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ARTICLE

Nonlinear Vibration Analysis of an Electrostatically Actuated Microbeam using Differential Transformation Method

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ABSTRACT

In this paper, nonlinear vibration of electrostatically actuated microbeam is analyzed using differential transformation method. The high level of accuracy of the analytical solutions of the method was established through comparison of the results of the solutions of exact analytical method, variational approach, homotopy analysis method and energy balance methods. Also, with the aid of the present analytical solution, the time response, velocity variation and the phase plots of the system are presented graphically. It is hope that the method will be widely applied to more nonlinear problems of systems in various fields of study.

1. Introduction

The applications of micro-electro-mechanical systems (MEMS) (batch-fabricated devices and structures at a microscale level [1]) in microswitches, transistors, accelerometers, biomechanics, consumer electronics sensors in aerospace, optical and biomedical engineering [2-4] show its tremendous importance in many areas. In these microelectromechanical systems, electrostatic actuation is the most popular actuation mechanism used. Such actuation can be modeled by an electrostatically driven microbeam and a pair of fixed electrodes. Understanding the mechanical behavior of microbeams [5-8] and microplates [9-11] is of great importance due to their various applications [12-14]. However, dynamic response of the beam is greatly influenced by the inherent nonlinearities in the system. These nonlinearities reveal that a collapse of the movable structure occurs at a critical voltage (pull-in instability), and the phenomenon can be used as change of ON or OFF state [15-19]. In order to investigate this chaotic behaviour, nonlinear analysis of dynamic and stability responses of the system have been presented [20-25] using different analytical and numerical methods. However, a combined advantage of simplicity and high accuracy were not able to be achieved through these methods. The high accuracy of the methods trade-off simplicity in approaches and principles. A further investigation revealed that the required combined advantage of simplicity and high accuracy of a solution method can be achieved using differential transformation method. Therefore, in this work, nonlinear vibration of electrostatically actuated microbeam is analyzed using differential transformation method. With the aid of the method, analytical solution is derived to analyze the behaviour of the system.

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2. Model for the Electrically Actuated Microbeam

Consider a fully clamped microbeam with uniform thickness h, length l, width b(b≥5*h*) as shown in Figure 1. By applying the Galerkin Method and employing the classical beam theory and taking into account of the mid-plane stretching effect as well as the distributed electrostatic force, the dimensionless equation of motion for the microbeam is derived as

$$\ddot{u}\left(a_1u^4+a_2u^2+a_3\right)+a_4u+a_5u^3+a_6u^5+a_7u^7=0 \tag{1}$$

the initial conditions are

$$u(0)=A,\quad \dot{u}(0)=0 \tag{2}$$

where u is the dimensionless deflection of the microbeam, a dot denotes the derivative with respect to the dimensionless time variable $t=\tau\sqrt{\frac{\bar{E}I}{\rho bhl^4}}$ with I and t being the second moment of area of the beam cross-section and time, respectively.

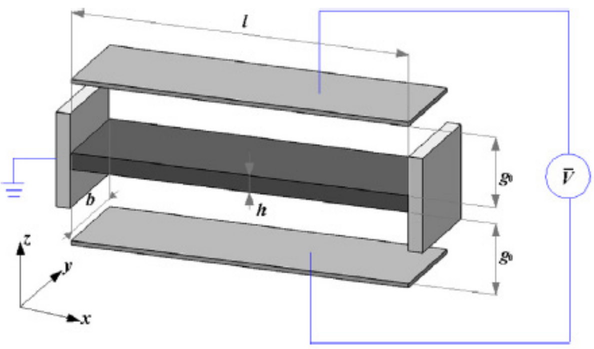

Figure 1. Schematics of a double-sided driven clamped-clamped microbeam-based electromechanical resonator

