

SHORT COMMUNICATION

Principal-agent Relationships: A Note on Biomass Depletion

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ABSTRACT

Public authorities frequently mandate public or private agencies to manage their renewable natural resources. Contrary to the agency, which is an expert in renewable natural resource management, public authorities usually ignore the sustainable level of harvest. In this note, we first model the contractual relationship between a principal, who owns the renewable natural resource, and an agent, who holds private information on its sustainable level of harvest. We then look for the Pareto-optimal allocations. In the situation of an imperfect information setting, we find that the Pareto-optimal contracting depends on the probability that the harvesting level stands outside the sustainability interval. The information rent held by the agent turns out to be unavoidable, such that stepping outside the sustainability interval implies the possibility of depletion of the renewable natural resource. This, in turn, compromises the maintenance of the ecological balance in natural ecosystems.

1. Introduction

States, regions and municipalities frequently mandate public or private agencies to manage their renewable natural resources^[4,11]. The following note analyzes these principal-agent-type relationships^[2,6,9].^① When it comes

^① The principal-agent problem occurs when an entity called the agent makes decisions on behalf of an entity called the principal. The problem exists in circumstances where agents act in their own best interests.

to natural resource management, principals and agents can have diverse preferences and objectives, be it on the harvest rates, the investments, the provision of natural and environmental amenities, etc. For instance, in the forest-based industry characterized by the exploitation of biomass, renewable natural resource owners frequently denounce the managing agency for withholding the information on the sustainable level of harvesting^[1]. In this case, what kind of properties condition sustainability?

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The prime consideration of this note is to understand how -- when it comes to managing renewable natural resources -- the agent's private information may come into conflict with the objective of a principal, be it a secondary topic that has been neglected in the literature. We thus model the contractual relationship between a principal and an agent as regards the renewable natural resource management. In the situation of an imperfect information setting, we find that the Pareto-optimal contracting depends on the probability that the level of harvesting stands outside the sustainability interval. This puts the Pareto-optimal allocations in jeopardy. The information rent held by the agent being unavoidable, stepping outside the sustainability interval implies the possibility of depletion of the renewable natural resource. This potential ecological overshoot, which occurs when human demand exceeds the regenerative capacity of a natural ecosystem, puts the Earth on an unsustainable trajectory.

After this starting section, we describe the model in Section 2, with a focus on the principal-agent relationship in a perfect information setting (Subsection 2.3) and when the agent owns private information on the sustainable level of harvesting (Subsection 2.4). Section 3 concludes.

2. A Principal-agent Model in Renewable Natural Resource Management

We consider a principal who delegates the management of a renewable natural resource of stock s to an agent. The principal and the agent have to agree upon a quantity to be harvested q from the renewable natural resource levels owned by the principal. Stock s is considered as a proxy of the natural resource amenities, which are all non-market goods and services related to the existence of this resource^[8]. The sustainable harvesting of renewable natural resources, such that the stock remains unchanged or $\frac{ds}{dq} = 0$, means that the level of harvest matches with the level of the resource's natural growth. Unsustainable harvesting can be envisaged in two ways. When $\frac{ds}{dq} > 0$, the scenario corresponds to the damages issued from the resource over-stocking^[3]. The absence of implementation of a management plan is one possible way to envisage such a context. When $\frac{ds}{dq} < 0$, the scenario implies the over-harvesting of the stock and the depletion of the renewable natural resource. Finally, the principal gives revenue share r to the agent for the renewable natural resource management, harvesting included, such that $\frac{dr}{dq} > 0$.

The principal and the agent can have different preferences on how to conduct the renewable natural resource management. For instance, the agent could have

stronger preferences for income from harvesting, while the principal could have stronger preferences for preserving the resource's natural and environmental amenities.

In the knowledge that the renewable natural resource harvesting must not jeopardize its sustainability, interval $[\underline{q}, \bar{q}]$ defines the bounding minimum and maximum levels of harvesting. It is the set of possible actions to be undertaken in purpose of the sustainable resource management. In detail, there is a maximum level of harvesting beyond which the resource is depleted. As a result, the resource cannot be renewed and no longer provides natural and environmental amenities. There is also a minimum level of harvesting below which amenities start to vanish. This can be explained by the absence of a management plan. Therefore, q has to lie somewhere within this interval. These assumptions implicitly mean that, in a specific range of harvesting levels, resources and natural and environmental amenities are complementary.

