

ARTICLE

Impedance, What It Is, How It Is Measured, and Why It Is Needed

V.K. Balkhanov*

Institute of Physical Materials Science of the Siberian Branch of the Russian Academy of Sciences, Ulan-Ude, Russia

ARTICLE INFO

Article history

Received: 18 April 2022

Accepted: 1 June 2022

Published Online: 8 June 2022

Keywords:

Impedance

MSI

Underlying medium

Logging

ABSTRACT

Impedance is the basic concept and quantity when measuring an electromagnetic field near the earth's surface. It is shown how the antennas of the IPI device are oriented, and how the coordinate system is set. It is established why the phase difference of the electromagnetic field component is limited to the limits from zero to minus ninety degrees. The introduction of the basic electrophysical characteristics of a continuous medium - dielectric constant and electrical conductivity - is considered. For a homogeneous medium, the dependence of impedance on electrophysical quantities is given. The Riccati equation for impedance is given. Not only the horizontal arrangement of the electrical cable is considered, but also the vertical one. The latter allows you to explore the electrical parameters of the media.

1. Introduction

In the theory of electromagnetism and geoprospecting, an important quantity is impedance, defined as the ratio of the components of the electric field to the magnetic field near the earth's surface. This definition introduces the Cartesian coordinate system. The importance of this value is manifested in the fact that the phase difference of the components of the electromagnetic field is exactly equal to the phase of impedance. The work will establish that the impedance phase is limited to the limits from 0 to minus ninety degrees. To deal with electromagnetic waves, the impedance must be much less than one, this is the Le-

onitovich-Shchukin condition. A homogeneous underlying medium has constant values of electrical conductivity and dielectric constant, which are related to impedance. The real underlying media are heterogeneous, in which the electrical conductivity and dielectric constant depend on the depth of the earth's rocks. Impedance allows you to enter effective values of electrical conductivity and dielectric constant. The Riccati equation for impedance is given. Not only the horizontal arrangement of the electrical cable is considered, but also the vertical one. The latter allows you to explore the electrical parameters of the environment. In order not to talk about geoelectrics, now we need to talk about impedance media.

*Corresponding Author:

V.K. Balkhanov,

Institute of Physical Materials Science of the Siberian Branch of the Russian Academy of Sciences, Ulan-Ude, Russia;

Email: ballar@yandex.ru

DOI: <https://doi.org/10.30564/nmms.v4i1.4640>

Copyright © 2022 by the author(s). Published by Bilingual Publishing Co. This is an open access article under the Creative Commons Attribution-NonCommercial 4.0 International (CC BY-NC 4.0) License. (<https://creativecommons.org/licenses/by-nc/4.0/>).

2. Electrical Conductivity, Resistivity and Dielectric Constant

First, let us recall the meaning of the basic quantities used in electromagnetism.

All continuous media consist of charged particles - electrons and nuclei, which are grouped into atoms and molecules, and are in perpetual motion. At distances much larger than the size of nuclei, all charged particles interact with each other through electric fields, which, in fact, charged particles bind together into atoms and molecules. We consider distances, much larger than the sizes of atoms and molecules, when it is necessary to take into account the laws of quantum mechanics, and it is possible not to be distracted (within the limits of our article!) by the laws of quantum mechanics.

Under the influence of an external electric field, all charges come into motion. Electrons begin to move away from atoms, molecules change their geometric shape. It is usually said that the continuous medium acquires dielectric properties. This property is described by the introduction of the dielectric constant of the substance that is denoted by how ϵ . The presence of dielectric properties leads to reduction of the vector of the external electric field E up to value E/ϵ in dielectrics.

The vector does not change direction, but decreases in magnitude, {but it is possible to create modern meta-materials that also change the direction of the vector}. It follows from this definition that the dielectric constant is a dimensionless quantity. If the dielectric constant is one ($\epsilon=1$), then the external electric field does not change, remaining unchanged in magnitude and direction. Such an environment is free space - air, atmosphere and all empty space. Since all continuous media are composed of charged particles, all substances are actually **dielectrics**. This leads to the fact that the light is refracted in the glass, and is delayed by the wall, through which we do not see anything. And to many other effects that we will not dwell on.

Let's talk a little about the value of the dielectric constant. Being on the street, we easily look around the surroundings through the air. Therefore, the dielectric constant of the air medium is one ($\epsilon=1$). Something is visible through a rare shrub, so its dielectric constant will be slightly greater than one (for example, equal to $\epsilon=1.1$).

