# ARTICLE <br> Experimentation on Optimal Configuration and Size of Thin Cylinders in Natural Convection 

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#### Abstract

In this paper, an experimental study of laminar, steady state natural convection heat transfer from heated thin cylinders in an infinite air medium has been reported. Two electrically heated cylinders having the same slenderness ratio (L/D) i.e. 6.1 but different diameters i.e. 3.8 cm and 5.08 cm were used. 105 experiments were carried out to study the effect of diameter and inclination angle of thin cylinder on natural convection heat transfer. After mandatory corrections of radiation and endcap heat losses, convective heat transfer results were presented in the form of local and average dimensionless numbers. For vertical configuration of thin cylinder, Nusselt number was varied from 52.99 to 95.10 corresponding to $1.28 \times 10^{8} \leq R a^{*}{ }_{L} \leq 1.08 \times 10^{10}$. While for horizontal configuration, Nusselt number was varied from 10.74 to 17.78 corresponding to $9.42 \times 10^{4} \leq R a^{*}{ }_{D} \leq 8.17 \times 10^{6}$. Results were compared with the published data and found satisfactory as the maximum percentage difference was only $3.09 \%$. The essence of research is that the heat transfer coefficient increases with decrease in diameter and increase in inclination angle. Smoke flow visualization was done to capture patterns of fluid flow. Finally, comparison was made to quantify increase in Nusselt number from slender cylinder as compared to the flat plate.


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## 1. Introduction

Heat transfer from slender (thin) cylindrical surfaces through natural convection has broad spectrum in engineering applications such as fuel rods of nuclear power plant, cylindrical tubes of steam generator and car radiator, electrical resistive heating components, nuclear reactor insulation design and many others. Slender cylinders are different from thick cylinders because in thin cylinders, thermal boundary layer thickness is comparable to the radius of the cylinder ${ }^{[1]}$. This characteristic effect heat transfer greatly. Popiel ${ }^{[2]}$ correlated the data of Cebeci ${ }^{[3]}$ in order to establish the criteria of thick and thin cylinder for isothermal vertical cylinders in laminar region as shown in Equation (1). It is valid for the range of Prandtl number from 0.01 to 100 .
$G r_{H}{ }^{0.25} \frac{D}{H} \leq 11.474+\frac{48.92}{P r^{0.5}}-\frac{0.006085}{P r^{2}}$
On the basis of orientation, thin cylinders can be divided into horizontal and vertical configuration. Available literature in this regard is mentioned below in a brief manner.

### 1.1 Thin Cylinders in Vertical Position

Vertical flat plate correlations are applicable to the vertical thick cylinders of length ' $L$ ' and diameter ' $D$ ', if they are fulfilled the condition given in Equation (2) ${ }^{[4]}$.
$\frac{D}{L} \geq \frac{35}{G r_{L}{ }^{1 / 4}}$
Churchill and Chu ${ }^{[5]}$, Patrick H. Oosthuizen and Jane T. Paul ${ }^{[6,7]}$, correlations are available for different boundary conditions and configurations of vertical flat plate. For thin cylinders, correlations depends upon slenderness ratio (L/D) to incorporate curvature effects. In this regard, widely used theoretical, numerical and experimental correlations are stated below.

Sparrow and Gregg ${ }^{[8]}$ presented first analytical solution for the laminar natural convection flow over the surface of vertical cylinder. They assumed cylindrical surface at constant temperature and solved conservation equations by applying similarity method and power series expansion. C.O Popiel et al. ${ }^{[2]}$ gave a correlation by performing transient analysis (Lumped capacitance method because Biot number was less than 0.01 ) to calculate average convective heat transfer coefficient. Experiments were performed on isothermal vertical slender cylinder for the Rayleigh number $1.5 \times 10^{8}$ to $1.1 \times 10^{9}$ and dimensionless height 0 $\leq \mathrm{H} / \mathrm{D} \leq 60$ in an infinite air medium.