$$\begin{aligned}
&a_1=\int_0^1\phi^6 d\xi,\qquad a_2=-2\int_0^1\phi^4 d\xi,\qquad a_3=\int_0^1\phi^2 d\xi,\\
&a_4=\int_0^1\left(\phi''''\phi-N\phi''\phi-V^2\phi\right)d\xi\\
&a_5=-\int_0^1\left(2\phi''''\phi^3-2N\phi''\phi^3+\alpha\phi''\phi\int_0^1(\phi')^2\,d\xi\right)d\xi,\\
&a_6=\int_0^1\left(\phi''''\phi^5-N\phi''\phi^5+2\alpha\phi''\phi^3\int_0^1(\phi')^2\,d\xi\right)d\xi\\
&a_7=-\int_0^1\left(\alpha\phi''\phi^5\int_0^1(\phi')^2\,d\xi\right)d\xi
\end{aligned} \tag{3}$$

where effective modulus $\bar{E}=\frac{E}{1-\nu^2}$, Young's modulus E, Poisson's ratio υ and density ρ .A is the initial angular displacement or the amplitude of the oscillations. The prime (') indicates the partial differentiation with respect to the coordinate variable x . The parameter N denotes the tensile or compressive axial load, g_0 is initial gap between the microbeam and the electrode, V the electrostatic load and ε_0 vacuum permittivity. The trial function is $\varphi(\xi)=\frac{16\xi^3}{(1-\xi)^2}$

3. Differential Transformation Method to the Nonlinear Problem

The application of differential transform method to the nonlinear problem is demonstrated in this section.

The DTM recursive relations for the governing equation of motion (Eq. (1)) of the system is

$$\begin{aligned}
&a_1\sum_{l=0}^{k}\sum_{p=0}^{k-l}\sum_{q=0}^{k-l-p}\sum_{r=0}^{k-l-p-q}\left(k-l-p-q-r+2\right)(\mathrm{k}-l-l-p-q-r+1)\\
&U[l]U[p]U[q]U[r]U[k-l-p-q-r+2]\\
&+a_2\sum_{l=0}^{k}\sum_{p=0}^{k}\left(k-l-p+2\right)(k-l-p+1)U[l]\,\mathrm{U}[p]U[k-l-p+2]\\
&+a_3(k+1)(k+2)U[k+2]\\
&+a_4U[k]+a_5\sum_{l=0}^{k}\sum_{p=0}^{k-l}U[l]U[p]U[k-l-p]\\
&+a_6\sum_{l=0}^{k}\sum_{p=0}^{k-l}\sum_{q=0}^{k-l-q}\sum_{r=0}^{k-l-q-r}U[l]U[p]U[q]U[r]U[k-l-p-q-r]\\
&+a_7\sum_{l=0}^{k}\sum_{p=0}^{k-l}\sum_{q=0}^{k-l-q}\sum_{r=0}^{k-l-q-r}\sum_{s=0}^{k-l-p-q-r-s}\sum_{v=0}^{k-l-p-q-r-s-v}\\
&U[l]U[p]U[q]U[r]U[s]U[x]U[k-l-p-q-r-s-v]=0
\end{aligned} \tag{4}$$

Alternatively, we can write the recursive equation for governing equation as

$$\begin{aligned}
&a_1\sum_{k_4=0}^{k}\sum_{k_3=0}^{k_4}\sum_{k_2=0}^{k_3}\sum_{k_1=0}^{k_2}U[k_1]U[k_2-k_1]U[k_3-k_2]\\
&U[k_4-k_3]\left(k-k_4+1\right)\left(k-k_4+2\right)U[k-k_4+2]\\
&+a_2\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2}U[k_1]U[k_2-k_1]\left(k-k_2+1\right)\left(k-k_2+2\right)U[k-k_2+2]\\
&+a_3(k+1)(k+2)U[k+2]+a_4U[k]\\
&+a_5\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2}U[k_1]U[k_2-k_1]U[k-k_2]\\
&+a_6\sum_{k_4=0}^{k}\sum_{k_3=0}^{k_2}\sum_{k_2=0}^{k_3}\sum_{k_1=0}^{k_4}U[k_1]U[k_2-k_1]U[k_3-k_2]U[k_4-k_3]U[k-k_4]\\
&+a_7\sum_{k_6=0}^{k}\sum_{k_5=0}^{k}\sum_{k_4=0}^{k}\sum_{k_3=0}^{k_4}\sum_{k_2=0}^{k_3}\sum_{k_1=0}^{k_2}U[k_1]U[k_2-k_1]U[k_3-k_2]U[k_4-k_3]\\
&U[k_5-k_4]U[k_6-k_5]U[k-k_6]=0
\end{aligned} \tag{5}$$

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subject to

$$U[0] = A,\ U[1] = 0, \tag{6}$$

From the recursive relation, the term by term solutions were obtained. For example,

$$U[2] = -\frac{\left(a_4 A + a_5 A^3 + a_6 A^5 + a_7 A^7\right)}{2\left(a_1 A^4 + a_2 A^2 + a_3\right)},\ U[3] = 0$$

The other term by term analytical expressions for the solutions are too long and huge to be included in this paper. Using the definition of DTM, the desired analytical solution was established.