If we assume that the principal has stronger preferences for the stock conservation than the agent, the objectives of the parties should differ. The principal's harvesting objective will tend to be a low level of harvest, hence running the risk of being below \underline{q} . On the contrary, the agent will seek to harvest beyond this level.[Ⓢ] The knowledge of the interval of the sustainable levels of harvesting is crucial. The key issue of the contract setting is the information gap as to this interval. We will thus consider that this interval can correspond either to common knowledge or to a private information.

2.1 The Principal: The Renewable Natural Resource Owner

Let V_p be the value function of a risk-neutral principal. This value function is decreasing in revenue transfer r paid to the agent. However, the effect of renewable natural resource stock s and harvesting q on the value function is ambiguous, since harvesting generates revenues but, at the same time, reduces utility by diminishing the levels of natural and environmental amenities. While the principal is willing to harvest less than the agent, the question is whether principal's optimal harvest level q_p^* lies or not within sustainability interval $[\underline{q}, \bar{q}]$. The principal's value function is defined by:

$$V_p = V_p(s(q), I_p(q), r(q)) \tag{1}$$

where $I_p(q)$ represents the principal's identity function such that:

$$I_p(q) := \begin{cases} q_p^* > \underline{q} \text{ if } V'_{pq} = \frac{\partial V_p(s,q,r)}{\partial q} \leq 0 \\ q_p^* \leq \underline{q} \text{ if } V'_{pq} = \frac{\partial V_p(s,q,r)}{\partial q} > 0 \end{cases} \tag{2}$$

[Ⓢ] The opposite assumption yields symmetric results with respect to maximum level \bar{q} .

with q_p^* the (optimal) harvest quantity that maximizes the function defined by Equation (1). In particular, Equation (2) states that $V'_{pq} > 0$, which means that the principal's optimal harvest level lies within the sustainability interval, and does not otherwise.

From the foregoing, it can be stated that:

- $V'_{ps} \equiv \frac{\partial V_p(s(q), I_p(q), r(q))}{\partial s} \geq 0$, based on the principal's preferences toward non-market goods and services;
- $V'_{pr} \equiv \frac{\partial V_p(s(q), I_p(q), r(q))}{\partial r} < 0$, for the principal pays the agent to execute the management plan;
- $V'_{pq} \equiv \frac{\partial V_p(s(q), I_p(q), r(q))}{\partial q} \geq 0$, depending on principal's identity function $I_p(q)$.

Let us now analyze the conditions in which principal's optimal harvest level q_p^* lies within sustainability interval $[q, \bar{q}]$.

Lemma 1 *The condition for the principal's optimal harvest level q_p^* to lie within $[q, \bar{q}]$ is $V'_{pq} = -V_{pr} \frac{dr}{dq}$.*

Proof See the Appendix.

Thereby, the optimal harvest level implies that the marginal value from the renewable natural resource harvest equals the marginal remuneration for the renewable natural resource management.

2.2 The Agent: The Renewable Natural Resource Manager

Let V_a be the value function of a risk-neutral agent. We have previously assumed that the agent in charge of the renewable natural resource management is more willing to harvest than the principal. We are interested in revealing agent's optimal harvest level q_a^* within the sustainability interval $[q, \bar{q}]$. The agent is endowed with the following value function:

$$V_a = V_a(s(q), r(q), c(q)) \quad (3)$$

where c expresses the harvesting costs and q depends on q , following the simple functional form $q = \underline{q} + v$, where v can be a positive or a negative deviation from this level of harvesting.

The agent's value function increases in resource stock s , harvest quantities q and in revenue transfer r , and decreases in harvest costs c . Put another way:

- $V'_{as} \equiv \frac{\partial V_a(s(q), r(q), c(q))}{\partial s} \geq 0$, based on the agent's preferences toward non-market goods and services;
- $V'_{ar} \equiv \frac{\partial V_a(s(q), r(q), c(q))}{\partial r} > 0$, for the agent is paid by the principal to execute the management plan;
- $V'_{ac} \equiv \frac{\partial V_a(s(q), r(q), c(q))}{\partial c} \leq 0$, which involves increasing returns from the resource management.