Yang ${ }^{[9]}$ performed experiments by keeping $q_{w}$ and $T_{w}$
constant respectively and derive empirical correlations. These correlations are valid for all fluids $(0 \leq \operatorname{Pr} \leq \infty)$ and for $R_{a H}<10^{9}$. Al-Arabi and Khamis ${ }^{[10]}$ published empirical correlation for isothermal cylinder surrounded by air to approximate average Nusselt number in laminar and turbulent region. Ali Riaz et al. ${ }^{[11]}$ experimentally determined local variation of heat transfer coefficient across the length of cylinder placed in infinite air medium.

### 1.2 Horizontal Heated Thin Cylinder

Limited coorelations are available in horizontal configuration as compated to vertical. Nusselt ${ }^{[12]}$ solved equation analytically to determine the heat transfer coefficients for the horizontal cylinders in air or liquids for $10^{4} \leq G r_{D}$ $\operatorname{Pr} \leq 10^{8}$. Morgan ${ }^{[13]}$ correlated the results of Koch for the range: $4 \times 10^{3} \leq G r_{D} \operatorname{Pr} \leq 6 \times 10^{6}$. Dyer ${ }^{[14]}$ proposed correlation for iso-flux horizontal cylinder for the range: $10^{3} \leq$ $G r_{D}{ }^{*} \operatorname{Pr} \leq 10^{10}$.

### 1.3 Inclined Heated Thin Cylinder

Al-Arabi and Khamis ${ }^{[10]}$ studied natural convection heat transfer from inclined thermal cylinders in detail. They plotted results in local and average dimensionless form inside the laminar and turbulent region. P.H. Oosthuizen and V. Mansingh ${ }^{[15]}$ investigated natural and forced convection heat transfer from short inclined cylinders having length to diameter ratio between 1.5 and 16.

By examining research studies, it is observed that prime focus of research was on vertical and horizontal configuration of heated thin cylinders in infinite air medium. Limited studies regarding iso-flux boundary condition on thin cylindrical element have been reported as compared to isothermal condition. Experimental results describing the effect of inclination on natural convection heat transfer is also limited. In this regard, only Al-Alarabi M. and Y. Salman ${ }^{[16]}$ and P.H. Oosthuizen and V. Mansingh ${ }^{[15]}$ experimental data are available, to the best knowledge of the authors. This was motivation behind this study.

In this research, experimental approach was used to generate benchmark data regarding thermal performance of thin cylinders in various configurations such as $0^{\circ}, 30^{\circ}$, $45^{\circ}, 60^{\circ}$ and $90^{\circ}$ inside the infinite air medium. Furthermore, experiments were conducted to analyze the effect of cylinder diameter on natural convection heat transfer.

During literature review, considerable difference in data regarding average Nusselt number over the surface of vertical cylinder by various researchers was observed ${ }^{[2,3,9,10]}$. The empirical correlations of heat transfer coefficient show a widespread and some percentage of error. Therefore, a large number of experiments were conduct-
ed to suggest which correlation is more appropriate in predicting results in stated range. At the end, results are reported in dimensionless form. This data can be used to perform numerical simulations and to formulate an empirical correlation of heat transfer.

## 2. Energy Balance

Heat supply to thin cylinders must be equal to radiation and natural convection heat transfer from the surface to ambient air plus heat losses through end caps as shown in Figure 1 and written in the form of Equations (3) to (8).


Figure 1. Energy balance on schematic view of heat source
$Q_{\text {total }}=Q_{\text {in }}=Q_{\text {convection }}+Q_{\text {radiation }}+Q_{\text {conduction }}$
$Q_{\text {in }}=I \times V=Q_{\text {total }}=h_{\text {combine }} A_{s}\left(T_{s}-T_{b}\right)+Q_{\text {loss }}$
where, $h_{\text {combine }}=h_{\text {convective }}+h_{\text {radiative }}$
$Q_{\text {loss }}=Q_{\text {conduction-end caps }}+Q_{\text {radiation-heat source }}$
As it is steady state experimental study, so mathematically it can be written as:

$$
\begin{equation*}
Q_{\text {in }}=Q_{o u t} \tag{7}
\end{equation*}
$$

Fourier law of heat conduction has been used to calculate conduction heat losses. It is given as ${ }^{[17]}$ :

$$
\begin{equation*}
Q=-k A \frac{d T}{d x} \tag{8}
\end{equation*}
$$

Heat losses in the form of radiation were calculated through Equation (9) ${ }^{[11]}$.