In some continuous media, some electrons feel free. Then any external electric field will set them in motion. We say that an electric current (usually referred to as I) has appeared in the medium. It can be easily measured with an ammeter! Media that have free electrons are called conductors. This property is described by electrical

conductivity (designated, how σ). Since one part of the charged particles is attached inside the medium, and the other part in it is free, all substances can have both properties at once, i.e. dielectrics can often conduct an electric current. In this case, they can be called semiconducting (not to be confused with semiconductors! - a separate class of materials in which charges of different signs play an equal role), in which electrical parameters ϵ and σ play equal roles. The movement of free charges is mainly influenced by the nuclei of atoms and molecules, which inhibit the movement of charges, thereby reducing the electric current. This phenomenon is described by the introduction of specific *electrical resistanc* (denoted by how ρ). Since the ability to conduct an electric current and resist it are the flip sides of a single phenomenon, conductivity and resistivity must be related to each other. In their distant time, they were defined as inverses of each other, i.e.

$$\sigma = 1/\rho. \quad (1)$$

In different formulas, you can use both notations in the same way, just without confusing them with each other. Some researchers are accustomed to using conductivity, others - resistivity. (And, sometimes, when talking about the same thing, they don't understand each other; don't be like them!).

We do not give in the form of tables the values of the dielectric constant and conductivity of various media and materials. They are easy to find on the Internet (previously, the researcher had to have solid reference books for this, they say, when asked whether he remembers certain or constant ones, A. Einstein replied that no - he takes the necessary values from reference books). But we can't resist, and we'll give these values for the two environments. For Baikal water (the purest water in nature!):

$$\begin{aligned} \text{dielectric permeability } \epsilon &= 81; \\ \text{specific resistance } \rho &= 150 \Omega \cdot m. \end{aligned}$$

For granitic rocks (the main rock that makes up the earth's crust and lithosphere):

$$\begin{aligned} \text{dielectric ermeability } \epsilon &= 5; \\ \text{specific resistance } \rho &= 10^6 \Omega \cdot m. \end{aligned}$$

Thus, we demonstrate the range of these quantities.

3. Electromagnetic Field and Wave Number

Everything that is under our feet is called the underlying media. These are forest vegetation, a layer of soil, water mass and sedimentary rocks. The Earth's atmosphere is literally filled with electromagnetic fields of different nature and different frequencies. They spread differently along the earth's surface and penetrate into the underlying media. It would be a sin not to use these fields to probe the terrestrial media. In the atmosphere, electromagnetic

waves propagate in the same way as in free space. These waves have electric vectors E and magnetic H fields simultaneously reach their maximum value and also simultaneously pass through zero (Figure 1). To put it another way, the electric and magnetic fields in free space are in phase with each other.

It turns out that electromagnetic waves, regardless of frequency in empty space, always propagate at the same speed - the speed of light c (in SI $c=10^8$). Thus, the frequency (denoted by f , measured in Hz = 1/sec) becomes of fundamental importance. No wonder, all electrical appliances, if anything, measure, but always on some a certain frequency. The distance between two zero values received, for example, by an electric vector, is called the wavelength (denoted by λ , and is measured in m). Moreover, the wavelength, frequency and speed of light are related by a simple ratio:

$$\lambda f = c. \tag{2}$$

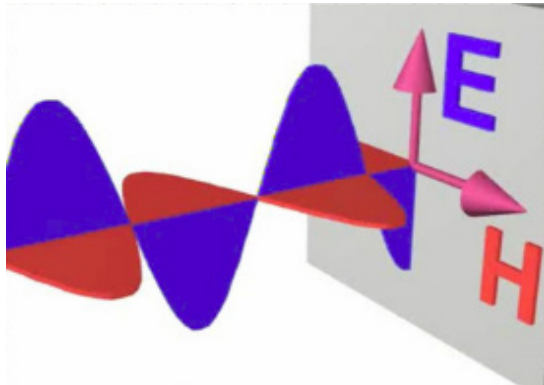


Figure 1. Electric E and magnetic H fields in free space.

Although the exact value of the speed of light in SI is known:

$$c = 2.9979 \times 10^8,$$

but for evaluation you can take

$$c = 3 \cdot 10^8 \text{ m/s}$$

(thus we make a mistake less than 0.3%). Then the electromagnetic wave with a frequency of 1 MHz will have a wavelength $c = \frac{3 \cdot 10^8}{10^6} = 300 \pm 1 \text{ m/s}$.

If we measure an electromagnetic wave, then it means that it came to us from somewhere, i.e. had a certain direction. To indicate this direction, you entered a **wave vector** k , the numerical value of which for a wave in free space is defined as $k = 2\pi / \lambda$. Together with the wave number, it is convenient to enter a circular frequency $\omega = 2\pi \cdot f$. Using a wave number and a circular frequency, the propagation of an electromagnetic wave along a positive direction, for example, the x -axis, will be described by the following

expression:

$$\exp(ikx - i\omega t). \tag{3}$$

This expression is called a *plane wave*.

An electromagnetic wave can propagate in a continuous medium, we see through glass. Therefore, in this case you can enter a wave number k , which has an electromagnetic wave in matter. It turns out that the square of a wave number in an impedance homogeneous medium is given by the following well-known expression ^[1]:

$$k^2 = \frac{\omega^2}{c^2} \varepsilon + \frac{i \mu_0 \omega}{\rho}. \tag{4}$$

Here in μ_0 - magnetic constant (in SI $\mu_0 = 4\pi \cdot 10^{-7}$). The rest are defined above. The two terms in this expression allow environments to be divided into two classes. If in some range of frequencies in the expression (4) the first term prevails, then the medium is called capacitive. If the second term is predominant, then the medium is considered inductive (or conductive). When both terms play the same roles, then the medium will be called impedance (although previously defined as semiconducting, but the name "impedance medium" is more successful).