Lemma 2 *The condition for the agent's optimal harvest level q_a^* to lie within $[q, \bar{q}]$ is $-V'_{ac} \frac{dc}{dq} = V'_{ar} \frac{dr}{dq}$.*

Proof See the Appendix.

The optimal harvest level implies that the marginal remuneration from the renewable natural resource management equals the marginal loss from the embedded management costs.

2.3 The Optimal Contract with Public Information on the Sustainability Interval

We assume that both the principal and the agent have the knowledge of sustainability interval $[q, \bar{q}]$. The principal then chooses a level of harvest such that he maximizes his utility, subject to the agent's participation constraint (or individual rationality (IR) constraint) and to minimum harvesting level \underline{q} issued from this sustainability interval:^③

$$\begin{aligned} \max_{q,r} V_p(s(q), I_p(q), r(q)) \\ \text{subject to (IR) } V_a(s(q), r(q), c(q)) \geq 0 \end{aligned} \quad (4)$$

The program means that the principal binds the agent to his participation constraint, that is, he sets the revenue transfer so that the harvest costs are covered and the agent is not in deficit. Although the principal might want to harvest less than the minimum sustainable level ($q_p^* < \underline{q}$), he has to comply with the sustainability constraint.

The Lagrangian can be written:

$$L = V_p(s(q), I_p(q), r(q)) + \lambda_{IR}[V_a(s(q), r(q), c(q))] \quad (5)$$

The first-order conditions implicitly give harvest level q^* and revenue transfer r^* :

$$\frac{\partial L}{\partial q} = \frac{\partial V_p(s(q), I_p(q), r(q))}{\partial q} + \lambda_{IR} \left[\frac{\partial V_a(s(q), r(q), c(q))}{\partial q} \right] = 0 \quad (6)$$

$$\frac{\partial L}{\partial r} = \frac{\partial V_p(s(q), I_p(q), r(q))}{\partial r} + \lambda_{IR} \left[\frac{\partial V_a(s(q), r(q), c(q))}{\partial r} \right] = 0 \quad (7)$$

$$\frac{\partial L}{\partial \lambda_{IR}} = 0 \quad (8)$$

When an additional level of harvesting provides, for the principal, an increasing marginal utility ($V'_{pq} > 0$), while it grants the agent with a diminishing marginal utility ($V'_{aq} < 0$), Equations (6) and (7) give:

$$\lambda_{IR} = -\frac{V'_{pq}}{V'_{aq}} \neq 0 \quad (9)$$

^③ This is minimum sustainable harvesting \underline{q} at stake, for the principal is willing to harvest less while the agent struggles to reach this level of harvesting.

$$\frac{V'_{pr}}{V'_{pq}} = \frac{V'_{ar}}{V'_{aq}} \quad (10)$$

From Equation (8), we know that the agent's participation constraint is binding, that is, the information rent is set to zero. The sign in Equation (9) indicates that an optimum exists. At last, Equation (10) means that the first-best contractual harvesting is when the principal's and the agent's ratios of marginal values equate.

The complete information efficient harvesting level is such that:

$$q^* < \underline{q} \quad (11)$$

Indeed, the principal cannot request to harvest outside the sustainability interval, so that the agent harvests up to the minimum sustainable level. Besides, harvesting diminishes the principal's marginal utility outside the sustainability interval.

The following proposition ensues.

Proposition 1 *In case of a perfect information setting, the Pareto-optimal contracting implies a level of harvest at least equal to the lower bound of -- and thus inside -- the sustainability interval, such that the principal's value function remains unchanged.*

Proof See the Appendix.

Proposition 1 uncovers the fact that the optimal contracting does not affect the principal's value function, so that the marginal costs and benefits from harvesting cancel each other.^④

2.4 The Optimal Contract with Private Information on the Sustainability Interval

In an imperfect information case, the principal does not know what the sustainable harvest interval is. The information is privately held by the agent. This time, the agent announces an interval $[\underline{q}, \bar{q}]$ to the principal, where the bounds depend on his own optimal levels of harvesting. The private knowledge of the sustainability interval gives the agent an opportunity to over-estimate the minimum harvesting level.^⑤ This in turn stimulates the principal to accept a more intensive level of harvesting.