$$
\begin{equation*}
Q_{\text {radiated }}=\varepsilon \sigma A_{s}\left(T_{s, \text { avg }}^{4}-T_{b}^{4}\right) \tag{9}
\end{equation*}
$$

Convective or radiative heat transfer coefficient can be calculated through Newton law of cooling written in Equation (10) ${ }^{[18]}$.
$Q_{\text {convection/radiation }}=h_{\text {conv.rad }} A_{s}\left(T_{s, a v g}-T_{b}\right)$
Physical properties of air have been calculated through following relation ${ }^{[19]}$ :
$P P=A+B T_{f}+C T_{f}{ }^{2}+D T_{f}{ }^{3}$
where, $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D are the thermodynamics constants and are stated in Table 1. $T_{f}$ is the film temperature, while 'PP' stands for physical properties such as $C_{p}, \mu, \mathrm{k}$ and $\rho$.

Table 1. Thermodynamic constants for physical properties of air

| Constants <br> Thermodynamic | A | B | C |
| :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Specific Heat }\left(C_{p}\right)- \\ J / K g K \end{gathered}$ | 1024.06 | -0.175768 | $\begin{gathered} 3.70976 \times \\ 10^{-4} \end{gathered}$ |
| Viscosity ( $\mu$ ) (Kg/m.sec) | $\begin{gathered} 2.12075 \times \\ 10^{-6} \end{gathered}$ | $\begin{gathered} 6.24755 \times \\ 10^{-8} \end{gathered}$ | $\begin{gathered} -2.6162 \times \\ 10^{-11} \end{gathered}$ |
| Thermal Conductivity <br> (k) <br> (W/m. K) | $2.317 \times 10^{-3}$ | $\begin{gathered} 8.7113 \times \\ 10^{-5} \end{gathered}$ | $\begin{gathered} -2.55188 \times \\ 10^{-8} \end{gathered}$ |

Density was calculated through ideal gas law and thermal expansion coefficient was approximated through reciprocal of absolute film temperature. All physical properties were calculated at average film temperature: $T_{f, \text { avg }}=\left(T_{s, \text { avg }}+T_{b}\right) / 2$. Average heat transfer coefficient for vertical configuration has been calculated as,
$\overline{h_{L}}=\frac{\sum_{i=1}^{3} h_{x i} \Delta x_{i}}{L}$
where $x_{i}$ is the distance from the bottom of heat source. For horizontal configuration, it can be calculated as,
$\overline{h_{L}}=\frac{\sum_{i=1}^{3} h_{x i} \Delta c_{i}}{2 \pi r}$
where ' $c_{i}$ ' is the circumference distance while ' $r$ ' is the radius of cylinder.

Average Nusselt number has been calculated as,
$N u=\frac{\overline{h_{L}} \times L}{k}$
As it is constant heat flux case, so average modified Grashof number and modified Rayleigh number has been calculated as follows:
$G r_{L}{ }^{*}=\frac{g \beta L^{4} q^{\prime \prime}}{k v^{2}}$
$R a_{L}{ }^{*}=G r_{L}{ }^{*} \cdot P r$
'L' should be replaced with 'D' in above relations for horizontal configuration. Precision of experimental data was stated after calculating standard deviation as shown in Equation (17), where, $\bar{x}$ is the mean value and ' $n$ ' is the total number of terms.
$S D=\sqrt{\sum|x-\bar{x}|^{2} / n}$

## 3. Experimental Setup

This section comprises on designing and manufacturing of thin cylinders along with wooden enclosure to provide undisturbed environment for natural convection study.

### 3.1 Designing

Designing of thin cylinders comprises on determining its diameter and length or L/D ratio. Depending upon scope of research study such as laminar; the length and diameter were selected. As one of the objectives of present study was to study the effect of thin cylinders diameter on natural convection in laminar region, so two different commercially available diameters were selected such as 3.8 cm and 5.08 cm . By considering range of present study i.e. $2.77 \times 10^{8} \leq R a^{*}{ }_{L} \leq 1.08 \times 10^{10}$ (laminar region), 31 cm length of cylinder was calculated. Designed range of modified Rayleigh number was achieved for designed length and diameter of cylinder through single phase power supply by varying heat flux from $9.39 \mathrm{w} . \mathrm{m}^{-2}$ to 638.99 w.m ${ }^{-2}$.