4. Impedance, Its Measurement and Coordinate System

Measurement of electromagnetic waves, their reception, is carried out by special equipment - SIM (surface impedance meter ^[2]), an integral attribute of which is an electric antenna that responds to an electric field, and a magnetic sensor in the form of a frame for measuring a magnetic field (Figure 2, Figure 3, Figure 4). The electric field is measured simply by an electrical wire isolated from contact with the ground. A magnetic sensor is a regular square frame on which another electrical wire is wound. The plane of the frame is oriented to the maximum reception of the electromagnetic signal and is taken as the y -axis. The electrical wire is orthogonal to the frame and is positioned in the direction of the signal source (usually the radio station), and this orientation is taken as the x -axis. The third axis of z coordinates is orthogonal to the x and y axes and is usually oriented from the surface of the earth into the atmosphere. Thus, a rectangular (Descartes) coordinate system was introduced.

Electromagnetic fields always have some source of their radiation, the characteristics of which are often unknown. Even if it is known that if a particular wave is emitted by a certain radio station, then the parameters of the wave at the place of its reception are strongly influenced by the presence of the earth itself. The terrain, the distribution of electrical parameters on the ground, all

this affects what wave we receive at the reception point. According to the book [3], “The medias that have to be dealt with in the theory of the propagation of radio waves along the earth are the soil and the atmosphere”. And further [3], “... the surface layer of the earth always behaves completely differently with respect to waves of different frequencies”. But in 1950, Academician A.N. Tikhonov pointed out that if the receiving equipment was tuned to measure not the components of the electromagnetic field themselves, but their ratio, then the parameters of the source would decrease. The resulting value of the ratio of the electric field to the magnetic field was called the surface impedance, and was designated by the symbol δ [4-6].

$$\delta = -\frac{1}{Z_0} \left(\frac{E_x}{H_y} \right)_{z=0} \quad (5)$$

Vacuum resistance is introduced here $Z_0 = \sqrt{\mu_0/\epsilon_0} = 376.3\Omega$ in order for the magnitude of the impedance to be dimensionless (therefore, it is sometimes called reduced when it is made of a dimensionless value).

The electromagnetic field equations (i.e., Maxwell’s equations) are linear, so their solution can be sought in a complex form; so, temporary dependence is immediately sought in the form of $\vec{E}, \vec{B} \sim e^{-i\omega t}$. This means that impedance is a complex quantity. It is written in two forms. Or as

$$\delta = |\delta| \exp(i\varphi_\delta), \quad (6)$$

where is $|\delta|$ module and φ_δ - impedance phase. Or as $\delta = \text{Re} \delta + i \text{Im} \delta$,

where is $\text{Re} \delta$ - valid and $\text{Im} \delta$ - imaginary parts of the impedance.

If we combine the formulas (5) and (6), we get:

$$E_x = Z_0 |\delta| H_y \exp(i\delta_\varphi), \quad (8)$$

(I’m more impressed with this record). The result means that in an electromagnetic wave, the magnetic field is phase ahead of the electric field. Moreover, this phase is equal to the impedance phase.

It follows from Figure 1 that there is no phase lag in free space, i.e. in this case the phase of the impedance of free space is equal to $\delta_\varphi = 0^\circ$. It is easy to establish that when the sine wave is shifted to $\lambda/4$ the absolute values of the fields will not change ($|E(x)| = |E(x \pm \frac{\lambda}{4})|$). This circumstance is equivalent to the fact that the phase difference between the electric and magnetic fields will be periodic in 90° .



Figure 2. Surface impedance meter kit (MSI-300). The number 300 indicates the measuring range in kHz. A reeled insulated cable and frame are shown.

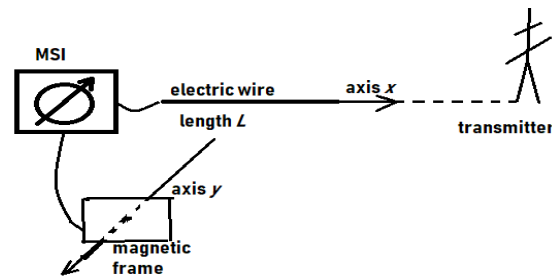


Figure 3. Surface impedance meter, orientation of electric and magnetic field meters along coordinate axes. The electric antenna lies on the ground, i.e. has a horizontal orientation.

Taking into account the sign of this phase difference, we come to the position that the phase δ_φ always extends from 0° before -90° . And this circumstance does not depend on the presence or absence of foreign bodies. The diversity of the structure of the earth’s surface means that, in general, the impedance phase can have any value - from zero to zero -. Hence 90° , the difference in the phase of electric and magnetic fields in Figure 1 will be any, depending on the presence of the underlying medium, and in general, on the presence of foreign bodies.