Following^[7], we assume that $F(\cdot)$ is a continuous distribution function, with a positive density $f(\cdot)$, that describes the prior of the principal over the set of potential minimum levels of sustainable harvesting $[0, s]$.^⑥ The principal maximizes his expected value function subject

to the agent's individual rationality (IR) and incentive compatibility (IC) constraints:

$$\begin{aligned} & \max_{q,r} \int_0^s V_p(s(q), I_p(q), r(q)) f(q) dq \\ & \text{subject to (IR)} \quad V_a(s(q), r(q), c(q)) \geq 0 \\ & \quad \quad \quad \text{(IC)} \quad V_a(s(q), r(q), c(q)) - V_a(s(\bar{q}), r(\bar{q}), c(\bar{q})) \geq 0 \end{aligned} \quad (12)$$

Based on the above, the principal is maximizing his surplus by integrating his payoff function over \underline{q} . Although this value is provided by the agent, the principal reveals his preferences regarding \underline{q} .

The risk is that the agent attempts to signal a level of \underline{q} which maximizes his payoff function at a cost to the principal. Indeed, the agent sends a signal of the minimal sustainable harvesting level, but the principal ignores whether this signal is an honest one. Since the agent knows that both players disagree on the harvest level, and that the interval defining the sustainable harvest is his private information, he might want to belie on \underline{q} and choose a level that can lower the principal's payoff. In consequence, the principal has to maximize his value function over density $f(\underline{q})$, knowing that the real $f(\underline{q})$ is somewhere between 0 and s .

Two cases are possible:

In case $q_p^* \geq q_a^*(\underline{q})$, the principal and the agent implicitly agree on the harvesting volume. Therefore, both the principal and the agent enter into a contract that optimizes their respective payoff functions. Indeed, $q_a^*(\underline{q})$ saturates the participation constraint.

In case $q_p^* < q_a^*(\underline{q})$, the principal and the agent implicitly disagree on the harvesting volume. Given that $q_a^*(\underline{q}) > 0$, for the natural resource management includes some non-null resource maintenance, we have $0 < q_p^* < q_a^*$. This time, the harvesting level does not give the agent an incentive to contract.

The IC constraint is a maximizing argument. Hence, we have to look at the optimal conditions. The optimal condition of the IC constraint, that is, $v(q(\underline{q}), q(\underline{q})) \geq v(q(\underline{q}), q(\bar{q})) \forall [0, s]$, is given by:

$$0 = \frac{\partial v(q(\underline{q}), q(\bar{q}))}{\partial q(\bar{q})} \Big|_{q(\bar{q}) = q(\underline{q})}$$

The second-order condition of the IC constraint is:

$$0 \geq \frac{\partial^2 v(q(\underline{q}), q(\bar{q}))}{\partial q(\bar{q})^2} \Big|_{q(\bar{q}) = q(\underline{q})}$$

We define the Hamiltonian of the associated maximization problem:

$$\begin{aligned} H = & V_p(s(q), I_p(q), r(q)) f(q) \\ & + \lambda_{IR} [V_a(s(q), r(q), c(q))] \\ & + \lambda_{IC} \left[\frac{\partial V_a(s(q), r(q), c(q))}{\partial q} \right] \end{aligned} \quad (13)$$

④ This property only holds because we study the case of renewable natural resources.

⑤ It is our only variable of interest, since we assume that the principal is less willing to harvest than the agent.

⑥ The possible set of harvest goes from no harvesting ($q=0$) to clear-cutting of the whole stock ($q=s$), even if the latter is never reached.

Contract variables q and r must satisfy:

$$\frac{\partial H}{\partial q} = \frac{\partial V_p(s(q), I_p(q), r(q))}{\partial q} f(q) + \lambda_{IR} \left[\frac{\partial V_a(s(q), r(q), c(q))}{\partial q} \right] + \lambda_{IC} \frac{\partial}{\partial q} \left[\frac{\partial V_a(s(q), r(q), c(q))}{\partial q} \right] = 0 \quad (14)$$

$$\frac{\partial H}{\partial r} = \frac{\partial V_p(s(q), I_p(q), r(q))}{\partial r} f(q) + \lambda_{IR} \left[\frac{\partial V_a(s(q), r(q), c(q))}{\partial r} \right] + \lambda_{IC} \frac{\partial}{\partial r} \left[\frac{\partial V_a(s(q), r(q), c(q))}{\partial r} \right] = 0 \quad (15)$$

The boundary at $q = 0$ is unconstrained, meaning that q could take any value on the interval. Hence the transversality condition is $\lambda_{IC}(q) = 0$, that is, the shadow price of q is zero.