### 3.2 Manufacturing and Fabrication

Heating rod having maximum 1000 watts power rating was placed at center of cylindrical casing having 5.08 cm diameter and 31 cm length. To control heat losses from ends, Perspex disk having 5 cm diameter and 1.27 cm thickness was used because of its low thermal conduc-
tivity. Center hole of 5 mm diameter was drilled at the Perspex disk to provide an electrical connection. Silica gel was used to completely seal the cylinders from fresh air. Manufactured thin cylinder with electric connections and attached thermocouples is shown in Figure 2. On a similar basis, another thin cylinder having 3.8 cm diameter and 23 cm length was manufactured.


Figure 2. Aluminum thin cylinder having five thermocouples at the inner surface
After manufacturing thin cylinders of aforementioned dimensions, recorded maximum temperature difference was 74.24 K corresponding to 638.99 w. $\mathrm{m}^{-2}$ convective heat flux value. Thus, through proper designing, temperature of thin cylinders can be controlled and research can be performed for designed range of modified Rayleigh number.

As rising plume in natural convection is sensitive to the outer disturbances so wooden enclosure was manufactured having front transparent wall made up of Perspex for natural convection study and smoke flow visualization. Complete experimental setup is shown in Figure 3.


Figure 3. Experimental setup having enclosure for infinite medium

### 3.3 Method and Measurements

Generally, in experimentation sensors disturb the field and thus affect the measurement. Therefore, special attention should be taken to attach thermocouples to the surface so that they attain the surface temperature rather than perturbed temperature. Thermocouple installation is one of the main tasks as briefly described below:
H. Shaukatullah and A. Claassen ${ }^{[20]}$ claimed that the best method of thermocouple attachment is through thermal conductive epoxy. Additionally, they reported that k-type thermocouples are better than T-type because of their lower equivalent thermal conductivity. Therefore, miniature k-type thermocouples were attached with thermal conductive epoxy to the inner surface of thin cylinder so that it cannot disturb the external thermal and hydrodynamic boundary layer. Three thermocouples were attached at the inner surface of the cylinder. Their positions at the surface of cylinder are shown in Figure 4. All dimensions are in centimeters. Figure 4 also shows the cross-sectional view of thin cylinder i.e. Nichrome wire, air gap, Aluminum sheath and Perspex end caps. Ambient temperature was measured by placing thermocouple at axially 15 cm away from the bottom of the thin cylinder inside the enclosure.


Figure 4. Schematic view of thin cylinder. All dimensions are in cm .
Variable power supply was used in experiments to supply variable voltages in order to perform experiments at various convective heat flux values. Upon reaching steady state, surface temperature values were recorded for fifty seconds with the help of TC-08 datalogger, and then average surface temperature was calculated. Steady state is defined as the state at which temperature change is smaller
than $\pm 0.1^{\circ} \mathrm{C}$ in a 2 min period ${ }^{[21]}$. All experiments were performed in an isolated and quiescent room by turning off the blowers and fans of the laboratory and it took almost two hours to reach steady state.

105 experiments were performed to completely expedite the problem. On average, calculated heat loss was $6.5 \%$ of the total heat flux. Seven experiments were performed for each thin cylinder configuration and diameter to calculate heat transfer coefficient for the designed range of modified Rayleigh number. Each experiment was repeated thrice to ensure repeatability and to measure standard deviation. This study is divided into four cases:

### 3.3.1 Case: A

Thin cylinder having 23 cm length and 3.8 cm diameter was hanged in a vertical position. Total seven experiments were performed for the range of convective heat flux $14.17 \mathrm{w}^{\mathrm{w}} \mathrm{m}^{-2}$ to the $515.22 \mathrm{w} . \mathrm{m}^{-2}$. Maximum temperature difference observed corresponding to the range of aforementioned convective heat flux was 49.27 K .

### 3.3.2 Case: B

Thin cylinder having 31 cm length and 5.08 cm diameter was vertically hanged. Seven experiments were performed for the range of convective heat flux $9.39 \mathrm{w} . \mathrm{m}^{-2}$ to $638.99 \mathrm{w}^{\mathrm{w}} \mathrm{m}^{-2}$. Maximum average temperature difference recorded during experimentation was 74.24 K .