An interesting and curious history of the appearance of the concept of impedance is given in the articles [4,5]. In short, the story goes like this. The concept of impedance was introduced into electrodynamics O. Heaviside and O. Lodge. The introduction of impedance can be traced back to electrical engineering, as S.A. Schelkunoff did back in 1938, and as detailed in Feynman’s Physics Lectures. In these lectures, impedance is called complex resistance Z , which is the coefficient of proportionality between the voltage in the circuit U and the current J : $U = Z J$.

The voltage is created by the electric field E in a section of length L , so that impedance (aka complex circuit resistance) $Z = U / J = EL / J$. But the magnitude J/L is a current per unit length and determines the strength of the magnetic field H , i.e. $Z = E / H$. By introducing a dimensional multiplier in the form of vacuum resistance with resistance dimension, we arrive at the surface impedance given by the ratio (3).



Figure 4. Modified IPI-300 kit for measuring the electro-physical characteristics of the forest media. Vertical orientation of the electric antenna. 1. Telescopic rod; 2. Rod support; 3. Bottom mount; 4. Electric antenna suspension; 5. Dipole Nadeenko; 6. Input antenna amplifier; 7. Shielded wire; 8. Magnetic antenna; 9. MSI-300.

5. Leontovich-Shchukin Condition for Impedance

Taking the underlying medium for electromagnetic waves homogeneous, with constant values of dielectric constant and resistivity, from Maxwell's equations we can find:

$$\delta = \left\{ 1 + \varepsilon + \frac{i\sigma}{\varepsilon_0\omega} \right\}^{-1/2} = \frac{1}{\sqrt{1 + \varepsilon + i / \varepsilon_0\omega\rho}}. \quad (9)$$

Here $\varepsilon_0\omega\rho$. all the values are already familiar to us. Let's take it as a postulate: **the impedance task completely determines the electromagnetic fields in the air and in the underlying medium**. Although not in every case it is easy to do. Looking at the formula (9), we notice that the impedance depends on the dimensionless parameter - $\varepsilon_0\omega\rho$. Let's evaluate it by putting the $\sqrt{-i} = (1 - i)/\sqrt{2}$ frequency at 50 kHz and $\rho = 150 \Omega \cdot m$ (typical value for Baikal water is taken):

$$8.85 \times 10^{-12} \times 2\pi \times 50000 \times 150 = 4 \times 10^{-4} \ll 1.$$

It turns out that for many practically important tasks of geophysics and electrical exploration, this parameter is almost always much less than one:

$$\varepsilon_0\omega\rho \ll 1. \quad (10)$$

The smallness of this dimensionless parameter makes it possible to greatly simplify both the calculation themselves and resulting formulas. So, for a homogeneous medium, in the first non-disappearing limit for a small parameter $\varepsilon_0\omega\rho$, its inverse value turns out to be much greater than one, and in any case, more than the component $1 + \varepsilon$. Neglecting it, the impedance of (9) remains:

$$\delta = \sqrt{-i\varepsilon_0\omega\rho}, \quad (11)$$

and for its module and phase:

$$|\delta| = \sqrt{\varepsilon_0\omega\rho}, \quad \phi_\delta = -\frac{\pi}{4} = -45^\circ. \quad (12)$$

In this form, the impedance (its module and phase) is usually recorded for a homogeneous conductive medium. It can be said that when propagating over a conductive medium, the electric field, in comparison with free space, lags behind the magnetic field in phase by one eighth of the complete displacement.

For completeness, we give the values of the actual and imaginary parts of the impedance in question. As $\sqrt{-i} = (1 - i)/\sqrt{2}$, that

$$\text{Re } \delta = \sqrt{\varepsilon_0\omega\rho / 2}, \quad \text{Im } \delta = -\sqrt{\varepsilon_0\omega\rho / 2}.$$

The imaginary part of the impedance turns out to be a negative value, and in magnitude it is equal to the actual part.

In 1938, specially for electromagnetic waves, surface impedance was introduced by M.A. Leontovich. At the same time, from (10) and (12) it turns out that electromagnetic wave the impedance modulus $|\delta|$ turns out to be noticeably less than one (don't forget that!):

$$|\delta| \ll 1. \quad (13)$$

This inequality will be called the Leontovich-Shchukin approximation (in 1940 Shchukin published a book ^[6], in which he made extensive use of impedance).

For practical purposes ^[7,8] it is sufficient to assume that

$$|\delta| \leq 0.3. \quad (14)$$

Only in this case can we speak of electromagnetic fields as waves, variables in space and time. For example, near the emitter, the antenna is a static electric dipole. Then the magnetic field turns to zero, and the impedance (9) diverges, i.e. turns to infinity. However, in reality, electromagnetic waves can always be measured near the emitter, only they will have a bizarre spatial characteristic (the dependence of the field on the distance from the emitter). And at the same time, the Leontovich-Shchukin condition (13) will be fulfilled.

