before absorbing into state (I-1,1) is denoted by v_{ij}. Note that the first state of system is (0,1). We can obtain the values of v_{ij} by using following formulations,

$$\alpha v_{I-1,1}=(1-\alpha)v_{I-2,1}$$
$$v_{i,1}=(1-\alpha)v_{i-1,1}+\gamma v_{i+1,4}+\beta v_{i+1,3} \qquad for\ i=-B+1,\ldots,I-2\ and\ i\neq 0$$
$$\alpha v_{0,1}=(1-\alpha)v_{-1,1}+\gamma v_{1,4}+\beta v_{1,3}\ +1$$
$$v_{-B,1}=\gamma v_{-B+1,4}+\beta v_{-B+1,3}\ +\gamma v_{-B,4}+\beta v_{-B,3}$$
$$\alpha v_{I-1,3}=\alpha_1 v_{I-2,1}$$
$$v_{I-1,4}=\alpha_2 v_{I-2,1}$$
$$v_{i,3}=(1-\beta)v_{i+1,3}+\alpha_1 v_{i-1,1} \qquad for\ i=-B+1,\ldots,I-2$$
$$v_{i,4}=(1-\gamma)v_{i+1,4}+\alpha_2 v_{i-1,1} \qquad for\ i=-B+1,\ldots,I-2$$
$$v_{-B,3}=(1-\beta)v_{-B+1,3}+\ (1-\beta)v_{-B,3}$$
$$v_{-B,4}=(1-\gamma)v_{-B+1,4}+\ (1-\gamma)v_{-B,4}$$

Proposed algorithm for computing expected number of visiting states comes in following,

Step1: set $v_{I-1,1}=1$, $v_{I-2,1}=\ \alpha/(1-\alpha)$,$v_{I-1,3}=\alpha_1{*}v_{I-2,1}$

And $v_{I-1,4}=\alpha_2{*}v_{I-2,1}$

Step2: set $v_{I-3,1}=(\frac{1}{1-\alpha})v_{I-2,1}-(\frac{\gamma}{1-\alpha})v_{I-1,4}-(\frac{\beta}{1-\alpha})v_{I-1,3}$

Step3: for $i=I-2$ to $-B+2$ step -1 set

$$v_{I,3}=(1-\beta)*v_{i+1,3}+\ \alpha_1*v_{i-1,1}$$
$$v_{i,4}=(1-\gamma)*v_{i+1,4}+\ \alpha_2*v_{i-1,1}$$
$$v_{i-2,1}=\left(\frac{1}{1-\alpha}\right)v_{i-1,1}-\left(\frac{\gamma}{1-\alpha}\right)v_{i,4}-(\frac{\beta}{1-\alpha})v_{i,3}$$

Step4: set $\ v_{0,1}=\ (1-\alpha)v_{-1,1}+\gamma v_{1,4}+\beta v_{1,3}\ \ +1$

$$v_{-B+1,4}=(1-\gamma)v_{-B+2,4}+\alpha_2 v_{-B,1}$$

$$v_{-B+1,3}=(1-\beta)v_{-B+2,3}+\alpha_1 v_{-B,1}$$

$$v_{-B,3}=\left(\frac{1-\beta}{\beta}\right)v_{-B+1,3}$$
$$v_{-B,4}=\left(\frac{1-\gamma}{\gamma}\right)v_{-B+1,4}$$
$$v_{-B,1}=\gamma(v_{-B+1,4}+v_{-B,4})+\beta(v_{-B+1,3}\ \ +v_{-B,3}\ )$$

4. Cost Equations

In this section we have presented the expected values of costs functions which should be considered in the system. Also we require the expected cycle time for each period for computing total cost per unit time.

4.1 Expected Cycle Time of Each Period

The expected time of each period is equal to the expected number of visiting each state (i,j) multiplied by the time of staying in that state; term $\frac{I}{d}$ in equation (1) corresponds to state j=2 which is the time of using when inventory is equal I.

Note that a cycle is the time which within this time the inventory level gets (I) and we force the machine to be idle and use all inventories until they all will be sold. Expected time of cycle is obtained in equation (1).

$$\mathrm{E(T)}=\sum_{i=-B}^{I-1}(\frac{v_{i,1}}{p-d}+\frac{v_{i,4}}{d}+\frac{v_{i,3}}{d})+\frac{I}{d} \qquad (1)$$

4.2 Inventory Cost Equation

The expected holding cost of inventory is equal to the cost of holding one item per time unit (h) multiplied by the expected number of visiting state ($i=0,\ldots,I-1$) multiplied by the number of inventories in each state multiplied with the time of staying in each inventory level; in the other words, it is the number of inventories (i) multiplied by the expected time of being in each level of inventory

 DOI: https://doi.org/10.30564/mmpp.v2i1.655

multiplied by its corresponding cost.