### 3.3.3 Case: C

Thin cylinder having 3.8 cm diameter was fixed in a horizontal position with the help of fixture. Total seven experiments were performed for the range of convective heat flux 14.33 w. $\mathrm{m}^{-2}$ to $517.78 \mathrm{w} . \mathrm{m}^{-2} ; 45.64 \mathrm{~K}$ was the maximum measured temperature difference. Three thermocouples were attached along the length of the cylinder. Thin cylinder was rotated along its axis to measure the circumferential variation in temperature. The maximum variation of temperature measured from thermocouples in the above mentioned range of heat flux was less than $1^{\circ} \mathrm{C}$ along the length and circumference of the cylinder. Range of convective average heat transfer coefficient for the horizontal thin cylinder configuration is $7.54 \mathrm{w} \cdot \mathrm{m}^{-2} \cdot \mathrm{k}^{-1}$ to $11.42 \mathrm{w} . \mathrm{m} .{ }^{-2} . \mathrm{k}^{-1}$ for the range of convective heat flux from $13.33 \mathrm{w} \cdot \mathrm{m}^{-2}$ to $517.78 \mathrm{w} \cdot \mathrm{m}^{-2}$.

### 3.3.4 Case: D

Thin cylinder having 31 cm length and 5.08 cm diameter was placed in a horizontal position with the help of a fixture. Maximum average temperature difference recorded during experimentation was 76.08 K. Six thermocou-
ples were attached to the inner surface of the thin cylinder. It was observed that when thin cylinder in horizontal orientation, the temperature varied only in circumferential direction not in axial. Calculations showed that maximum thermal losses were $8.06 \%$ of the total heat flux. Range of convective heat transfer coefficient was from $3.81 \mathrm{w} . \mathrm{m}^{-2} . K^{-1}$ to $10.89 \mathrm{w} \cdot \mathrm{m}^{-2} . K^{-1}$ corresponding to the range of heat flux from $9.31 \mathrm{w}^{2} \mathrm{~m}^{-2}$ to the $643.11 \mathrm{w} . \mathrm{m}^{-2}$.

## 4. Results and Discussions

Comparison of results with the already published results is a necessary step in order to validate experimental data. Validation confirms the authenticity of the research and helps to obtain reliable results. For validation, initial experiment was performed and results were compared with the already published results of Alarabi and Khamis ${ }^{[10]}$. Alarabi and Khamis performed experiment on thin cylindershaving 50.8 mm diameter and 30 cm length and the temperature difference between ambient and surface temperature was $70 \pm 4{ }^{\circ} \mathrm{C}$. Thin cylinder having same dimensions was manufactured and similar experimental settings were achieved in the laboratory for comparison of experimental results. Comparison of results in the form of average heat transfer coefficient and Nusselt number are shown in Table 2.

Table 2. Comparison of experimental data with Alarabi and Khamis ${ }^{[10]}$

| Source | $\boldsymbol{N} \boldsymbol{u}_{\boldsymbol{L}}$ |
| :---: | :---: |
| Al Arbi and Khamis ${ }^{[10]}$ | 102.41 |
| Present Study | 99.24 |
| Precentage Difference | $3.09 \%$ |

Temperature variation in Alarabi experiment was $\pm 4^{\circ} \mathrm{C}$ and they did not clearly mention the ambient and surface temperature. One can obtain the same temperature difference i.e. $70{ }^{\circ} \mathrm{C}$ through number of ambient and surface temperature values. This ambiguity yields into different film temperature which resulted into different physical properties of fluid. This may be a reason of difference between the present and Al Arabi and Khamis experimental results. At last, least count and sensitivity of the sensors used in experimental study may also contribute to the error. In the light of above discussion, one can confidently say that present experimental results are in good agreement with the published results. Surface and ambient temperature of the first experiment of case: A is shown in Figure 5. After achieving steady state, temperature was recorded at interval of one second for fifty seconds with the help of datalogger.