To determine the components of an electromagnetic

field, the so-called wave equations are usually derived from Maxwell's equations. They are usually solved either by separating variables or by using the Fourier-Hankel transform^[9]. But if we are interested in a directly measurable quantity - impedance, then we can establish an equation for it:

$$\frac{d\delta}{dz} = \frac{i\omega}{c} + \mu_0 c \sigma(z) \delta^2. \quad (15)$$

Mathematically, this is the Riccati equation, which is usually studied in mathematical physics courses (I may discuss it in one of the next articles).

6. Determination of Electrical Parameters for Impedance Measurement

The earth's interior is highly heterogeneous. According to the basic tenets of the theory of global tectonics, the lithosphere, the outer solid surface of the Earth, is a relatively rigid shell "floating" on the surface of a very viscous mantle. The shell is divided by tectonic disturbances into large and durable lithospheric megablocks - plates, the linear dimensions of which reach several thousand kilometers. Large tectonic plates are made up of smaller structural blocks, which in turn are broken down into many even smaller blocks. The real rock mass of the Earth's lithosphere is thus a complex hierarchical tile system of blocks decreasing in size. The hierarchical structure of the surface layer of the earth will also affect the response of the electrical parameters of the system to the external electromagnetic field. When measuring the impedance, we will obtain some effective values of the electrical conductivity σ_{ef} and dielectric conductivity ε_{ef} . In 1952, to study heterogeneous media, the French researcher L. Cagniard introduced apparent resistance. The idea of introducing it is very simple. If the impedance modulus was measured $|\delta|$, then, according to the formula (9) $|\delta| = \sqrt{\varepsilon_0 \omega \rho}$, convenient to enter

$$\rho_{eff} = \frac{|\delta|^2}{\varepsilon_0 \omega}. \quad (16)$$

This is the meaning of apparent resistance that the medium would possess if it were homogeneous. It is not difficult to obtain if the impedance phase and the effective value of the dielectric constant are known. To do this, taking into account the replacement of the dielectric constant and resistivity with their effective values, we will rewrite (9) in the following form:

$$\delta^2 = \frac{1}{1 + \varepsilon_{eff} + i/\varepsilon_0 \omega \rho_{eff}} = |\delta|^2 \exp(2i\varphi_\delta)$$

Then we separate the actual and imaginary parts. The resulting two equations are solved with respect to $1 + \varepsilon$.

Result (17) clarifies the definition (16). Because for a homogeneous conduction media. Then we separate the actual and imaginary parts. The resulting two equations are solved with respect to $1 + \varepsilon$ and $\sigma/\varepsilon_0 \omega$, then we find:

$$\rho_{eff} = \frac{|\delta|^2}{\varepsilon_0 \omega} \frac{1}{|\sin 2\varphi_\delta|}, \quad (17)$$

$$1 + \varepsilon_{eff} = \frac{\cos 2\varphi_\delta}{|\delta|^2}. \quad (18)$$

Result (17) clarifies the definition (16). Because for a homogeneous conduction media $\varphi_\delta = -45^\circ$, then (17) goes to (16).

Similar to the introduction of effective resistivity and dielectric constant, an effective skin layer can be introduced $H_s = \sqrt{2\rho/\mu_0 \omega}$ replacement ρ on ρ_{eff} :

$$H_s = \sqrt{\frac{2\rho_{eff}}{\mu_0 \omega}}. \quad (19)$$

We encounter this value in problems related to the heterogeneity of the media. Now, taking into account the replacement of electrical parameters with their effective values, the wave number will change, become effective:

$$\tilde{k} = k_0 \sqrt{\varepsilon_{eff} + \frac{i}{\varepsilon_0 \omega \rho_{eff}}}. \quad (20)$$

By combining impedance (4.1), only with the replacement of electrical parameters with their effective values,

$$\delta = \frac{1}{\sqrt{1 + \varepsilon_{eff} + i/\varepsilon_0 \omega \rho_{eff}}}, \text{ with a ratio (20), find}$$

$$\tilde{k} = k_0 \frac{\sqrt{1 - \delta^2}}{\delta}. \quad (21)$$

This relation expressed the wave number in the impedance medium through the impedance itself.

It has already been mentioned above that real natural environments are heterogeneous. Just now, formulas (16) to (21) simulated inhomogeneous media with effective electrical parameters. Their dielectric and conductive properties randomly change from one location to another. This situation is often modeled with a layered medium, when in a wide range of frequencies each layer has homogeneous electrical parameters. After a homogeneous environment, the next model is a two-layer model. Then three-layer, etc. In certain frequency ranges, many terrestrial surfaces are enough to simulate with a two-layer medium. Such environments are quite widely represented on the ground. For example, ice on the water, forest on the ground, etc. So, in winter on the Northern Sea Route and on the salt lakes of Eurasia (Southern Siberia, Mongolia and China) Radio routes can be simulated as a two-layer medium, where the first medium (ice, forest) is a dielectric

layer, and the second (sea, salt water, soil) in a wide frequency range of radio waves is the underlying conductive medium. Now we can see what values of the impedances of real media correspond to the effective electrical parameters and the skin layer. We have formulas (17) to (20) for this.