Therefore, $\frac{\partial H}{\partial \lambda_{IC}} = f(q)$ yields $\lambda_{IC}(q) = \int \frac{\partial H}{\partial \lambda_{IC}(q)} = F(q)$, which gives:

$$\lambda_{IR} = - \frac{\frac{\partial V_p(s(q), I_p(q), r(q))}{\partial q} f(q) + \frac{\partial^2 V_a(s(q), r(q), c(q))}{\partial q^2} F(q)}{\frac{\partial V_a(s(q), r(q), c(q))}{\partial q}} \quad (16)$$

The second-best optimal condition is therefore:

$$\frac{V'_{pr}}{V'_{pq}} - \sigma = \frac{V'_{ar}}{V'_{aq}} \quad (17)$$

where

$$\sigma = \frac{V''_{aqq} - V''_{arq} V'_{aq} F(q)}{V'_{aq} f(q)} \quad (18)$$

In comparison to the perfect information case, an additional term σ arises. The optimal condition for contracting is when the principal's and the agent's ratios of marginal values equate modulo σ . As the sustainability interval is not known to the principal, the agent announces a level which maximizes his own payoff. In this sense, σ represents the information rent captured by the agent or the loss in the payoff inflicted to the principal.

The second-best optimal harvesting q^{SB} induces the following probability over q :

$$F(q) = \left(\frac{V'_{pr}}{V'_{pq}} - \frac{V'_{ar}}{V'_{aq}} \right) \frac{V'_{aq} f(q)}{V''_{aqq} - V''_{arq} V'_{aq}} \quad (19)$$

If $F(q) = 0$, we fall on the perfect information case and $q^* = q$. Should this not be the case, the consequential proposition can be stated.

Proposition 2 *In case of an imperfect information setting, the Pareto-optimal contracting depends on the probability that the level of harvest is less than or equal to the lower bound of -- and thus outside -- the sustainability interval.*

Proof See the Appendix.

The rent represents the agent's use of his private information on the sustainability interval. It is increasing under the Jensen's inequality^[10,5]:

$$\frac{V''_{arq}}{V''_{aqq}} \geq \frac{1}{V'_{aq}} \quad (20)$$

Corollary 1 *Given the agent's preferences toward contracting, there is an unavoidable information rent.*

Proof See the Appendix.

On the assumption that $F(q) \neq 0$, the only way to fall on the perfect information case and thus to fall on the lower bound of the sustainability interval is to set $\sigma=0$. This implies that the Jensen's relative inequality becomes a strict equality. To get there, the inverse of the agent's marginal value from harvesting should be equal to his ratio of critical points over the revenue transfer and harvesting. In consequence, V_a should be linear, be it a condition hard to meet in reality.^⑦

3. Conclusions

The management of renewable natural resources often takes the form of a delegation from the resource owner to the resource manager. In presence of asymmetric information on the sustainable harvesting interval, a principal-agent formulation may be put to good use. In this note, we show that if the renewable natural resource manager has a higher propensity to harvest than the resource owner, he may be tempted to manipulate, against the owner, his private information on the true sustainability interval. Would it be the case, the latter ends up accepting levels of harvesting that do not match his preferences. Our model ultimately shows the possibility of occurrence of an ecological deficit when the renewable natural resource owner's fondness for sustainability is concealed for profit motives.

The modelling outcomes give the insight that, in a world where owners mandate agents holding private information to manage their renewable natural resources, improving the access to information is necessary and sufficient to decrease the extend of moral hazard problems. Therefore, further involving the natural resource owners in decision making might be a sound way to guarantee the needed common-knowledge perspectives.

⑦ Assuming the concavity of the agent's value function with respect to q and $V''_{arq} > 0$ would imply that $\sigma < 0$.

Author Contributions

Each author has made substantial contributions to the conception or design of this work.