Note that last term in equation (2) corresponds to the state which inventory gets equal to level I and we force the machine to be idle.

$$E(IC) = h\left(\sum_{i=0}^{I-1} \frac{iv_{i,1}}{p-d} + \frac{iv_{i,4}}{d} + \frac{iv_{i,3}}{d} + \frac{I(I+1)}{2d}\right) \quad (2)$$

4.3 Backordering Cost

To compute the backordering cost, we define π_1 and π_2 as the cost of occurring backordering event and cost of backordering one unit per time unit respectively. Expected backordering cost is obtained as follows,

$$E(BC) = \pi_1\left(\sum_{i=-B+1}^{0} v_{i,4} + v_{i,3}\right) + \pi_2\left(\sum_{i=-B}^{0} \frac{iv_{i,1}}{p-d} + \frac{iv_{i,4}}{d} + \frac{iv_{i,3}}{d}\right) \quad (3)$$

4.4 Lost Sales Cost

To compute the lost sales cost, we define (s) as the cost of occurring lost sale event. The expected lost sales cost is obtained as follows,

$$s\left(v_{-B,4} + v_{-B,3}\right) \quad (4)$$

4.5 Maintenance Cost

To compute maintenance cost, we define m_{f1} , m_{f2} as the fixed costs per each type of failure and m_{t1} , m_{t2} as the costs per time unit of repairing and replacing the machine. The expected maintenance cost is obtained as follows,

$$E(MC) = m_{f_1}\alpha_1\left(\sum_{i=-B}^{I-1} v_{i,1} + m_{f_2}\alpha_2\left(\sum_{i=-B}^{I-1} v_{i,1}\right) + m_{t_1}\left(\sum_{i=-B}^{I-1} \frac{1}{d} * v_{i,3}\right) + m_{t_2}\left(\sum_{i=-B}^{I-1} \frac{1}{d} * v_{i,4}\right)\right.$$

4.6 Total Cost Per Unit Time

Total cost per time unit can be determined as follows,

$$TC = \frac{E(IC) + E(BC) + E(LC) + E(MC)}{E(T)}$$

5. Numerical Results

To study the behavior of the developed model above, We start by examining the behavior of the cost components with Abboud [1]'s set of parameters: p=100,d=60, B=1, a1=0.05, a2=0.05 , b=0.7, g=0.2,K=500, h=15, π1=22, π2=23, s=100,mf1=37, mf2=38, and mt1=12, mt2=13. We should determine optimal value of I and possible integer value for I is assumed to be within interval 5-20. As Figure2, It is denoted that by increasing I from 5 to 20, all cost components behaved as expected. The optimal inventory's cost in Abboud [1]'s model is more than the cost of our model. Each cost functions were drawn separately in Figs. 3, 4, 5 and 6. Inventory cost has significant effect on the total cost. The backordering, maintenance and lost sales costs were significantly less than the inventory cost. In contrast with the findings of Abboud [1], the maintenance cost increases by increasing lot size. To describe the behavior of Total cost as failure rate changes, we have tested the model for the following values of α=0.0, 0.2, 0.4, 0.6 and 0.8. The results of this experiment are shown in Figure7 along with the corresponding values of I. In contrast with the findings of Abboud [1], we see that lot size does not increase by increasing the probability α. Since the proposed model is a generalization of Abboud [1]'s model by considering the replacement decisions thus the results of Abboud [1]'s model is not confirmed in this research. It is observed that Inventory cost is a convex function of I but Backordering cost and Lost Sales cost and Maintenance cost are concave function of I. Also it is seen that the objective function is a convex function of I and we can be sure that optimal solution is obtained. Since all results are obtained by Markov modeling of production process thus the results are reliable and they can be verified in practical environment.