Example 1. We have a radio route “ice-salt water” near the village of Tiksi, at a frequency of 10 MHz ($\lambda_0 = 30$ m), impedance was measured $\delta = 0.185 \exp(-82.5^0)$. Now we can consistently find:

$$\rho_{eff} = 1.8 \cdot 10^7 \frac{0.185^2}{10^4 |\sin 165^0|} = 238 \Omega \cdot m,$$

$$\epsilon_{eff} = -1 + \frac{\cos 165^0}{0.185^2} = -29.2,$$

$$H_s = 15.9 \sqrt{\frac{238}{10^4}} = 2.45 m.$$

Example 2. Lake Sulfate: frequency 10 MHz ($\lambda_0 = 30$ m), $\delta = 0.152 \exp(-65.6^0)$,

$$\rho_{eff} = 55.3 \Omega \cdot m, \epsilon_{eff} = -29.5, H_s = 1.2 m.$$

Note that the effective value of the dielectric constant for some media has not only a negative value, but also a large absolute value.

7. Impedance Measurement in Forest and Mine (Logging)

The magnetic frame is always positioned in its plane orthogonally to the direction of the radio station, as in Figures 2-4. If the electrical cable is spread on the ground, then we can calculate from the formulas (17) and (18) the electrical parameters of the underlying medium. However, the electrical cable can also be positioned vertically. Then we will be able to restore the electrical parameters of not the underlying environment, but the environment, for example, the forest layer in Figure 4.

Let's place a measuring complex in the forest ^[10] with vertical orientation of the electrical cable. The measured impedance, which we call normal, will be (try to get it yourself, we will do it below):

$$\delta^n = \frac{\omega \lambda}{ck^2}, \quad (22)$$

where k is the square of the wave number of the forest-layer

$$k^2 = \frac{\omega^2}{c^2} \epsilon_w + i \mu_0 \omega \sigma_w. \quad (23)$$

In here σ_w and ϵ_w - conductivity and dielectric constant respectively of forest vegetation.

Formulas (22) and (23) can be used only for a homoge-

neous continuous medium, as the forest layer appears in the LW-HW ranges. Substituting $\lambda = \omega/c$ and (22) to the ratio (23), we get

$$\delta^n = \frac{1}{\epsilon_w}, \quad (24)$$

where is the complex permeability

$$\epsilon_w = \epsilon_w + \frac{i}{\epsilon_0 \omega \rho_w}. \quad (25)$$

Here is the resistivity $\rho_w = 1/\sigma_w$. As $\delta = |\delta|e^{i\varphi_\delta}$ then, separating the actual and imaginary parts, from (24) and (25) we find the calculation formulas for determining ρ_w and ϵ_w :

$$\epsilon_w = \frac{\cos \varphi_\delta}{|\delta|}, \rho_w = \frac{|\delta|}{\epsilon_0 \omega \sin \varphi_\delta}. \quad (26)$$

For the first time the formula (24) was obtained by V.A. Egorov. Thus, it was established that the specific electrical resistance of the forest layer in question $\rho_w = 37 \pm 12 k\Omega \cdot m$, and dielectric constant $\epsilon_w = 1.6 \pm 0.3$.

Since the beginning of the last century, at the instigation of the French brothers of Schlumberger engineers, the method of VES (vertical electric sounding) has been widely used to find oil and gas fields, when electrodes are driven into the surface of the earth, and a direct or low-frequency electric current is supplied through them ^[11]. By measuring the electrical potential between the electrodes, it is possible to judge the electrical conductivity of the earth's depths, and already from these values to predict the presence and power of minerals. However, this ignores another useful electrical characteristic of rocks, this is their dielectric constant. The values of the dielectric constant, coupled with conductivity, can and should also characterize the presence of certain rocks in the depths of the earth's interior. As we show, for this it is necessary to use natural or artificial variable electromagnetic fields from various sources - radio stations and specialized emitters. At the same time, it is possible to use already existing measuring complexes, such as MSI, with the addition of a magnetic sensor to them - frames with wound electrical wires. The dimensions of the frame only have to be narrow to fit into the well. After all, wells are often drilled on the surface of the earth to study the subsoil, and it is a sin not to use new and old wells.

First, the reduced surface impedance is measured δ equation (5). In this case, a frame is used, which is usually included in the complex of MSI equipment. Next, a similar frame is made, but having other dimensions to fit freely in the well. Calibration of the new frame is made by measuring the known reduced surface impedance. Next, a new frame and an ungrounded insulated electrical

wire are lowered in the form of a probe to a depth of the well. A new frame measures the horizontal component of the magnetic field H_φ , and vertical electric wire vertical component of electric field E_z . Component является горизонтальной, и ортогональной осям r и z . Reduced surface impedance δ can be called tangential, since it is located along the horizontal components of the magnetic field H_φ and electric field E_r , and is calculated according to the formula known to us equation (5):

$$\delta = -\frac{1}{\mu_{0m}c} \frac{E_{r0}}{H_{\varphi 0}} \Big|_{z=0}. \quad (27)$$

In here μ_{0m} is magnetic constant, and c is the speed of light.