Figure 5. Steady state temperature at $14.17 \mathrm{W.m}^{-2}$ (Case: A)
Temperature difference between ambient and average surface value at various heat fluxes for thin cylinders of case $A$ and $B$ is shown in Figure 6. At the same heat flux value, the temperature difference was higher for case $B$ as compared to case A. This is because of functional dependency of heat transfer coefficient on physical parameter of thin cylinder such as diameter. As the diameter decreases the heat transfer coefficient increases and temperature difference reduces. Furthermore, one can easily observe that temperature increment with convective heat flux is almost linear in both cases, the difference lies in slope of the line. Average Nusselt number as a function of convective heat flux is shown in Figure 7.


Figure 6. Temperature difference versus total heat flux
Variation of average Nusselt number as a function of modified Rayleigh number - calculated through convective heat flux - for vertical and horizontal configuration of thin cylinder is presented in semi log graph as shown in Figure 8 and Figure 9 respectively. Logarithmic increment in average Nusselt number with modified Rayleigh num-
ber can be observed from both figures. Range studied for vertical configuration is $1.28 \times 18^{8} \leq R a_{H}^{*} \leq 1.08 \times 10^{10}$ while for horizontal configuration the range is $1.99 \times 18^{5} \leq$ $R a^{*}{ }_{D} \leq 8.15 \times 10^{6}$.


Figure 7. Variation of average Nusselt number with convective heat flux


Figure 8. Average Nusselt number as a function of modified Rayleigh number in vertical configuration


Figure 9. Average Nusselt number as a function of modified Rayleigh number in horizontal configuration

Design and experimental parameters ranges are reported in Table 3.

Table 3. Critical design and experimental parameters

| S. No. | Parameters | Values |
| :---: | :---: | :---: |
| 1 | Diameter of thin cylinders | $3.8 \mathrm{~cm}, 5.08 \mathrm{~cm}$ |
| 2 | Length of thin cylinders | 31 cm |
| 3 | Heat flux range | 9.39 to $638.99 \mathrm{~W} . \mathrm{m}^{-2}$ |
| 4 | Max. temperature difference | 74.24 K |
| 5 | Modified Rayleigh number range <br> for vertical configuration | $2.77 \times 10^{8}$ to $1.08 \times 10^{10}$ |
| 6 | Nusselt number range for vertical <br> configuration | 52 to 95 |
| 7 | Modified Rayleigh number range <br> for horizontal configuration | $1.8 \times 10^{5}$ to $1.9 \times 10^{7}$ |
| 8 | Nusselt number range for <br> horizontal configuration | 9.6 to 18 |

### 4.1 Effect of Thin Cylinder Diameter

Nusselt number on the surface of thin cylinder is functionally dependent on length to diameter ratio, and the Rayleigh number (for of isothermal case) or modified Rayleigh number (for iso-flux case). Diameter of thin cylinderis the prime parameter that significantly affects the natural convection heat transfer phenomena. Heat transfer coefficient varies significantly from heated flat plate (Diameter $\rightarrow \infty$ ) to the slender cylinder having few millimetres diameter. One of the main objectives of this research was to experimentally examine the effect of thin cylinder diameter, therefore, 38 mm and 50.8 mm diameter were used.

As the length of the two thin cylinders was different so heat transfer coefficient was compared against the same Rayleigh number. It was observed that heat transfer coefficient varied decisively by changing the diameter as shown in Figure 10. From graph, it is obvious that as diameter decreases the average heat transfer convective coefficient increases. It can also be observed that the maximum increment in heat transfer coefficient decreases as Rayleigh number increases. This observation shows that diameter has more pronounced effect in laminar region as compared to the turbulent region.

At modified Rayleigh number $3.05 \mathrm{E}+08$, the increase in heat transfer coefficient from 5.08 cm diameter to 3.8 cm diameter was $41.86 \%$ while this increment reduced to $31.43 \%$ at $2.44 \mathrm{E}+09$ modified Rayleigh number. As the diameter decreases the curvature effect becomes more
significant and affecting the development of thermal boundary layer which results into an increment in heat transfer coefficient. The same trend was also observed by Fumiyoshi KIMURA et al. ${ }^{[22]}$ and Al Arabi and Khamis ${ }^{[10]}$. This is primarily due to thinning of thermal boundary layer around the cylinder.