4. Results and Discussion

The accuracy of the differential transformation method is shown in Table 1. The Table depicted the high level of accuracy and agreements of the symbolic solutions of the DTM when compared to the exact analytical method, homotopy analysis method (HAM), variational approach (VA), and energy balance method (EBM). From the results in the Table, it could be stated that the differential transformation method gives highly accurate results as homotopy analysis method and agrees very well with the exact analytical solution. The DTM is comparably very simple and avoids any numerical complexity. Also, the higher accuracy of the differential transformation method over variational approach and energy balance method is shown. It is shown that the results obtained by EBM and VA are not reliable.

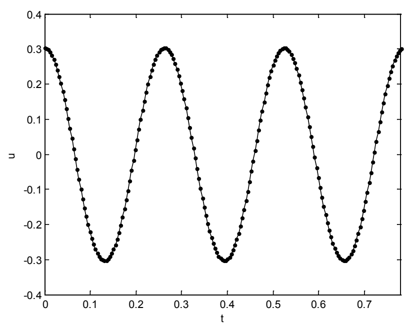

Figure 2. Time response of the system when $A = 0.3$

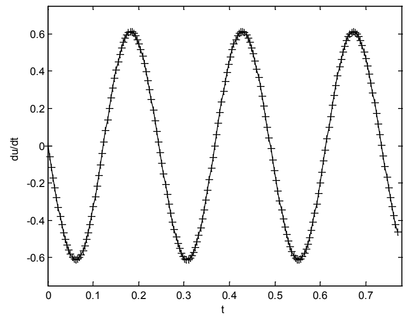

Figure 3. Velocity variation with time when $A = 0.3$

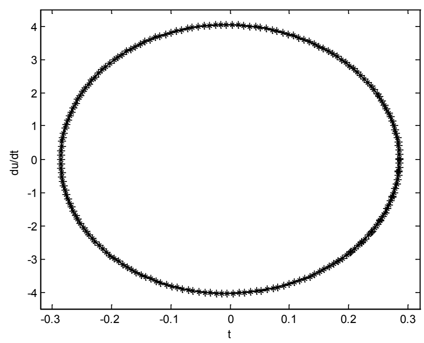

Figure 4. Phase plots of the system

Figure 2 shows the time response of the system while Fig, 3 displays the velocity variation of the system with time. It could be seen that for the relatively large initial displacement value, it can be seen that the time-displacement graphs have a consistent harmonic pattern.

Figure 4 shows the phase plots of the system. The circular curve around (0,0) in figure shows that the system goes into a stable limit cycle. The plot agrees very well with the past works. It is therefore established that, DTM provides a good analytical solution to non-linear equation of motion of the system.

5. Conclusion

In this work, the effectiveness and convenience of differential transformation method to the nonlinear vibration of electrostatically actuated microbeam has been displayed. The analytical solution was verified through comparison with the solutions with the exact analytical method, variational approach, homotopy analysis method and energy

Table 1. Comparison of results of frequency corresponding to various parameters of the system

A	N	a	V	Exact [24]	HAM [24]	VA [25]	EBM [21]	DTM
0.30	10	24	0	26.8372	26.8372	26.3644	26.3867	26.8372
0.30	10	24	10	16.6486	16.6486	16.3556	16.3829	16.6486
0.30	10	24	10	28.5382	28.5368	26.1671	26.5324	28.5382
0.30	10	24	20	18.5902	18.5902	17.0940	17.5017	18.5902

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balance methods. The differential transformation method was shown to be very efficient, simple, suitable and useful as a mathematical tool for solving the nonlinear problems.

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