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

Appendix A -- Proof of Lemma 1

The total differentiation of V_p with respect to q yields:

$$\frac{\partial V_p(s(q), I_p(q), r(q))}{\partial s} \frac{ds}{dq} + \frac{\partial V_p(s(q), I_p(q), r(q))}{\partial q} \frac{dq}{dq} + \frac{\partial V_p(s(q), I_p(q), r(q))}{\partial r} \frac{dr}{dq} = 0$$

Ceteris paribus, the marginal rate of technical substitution (MRTS) between the natural resource stock and harvesting can be analyzed. We have:

$$\frac{ds}{dq} = - \frac{V'_{pq} - V'_{pr} \frac{dr}{dq}}{V'_{ps}}$$

When $\frac{ds}{dq} = 0$, we fall on $V'_{pq} = -V'_{pr} \frac{dr}{dq}$.

A contrario, we have $\frac{ds}{dq} > 0$ for $q_p^* < \underline{q}$ or $\frac{ds}{dq} < 0$ for $q_p^* > \underline{q}$. When $\frac{ds}{dq} > 0$, we have $V'_{ps} > 0$ for $-V'_{pq} > 0$ and $-V'_{pr} \frac{dr}{dq} > 0$. When $\frac{ds}{dq} < 0$, we have $V'_{ps} < 0$ for $V'_{pq} < -V'_{pr} \frac{dr}{dq}$.

Appendix B -- Proof of Lemma 2

We are interested in the variation of V_a with respect to q . Hence:

$$\frac{\partial V_a(s(q),r(q),c(q))}{\partial s} \frac{ds}{dq} + \frac{\partial V_a(s(q),r(q),c(q))}{\partial c} \frac{dc}{dq} + \frac{\partial V_a(s(q),r(q),c(q))}{\partial r} \frac{dr}{dq} = 0$$

Ceteris paribus, the relationship between the natural resource stock and harvesting can be analyzed. From the total differentiation, we obtain:

$$\frac{ds}{dq} = - \frac{V'_{ac} \frac{dc}{dq} + V'_{ar} \frac{dr}{dq}}{V'_{as}}$$

As in the case of the principal, sustainable harvesting corresponds to $\frac{ds}{dq} = 0$. We fall on $-V'_{ac} \frac{dc}{dq} = V'_{ar} \frac{dr}{dq}$.

Conversely, $\frac{ds}{dq} > 0$ is verified for $q_a^* < \underline{q}$ and $\frac{ds}{dq} < 0$ for $q_a^* > \underline{q}$. When $\frac{ds}{dq} > 0$, we have $V'_{as} > 0$ for $-V'_{ac} \frac{dc}{dq} > V'_{ar} \frac{dr}{dq}$. When $\frac{ds}{dq} < 0$, we have $V'_{as} < 0$ for $-V'_{ac} \frac{dc}{dq} < V'_{ar} \frac{dr}{dq}$.

Appendix C -- Proof of Proposition 1

By Lemma 1, we know that $V'_{pq} = -V'_{pr} \frac{dr}{dq}$. Equation (10)

can thus be written as $-\frac{dr}{dq} = \frac{V'_{aq}}{V'_{ar}}$. Given that r and q vary in a complementary way, their MRTS equals zero, which implies that $-\frac{dr}{dq} = 0 = dV_p$.

Appendix D -- Proof of Proposition 2

Straightforward from equation (18). We know that $\sigma=0$ if $\frac{F(q)}{f(q)}=0$ or $\frac{V'_{aqq} - V'_{arq} V'_{aq}}{V'_{aq}} = 0$. If one of these two conditions were verified, Lemma 1 and Lemma 2 complete the proof.

Appendix E -- Proof of Corollary 1

The Jensen's inequality is verified when V_a is convex over both of its arguments. From equation (20), we have

$$\frac{V''_{arq}}{V'_{aqq}} \geq \frac{1}{V'_{aq}} \Leftrightarrow V''_{aqq} \leq V'_{aq} V''_{arq} \Leftrightarrow \frac{V''_{aqq}}{V'_{arq}} \leq V'_{aq}$$

Through complementarity, we know that $V''_{arq} > 0$. When $V'_{aq} > 0$, the opening relative inequality implies that $V''_{aqq} \geq 0$. Ergo, both arguments give the convexity of V_a , which ends the proof. When $V'_{aq} < 0$, the information rent is useless, for the agent does not wish to harvest more than what has been fixed, as a lower sustainability bound, in the management plan. In that stance, the only risk involved is an insufficient level of harvesting.