Figure 10. Heat transfer coefficient as a function of Rayleigh and diameter having same H/D ratio
Combined (convective and radiative) heat transfer coefficient was calculated by not subtracting the radiation heat losses from the total heat flux. The variation of combined heat transfer coefficient at various total heat flux values for two different diameter thin cylinders having same H/D ratio is shown collectively in Figure 11. Logarithmic trend was observed in plotted values of Figure 11.


Figure 11. Combined heat transfer coefficient as a function of diameter and total heat flux

### 4.2 Effect of Thin Cylinders Inclination

Variation of average heat transfer coefficient with inclination angle - measured from vertical axis - is shown
in Figure 12 and Figure 13 respectively. Both figures depicted that at same heat flux level, heat transfer coefficient increases with increase in inclination angle. It can also be observed that heat transfer increases with increment in Grashof number. Rise in heat transfer coefficient was more profound from $0^{\circ}$ to $45^{\circ}$ as compared to $45^{\circ}$ to $90^{\circ}$ inclination of cylinders. Same trend was observed by Al-Arabi and Y. Salman ${ }^{[16]}$ and P.H. Oosthuizen and Mansingh, Vivek ${ }^{[15]}$. Development of thermal boundary layer in various configuration is the reason behind this experimentally observed trend as shown in smoke flow visualization section. In horizontal configuration, thickness of thermal boundary increases as one traversed from bottom to upward. This results in decrement of heat transfer coefficient.


Figure 12. Variation of average heat transfer coefficient versus inclination angle of 3.8 cm diameter of thin cylinder


Figure 13. Variation of average heat transfer coefficient versus inclination angle of 5.08 cm diameter of thin cylinder

### 4.3 Uncertainty Analysis

Maximum uncertainty in temperature measurement is $\pm 0.5^{\circ} \mathrm{C}$ calculated by comparing with mercury digital thermometer. This is mainly due to the least count of temperature measuring system i.e. thermocouple and dat-
alogger. Although to minimize temperature measurement error to much extent a calibration process was done but experimentally one cannot achieve $100 \%$ accuracy due to limitations in sensors. Additionally, maximum uncertainty in reporting dimensions i.e. thin cylinder length, diameter and thermocouple locations is $\pm 2 \mathrm{~mm}$. Uncertainty in results was calculated by performing each experiment thrice. Experimental results are reported with $96.8 \%$ precision and average standard deviation from reported value is $1.61 \%$.

### 4.4 Smoke Flow Visualization

Pakistani burning incense stick was used for the visualization of smoke flow. This exercise was done just for
the sake of visualization of streamlines inside the laminar thermal boundary layer at different configurations. Temperature was recorded at steady state and then smoke flow pattern was captured.

Axial flow pattern was observed at the surface of vertical configuration of thin cylinder while cross flow pattern was observed in horizontal configuration. It can be seen from Figure 14 that smoke flow was thinner at the bottom and thicker at the top. Cross and axial flow pattern were observed in inclined $\left(45^{\circ}\right)$ configuration as shown in Figure 15. From Figure 16, it can be easily observed that how hot plume raised in a laminar fashion till the end of the cylinder, which is also supported by the experimental data as the Rayleigh number is $8.03 \mathrm{E}+06$ (laminar region).


Figure 14. Smoke flow visualization at $R a^{*}=1.88 \mathrm{E}+06$


Figure 15. Smoke flow visualization at $R a_{L}=2.89 \mathrm{E}+09$


Figure 16. Smoke flow visualization at $R a_{L}=5.07 \mathrm{E}+08$

## 5. Validation of Results

Experimental results are validated after comparing with published data. Comparison of present data with theoretical data of H.R. Lee et al. ${ }^{[22]}$ and experimental results of S. Jarall and A. Campo ${ }^{[23]}$ is shown in Figure 17. Ratio of Nusselt number of cylinder to flat plate was plotted against a dimensionless parameter ' $\lambda$ '. Correlation equation by Raithby and Hollands ${ }^{[24]}$ was used to calculate local Nusselt number at vertical flat plate. This comparison also validated the present study as results are in between the published experimental and theoretical results. However present results are slightly higher than the theoretical


Figure 17. Comparison of present experimental results with H.R. Lee et al. published data

## 6. Conclusions

In this detailed experimental investigation, an effect of thin cylinder inclination and diameter on natural convective heat transfer coefficient over the surface of electrically heated slender cylinders has been studied. Average value of Nusselt number was calculated for different configurations of thin cylinder. Study shows that heat transfer coefficient depends upon both the diameter and inclination of thin cylinders. Information in dimensionless form in this study may be useful for thermal design engineers in decision making processes. Main findings of the present study are given below:

1) Heat transfer coefficient increases significantly with decrease in diameter due to decrease in thermal boundaey layer.
2) Almost $30 \%$ increment in average Nusselt number
results; this may be because of disturbance caused by sensors during measurement.