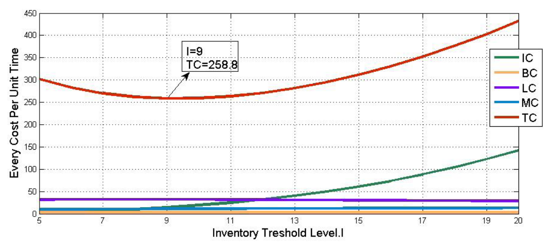

Figure 2. All costs as functions of I

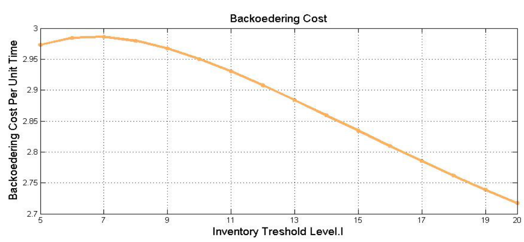

Figure 3. Backordering cost as function of I

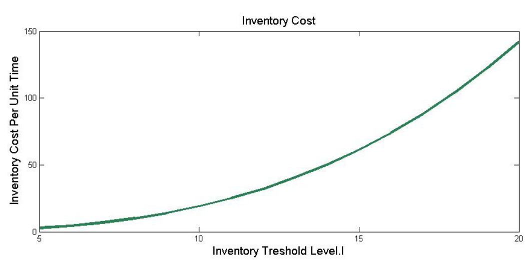

Figure 4. Inventory cost as function of I

DOI: https://doi.org/10.30564/mmpp.v2i1.655

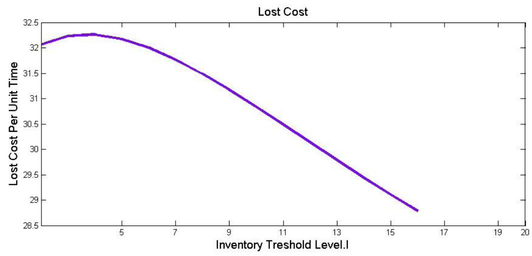

Figure 5. Lost Sales cost as function of I

Figure 6. Maintenance cost as function of I

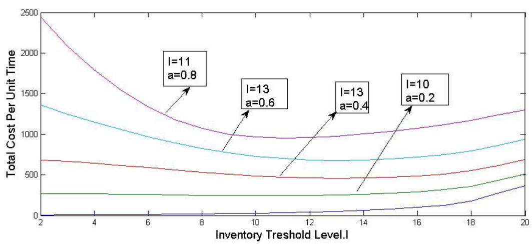

Figure 7. Total cost as a function of α and I

6. Conclusion

In this research, a Markov model is developed to obtain production repair decision. The required probabilities are obtained by formulations of Markov chains. Then a cost objective function is developed. Since the decision variables are integer thus objective function is solved by numerical methods. Also it is seen that the objective function is a convex function of decision variable thus we can be sure that optimal solution is obtained. Since all results are obtained by Markov modeling of production process thus the results are reliable and it can be verified in practical environment.

References

[1] Abboud.N.E. A discrete-time Markov production-inventory model with machine breakdowns. Computers and Industrial Engineering, 2001, 39: 95-107. https://doi.org/10.1016/S0360-8352(00)00070-X

[2] Grabski.F. The reliability of an object with semi-Markov failure rate. Applied Mathematics and Computation, 2003, 135: 1-16. https://doi.org/10.1016/S0096-3003(01)00271-5

[3] Prowell.S.J, Poore.J.H. Computing system reliability using Markov chain usage models. Journal of Systems and Software, 2004, 73(2): 219–225. https://doi.org/10.1016/S0164-1212(03)00241-3

[4] Grabski,F. Semi-Markov failure rates processes. Applied mathematics and computation, 2003, 217: 9956-9965. https://doi.org/10.1016/j.amc.2011.04.055

[5] Veeramany.A, D. Pandey.M. Reliability analysis of nuclear component cooling water system using semi-Markov process model. Nuclear Engineering and Design, 2011, 241(5): 1799-1806. https://doi.org/10.1016/j.nucengdes.2011.01.040

[6] Lisnianski.A, Elmakias.D, Laredo.D, Ben Haim.H. A multi-state Markov model for a short-term reliability analysis of a power generating unit. Reliability Engineering & System Safety, 2012, 98(1): 1-6. https://doi.org/10.1016/j.ress.2011.10.008

[7] Wang.X, Liu.W. Research on Air Traffic Control Automatic System Software Reliability Based on Markov Chain. International Conference on Applied Physics and Industrial Engineering , 2012, 24: 1601-1606. https://doi.org/10.1016/j.phpro.2012.02.236

[8] Liu.B, Cui.L, Wen.Y, Shen.J. A performance measure for Markov system with stochastic supply patterns and stochastic demand patterns. Reliability Engineering and System Safety, 2013, 119: 294-299. https://doi.org/10.1016/j.ress.2013.07.001

 DOI: https://doi.org/10.30564/mmpp.v2i1.655