Measured impedance by value of the component of the electromagnetic field in the well (horizontal magnetic field H_φ and vertical electric field E_z), can be called normal impedance δ^n . It is calculated by the formula

$$\delta^n = -\frac{1}{\mu_{0m}c} \frac{E_z}{H_\varphi}, \quad (28)$$

and turns out to be equal

$$\delta^n = \frac{k_0^2}{k^2} \sqrt{1-\delta^2} = \frac{\delta^2}{\sqrt{1-\delta^2}}. \quad (29)$$

Hence, in particular, neglecting the square of the impedance in the square root, we come to the formula (22). Here is the wave number in the atmosphere $k_0 = \omega/c$, and the square of the wave number in the terrestrial media.

$$k^2 = \frac{\omega^2}{c^2} \left(\varepsilon + \frac{i\sigma}{\varepsilon_0\omega} \right). \quad (30)$$

In here ε and σ - the desired dielectric constant and electrical conductivity of the earth's rock at depth. Normal impedance, as shown by the result (29), does not depend on the vertical coordinate. If the medium is homogeneous, then regardless of the depth of the probe's descent into the well, the normal impedance will always be the same. However, the real terrestrial environment is heterogeneous, so the normal impedance measured at different depths of the probe will be different. This circumstance will make it possible to judge the properties of rocks lying in the depths of the lithosphere, in particular the presence of minerals.

When the normal impedance is known from the measurements, it is now easy to reconstruct the dielectric constant values from the result equation (29):

$$\varepsilon = \text{Re} \frac{\sqrt{1-\delta^2}}{\delta^n}, \quad (31)$$

and electrical conductivity:

$$\sigma = \varepsilon_0\omega \text{Im} \frac{\sqrt{1-\delta^2}}{\delta^n}. \quad (32)$$

In here Re is valid part, Im is imaginary part.

If the meanings are known for a priori reasons ε and σ , and with them the square of the wave number (30), then the given surface impedance can be found from the relation (resulting from expression (29)).

$$\delta = \frac{k_0}{k} \sqrt{\frac{1}{2} \left(\sqrt{1 + \left(\frac{2k}{k_0} \right)^2} - 1 \right)}. \quad (33)$$

Task. Try to find out whether from such measurements it is possible to establish the magnetic permeability of the earth's interior.

Hint. Start with Maxwell's equations containing magnetic permeability.

8. How the Electromagnetic Field Penetrates into the Impedance Medium

Two methods can be used to calculate the components of an electromagnetic field. The first is to construct a wave equation with a source for the vector potential. For the latter, the solution is expressed as the Sommerfeld integral, from where all the components of the fields are further located in article [12]. The second simpler method is to construct a wave equation for the single non-zero component of the magnetic field H_φ . This method was carried out by solving the wave equation by the method of separating variables in article [13]. For the convenience of references, we present the results of calculations for non-zero components of the electromagnetic field in a continuous medium, expressed through the given surface impedance (K is a constant that does not play a role in determining the impedance):

$$H_\varphi = -ik_0\sqrt{1-\delta^2} \frac{K}{\sqrt{r}} \exp\left(-i\omega t + ik_0r\sqrt{1-\delta^2} - ik_0\frac{1-\delta^2}{\delta}z\right). \quad (34)$$

$$E_r = i\omega\delta\sqrt{1-\delta^2} \frac{K}{\sqrt{r}} \exp\left(-i\omega t + ik_0r\sqrt{1-\delta^2} - ik_0\frac{1-\delta^2}{\delta}z\right). \quad (35)$$

$$E_z = i\omega\delta^2 \frac{K}{\sqrt{r}} \exp\left(-i\omega t + ik_0r\sqrt{1-\delta^2} - ik_0\frac{1-\delta^2}{\delta}z\right). \quad (36)$$

Now, according to (5), we find that the magnitude δ is just the superficial impedance given. And from the formula (5) follows the result formula (4). Evaluation of the established fields shows that in free space and in the dielectric layer pattern, electric fields do not change, although in a dielectric electric fields are less than them in free space in $\frac{1}{\varepsilon_1}$, where is ε_1 is dielectric constant of dielectric medium.

In the conductive base, the electric field has basically

only a horizontal component: if taken $E_{z0} = 1$, that $E_{z1} = \frac{1}{\epsilon_1}$ and $E_{z2} = \delta^2$. The reduction of the electric field in the dielectric layer is consistent with the general concept of introducing a dielectric constant. Further $E_{r0} = \delta$, $E_{r1} = \frac{\delta}{\epsilon_1}$ и $E_{r2} = \delta$.

For an ideal conductor, its resistivity tends to zero. Along with it, impedance will tend to zero. Then from the obtained estimates it follows that in the conductor the longitudinal and vertical components of the electric field, together with the impedance, rush to zero. Together with them, the entire vector of the electric field will be zero, as required by the determination of the conductive material - the electric field does not penetrate into the conductor in book [14]. We now know how an electric field does not penetrate a conductor.