Average surface temperature was assumed over the whole cylindrical surface in order to make comparison with Al-Arabi and Khamis correlation ${ }^{[10]}$ as it was proposed for isothermal case. Present results are highly in agreement with the published data as shown in Figure 18. It was also observed that Al-Arabi and Khamis correlation can be used with confidence for: $1.28 \times 10^{8} \leq R a^{*}{ }_{L} \leq 1.08 \times$ $10^{10}$. This exercise assured that Al-Arabi and khamis results are more credible than other mentioned correlations under the sub-heading of 'vertical thin cylinder'.


Figure 18. Comparison of present experimental results with Al Arabi and Khamis data
was observed on vertical cylinder having 6.1 slenderness ratio as compared to the vertical flat plate.
3) Al-Arabi and Khamis correlation can be used with confidence for iso-flux heated cylinders having minimal end losses for $1.28 \times 10^{8} \leq R a^{*} \leq 1.08 \times 10^{10}$.
4) Heat transfer coefficient increases with increase in inclination angle - measured from vertical axis.
5) Experimental data is in good agreement with the already published data.
6) Axial smoke flow was observed in vertical configuration; cross smoke flow was observed in horizontal configuration while both type of flows were observed in inclined configuration of thin cylinder.
7) A slight external disturbance at the vicinity of thin cylinder may significantly increase the natural convection heat transfer. Therefore, it is advised to perform research study inside the enclosure.

## Nomenclature

| $A_{s(m)}{ }^{2}$ | Surface area | $T_{b}(\mathrm{~K})$ | Ambient or Bulk stream temperature |
| :---: | :---: | :---: | :---: |
| $C_{p(J . K g . K}{ }^{-1}$ ) | Specific heat at constant pressure | $T_{f(K)}$ | Film Temperature |
| D (m) | Outer surface diameter | $T_{s(K)}$ | Surface temperature |
| $G r^{*}$ | Modified Grashof number | $V(v)$ | Voltage |
| $g\left(m \cdot s^{-2}\right)$ | Gravitational acceleration | $x$ (m) | Distance from bottom of cylinder |
| $h\left(W \cdot m^{-2} \cdot K^{-1}\right)$ | Convective heat transfer coefficient | $\Theta\left({ }^{\circ}\right)$ | Angel of inclination of cylinder (from the vertical position) |
| I (amp) | Current | $B\left(K^{-1}\right)$ | Coefficient of volumetric expansion |
| $k\left(W \cdot m^{-1} \cdot K^{-1}\right)$ | Thermal conductivity | $P\left(\right.$ Kg.m ${ }^{-3}$ ) | Density |
| $L_{c(m)}$ | Characteristic length | $A\left(m^{2} \cdot s^{-1}\right)$ | Thermal diffusivity |
| $\mathrm{Nu}_{\text {L }}$ | Average Nusselt number | $\mu\left(\mathrm{kg} \cdot \mathrm{m}^{-1} \cdot \mathrm{~s}^{-1}\right)$ | Dynamic viscosity |
| Pr | Prandtl number | $E$ | Emissivity |
| $Q(W)$ | Thin cylinder power | $v\left(m^{2} \cdot s^{-1}\right)$ | Kinematic viscosity |
| Ra* | Modified Rayleigh number | $O\left(W \cdot m^{-2} \cdot K^{-4}\right)$ | Stefan Boltzmann constant |
| $T_{b}(\mathrm{~K})$ | Ambient or Bulk stream temperature | $\Lambda$ | Dimensionless independent variable $\left[(y / R)^{0.8} / G r_{R}\right]^{1 / 8}$ |

## Conflict of Interest

There is no conflict of interest.

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