The magnitude of magnetic induction in all media has not changed much; if $B_{y0} = 1$ then and $B_{y1} = 1$, $B_{y2} = 1$. Qualitatively, the picture for the electric field is shown in Figure 5.

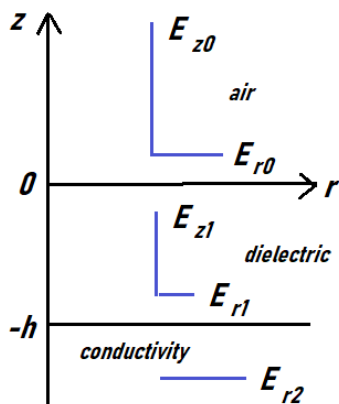


Figure 5. Horizontal and vertical components of the electric field (in relative units).

9. Conclusions

The paper introduces the basic concept and magnitude when measuring the electromagnetic field near the earth's surface - impedance. It is shown how the antennas of the IPI device are oriented, and how the coordinate system is set. It is established why the phase difference of the electromagnetic field component is limited to the limits from zero to minus ninety degrees. The introduction of the basic electrophysical characteristics of a continuous medium - dielectric constant and electrical conductivity - is considered. For a homogeneous medium, the dependence of impedance on electrophysical quantities is given. The Riccati equation for impedance is given. Not only

the horizontal arrangement of the electrical cable is considered, but also the vertical one. The latter allows you to explore the electrical parameters of the media.

Funding

Work done in Laboratory Electromagnetic Diagnostics Institute of Physical Materials Science of the Siberian Branch RAS on the state budget project "Development of the fundamental foundations of diffusion HF-VHF and UHF radio waves in inhomogeneous impedance channels".

Conflict of Interest

There is no conflict of interest.

References

- [1] Balkhanov, V.K., Bashkuev, Yu.B., 2022. Electromagnetic surface impedance, geoelectric homogeneous, two-layer and gradient media, Sommerfeld integral, 2nd ed.
- [2] Parfentyev, P.A., Pertel, M.I., 1991. Device for measuring surface impedance in the SDV-SV band in the Earth-Ionosphere waveguide, Gylym, 133, Alma-Ata.
- [3] Feinberg, E.L., 1999. Propagation of radio waves along the earth's surface. Fizmatlit, Moscow.
- [4] Berdnik, S.L., Penkin, D.Yu., Katrich, V.A., et al., 2014. The use of the concept of surface impedance in electrodynamic problems (75 years later). Radiophysics and Radio Astronomy. 19(1), 57-80.
- [5] Miller, M.A., Talanov, V.I., 1961. The use of the concept of surface impedance in the theory of surface electromagnetic waves. Izvestiya vuzov, Ser. Radiophysics. 4(5).
- [6] Shchukin, A.N., 1940. Propagation of radio waves. Military-Maritime, Moscow; Leningrad, Publishing House.
- [7] Balkhanov, V.K., Bashkuev, Yu.B., Angarkhaeva, L.Kh., 2017. New Horizons in Mathematical Physics. 1(3). DOI: <https://dx.doi.org/10.22606/nhmp.2017.13002>
- [8] Balkhanov, V.K., Bashkuev, Yu.B., Angarkhaeva, L.Kh., 2018. Surface Impedance of a Highly Inductive Two-layer Medium. Technical Physics. 63(3), 438-442. DOI: <https://doi.org/10.1134/S1063784218030015>
- [9] Markov, G.T., Chaplin, A.F., 1983. Excitation of electromagnetic waves. Radio & Communications, Moscow.
- [10] Balkhanov, V.K., Advokatov, V.R., Bashkuev, Yu.B., 2014. Deformation of a fresh-water ice cover due to

- the capillary oscillations of underlying water. *Technical Physics*. 59(8), 1255-1256.
DOI: <https://doi.org/10.1134/S1063784214080040>
- [11] Fock, V.A., 1933. *Theory of Determining the Resistance of Rocks by the Method of Logging*. Moscow-Leningrad, GTTI.
- [12] Balkhanov V.K., Bashkuev, Yu.B., Angarkhaeva, L.Kh., 2019. Vector Potential of Electromagnetic Wave above Strongly Inductive Two-Layer Earth Surface. *Technical Physics* September. 64(9), 1361-1366.
DOI: <https://doi.org/10.1134/S1063784219 090032>
- [13] Balkhanov, V.K., Advocates, V.R., Bashkuev, Yu.B., 2014. Averaged electrical characteristics of the “forest layer” and the height of the forest cover. *Technical Physics*. 84(8), 132-135.
DOI: <https://doi.org/10.1134/S1063784214080039>
- [14] Landau, L.D., Lifshitz, E.M., Pitaevskii, L.P., 1984. *Electrodynamics of Continuous Media*. 8. (2nd ed. Butterworth-Heinemann).